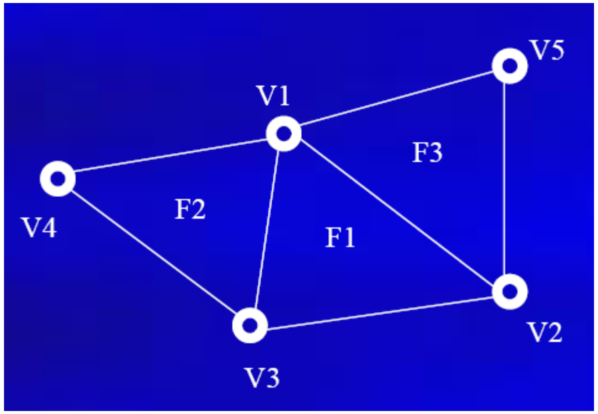
Lecture 2
Triangle Mesh Representation and its Data Structure

Overview

- To study the storage and data structure of the widely used triangle mesh in representing 3D surfaces
  - triangle meshes can adaptively approximate the continuous surfaces using a finite number of vertices and triangles
  - a piecewise linear representation

Storage of Triangle Meshes: Polygon Soup



Triangle Set		
(x_{11},y_{11},z_{11})	(x_{12},y_{12},z_{12})	(x_{13},y_{13},z_{13})
(x_{21},y_{21},z_{21})	(x_{22},y_{22},z_{22})	(x_{23},y_{23},z_{23})
(x_{31},y_{31},z_{31})	(x_{32},y_{32},z_{32})	(x_{33},y_{33},z_{33})

Polygon (Triangle) Soup: A collection of unorganized triangles

- Example: the Stereolithography (STL) Format (widely used in computer-aided design/manufacturing software) is a type of polygon soup

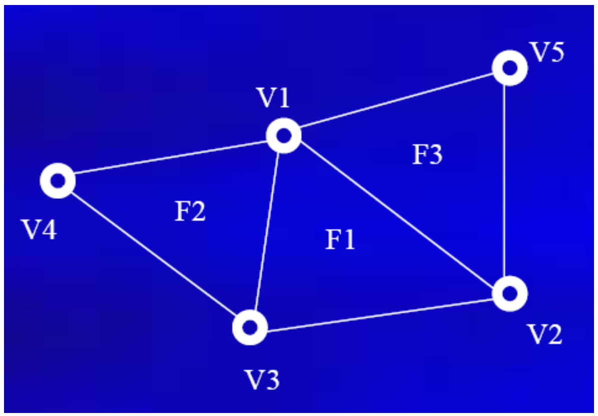
Storage Cost:

- If using x (e.g., 32-bits or 4 bytes) bits to represent a vertex coordinate (float)
- Then each triangle needs 3*3*4 = 36 bytes
- A mesh with n triangles needs 36n bytes

Pros and Cons:

- ✓ Efficient rendering
- No connectivity info stored
- Inefficient for many geometric computing: e.g. traversing local adjacency information
- Vertex positions replicated as many times as the degree of the vertices

Storage of Triangle Meshes: Indexed Vertex Tables



Vertex table	
V_1	(x_1,y_1,z_1)
V_2	(x_2,y_2,z_2)
V_3	(x_3,y_3,z_3)
V_4	(x_4,y_4,z_4)
V_5	(x_5,y_5,z_5)

Face table	
F_1	V_1,V_3,V_2
F_2	V_1,V_4,V_3
F_3	V_5,V_1,V_2

Using a an indexed vertex table, then a face table

- Examples: OFF, OBJ, VRML, M formats

Storage Cost:

- If using x (e.g., 32-bits or 4 bytes) bits to represent a vertex coordinate (float), and x bits to represent a vertex index (int)
- Then each vertex needs 3*4=12 bytes, and each triangle needs 3*4 = 12 bytes
- A mesh with n triangles needs 12n+n/2*12 = 18n bytes

Pros and Cons:

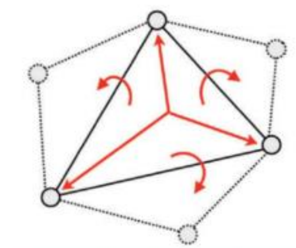
✓ Efficient storage and rendering

❑ Inefficient for local traversal

Mesh Representation in Memory for Efficient Computation in CG Tasks

What operators do we usually need?

- Access to individual elements (vertices edges, and faces): enumeration of all elements
- Local traversal, e.g.:
  - What are the edges in a given face;
  - What are the vertices in a given face or edge;
  - What are the one-ring primitives of a geometric primitive
    - E.g. incident faces/edges/vertices of a given vertex
    - E.g. incident faces of a given edge
- Example: Modifying the last page's data structure for local traversal
  - For each face: store references to its 3 vertices + neighboring triangle
  - For each vertex: store 3 coordinates + a reference to its neighboring triangle
  - Used in CGAL for representing 2D Triangulation, 32 bytes / triangle
    - CGAL = an open-source computational geometry algorithm libra Google "CGAL"
  - Limitations:
    - But enumerating the one-ring vertices of a vertex is not easy
    - Not easily extendable to general/mixed polygonal meshes



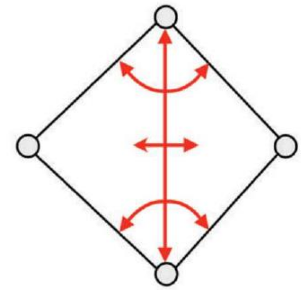
Vertex	
Point	position
FaceRef	face

Face	
VertexRef	vertex[3]
FaceRef	neighbor[3]

Edge-based Data Structure

- A more generally used data structure, since the connectivity is a graph, directly relates to the mesh edges
- Many well known methods: winged-edge [Baumgart 72], quad-edge [Guibas and Stolfi 85], and variants [O'Rourke 94]

- *An example: Winged-edge structure
  - Each edge: stores references to its endpoint vertices + two incident faces + next and previous edge within the left and right faces
  - Each vertex: stores a reference to one of its incident edges
  - Each face: stores a reference to one of its incident edges
  - **Storage Cost**: A mesh with n faces needs 60n bytes
- Limitations: Still not easy for some local traversal
  - e.g. to traverse the one-ring of a vertex, how do you know if it is the first or second vertex of an edge?



Vertex	
Point	position
EdgeRef	edge

Face	
EdgeRef	edge

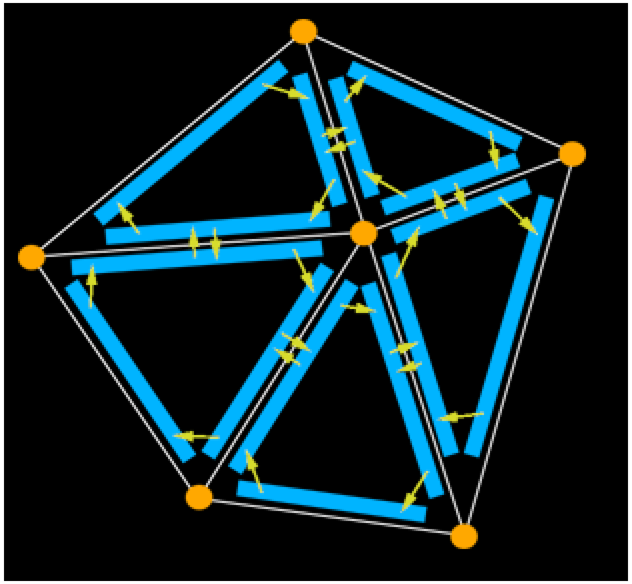
Edge	
VertexRef	vertex[2]
FaceRef	face[2]
EdgeRef	next[2]
EdgeRef	prev[2]

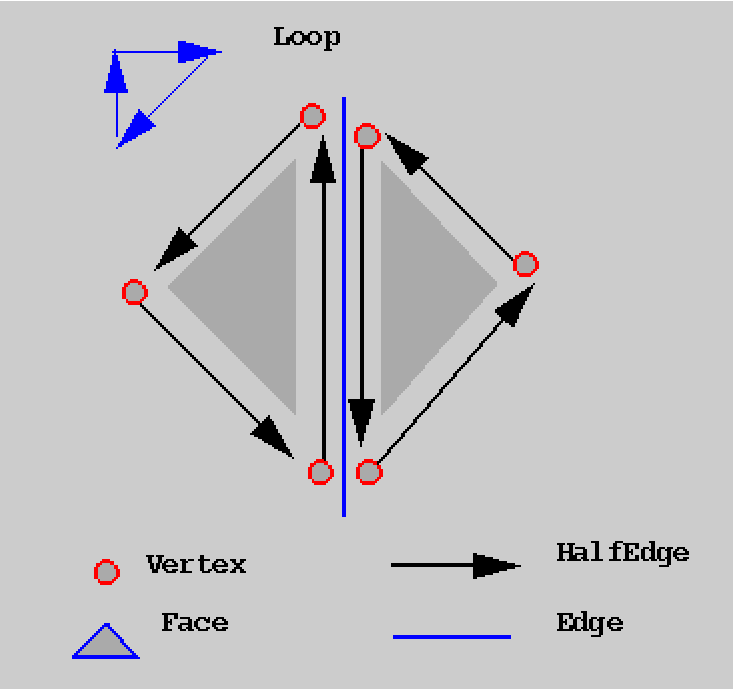
Half-Edge Data Structure

- (What?) A common way to represent triangular mesh for geometric processing
  - We first focus on triangle-mesh, (it works for general polygonal mesh).
  - 3D analogy: half-face data structure for tetrahedral mesh
- (Why?) Effective for maintaining incidence information of vertices
  - Efficient local traversal
  - Relatively low spatial cost
  - Supporting dynamic local updates/manipulations (edge collapse, vertex split, etc.)
- (Resources?) Codes are provided. After this class, please go through them carefully, we will work on it during the whole semester.

Half-Edge Data Structure (cont.)

- Consider each edge by splitting it into two halfedges
  - Primitives: Face, Edge, Halfedge, Vertex
- Store all adjacency information between primitives on halfedges
  - Each edge has 2 halfedges (the boundary edge has only 1)

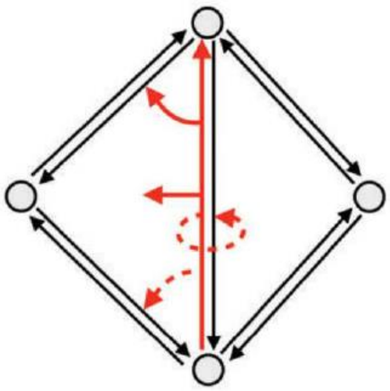




Half-Edge Data Structure (cont.)

- Halfedges are oriented consistently in counterclockwise order around each face
- Each halfedge designates a unique corner on each face (can be used to store texture coordinates, later in texture mapping)

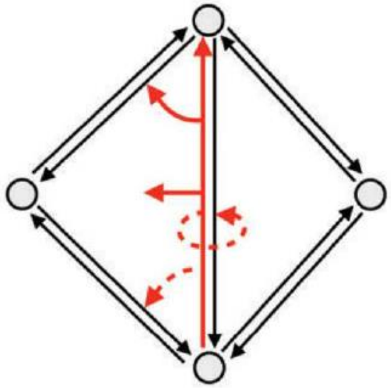
- On each halfedge, we store:
  - the vertex it points to (its target);
  - the face this halfedge locates;
  - the next halfedge on the face;
  - the previous halfedge on the face;
  - $^{*(1)}$ its twin halfedge;
- For each vertex: store one of its incident incoming halfedges
- For each face: store one of its halfedges
- $^{*(2)}$ for each edge: store its two halfedges



- $^{*(1)}$ and $^{*(2)}$: keep either one and we can get the other easily
- **Storage Costs:**
  - A mesh with n triangles needs ?? Bytes
  - Hint: # of halfedges H is about 6 times of V

Half-Edge Structure Implementation

Read through the provided source codes for the implementation of halfedge data structure:



- Check Halfedge.h, each halfedge class stores:
  - → target(): the target vertex;
  - → face(): adjacent face;
  - → next(): the next halfedge on the face;
  - → prev(): the previous halfedge in the face;

  And you can use some other implmentation:
  - → twin(): its twin halfedge;
  - → source(): the source vertex;
- Check Edge.h, Vertex.h, Face.h, and finally Mesh.h

Half-Edge Data Structure (example)

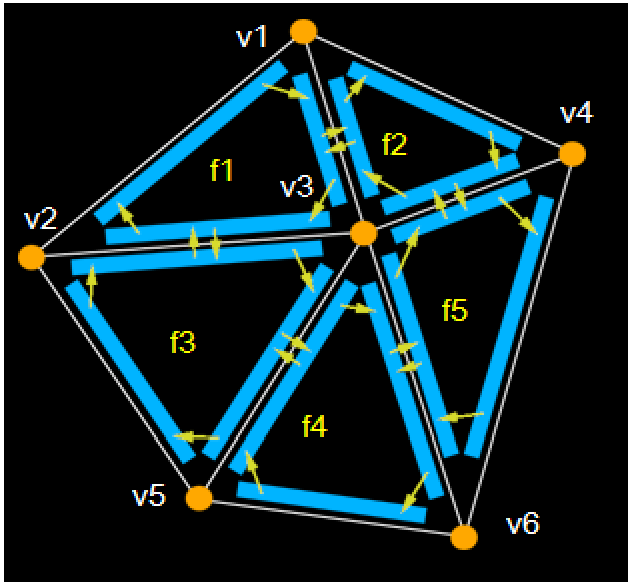
1). In Mesh.h, four containers used to store primitives:

The Vertex Container*	v1 … v6
The Half-Edge Container	[v1,v2], [v2, v3], [v3, v1], [v1, v3], [v3, v4], …
The Edge Container	[v1,v3], [v1,v2], [v2,v3], [v1,v4], [v3,v4], …
The Face Container	f1[v1,v2,v3] … f5[v4,v3,v6]

$[v_1, v_2]$ means: a halfedge from v_1 to v_2

Halfedges: $[v_1, v_2] \neq [v_2, v_1]$

The orientation of all the halfedges should be consistent: CounterClockWise (CCW) in our configuration



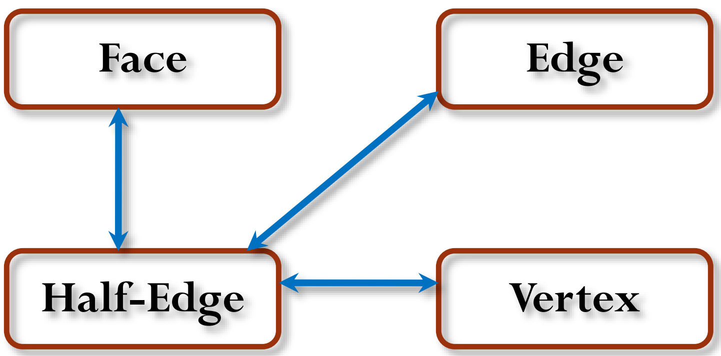
Note*: the container could be array, list, binary search tree…
(it depends, our sample codes used list)

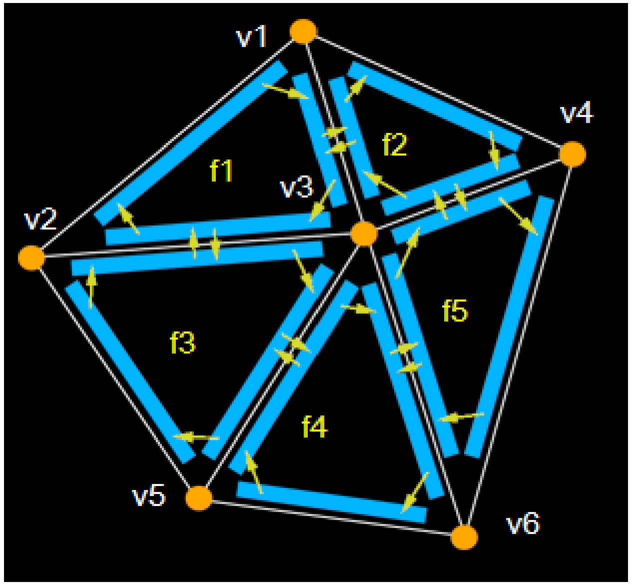
Half-Edge Data Structure (example)

1). Containers store primitives:

The Vertex Container*	v1 … v6
The Half-Edge Container	[v1,v2], [v2, v3], [v3, v1], [v1, v3], [v3, v4], …
The Edge Container	[v1,v3], [v1,v2], [v2,v3], [v1,v4], [v3,v4], …
The Face Container	f1[v1,v2,v3] … f5[v4,v3,v6]

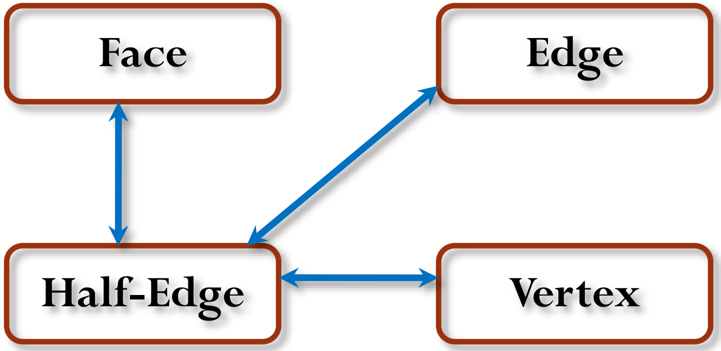
2). Relationship between primitives:

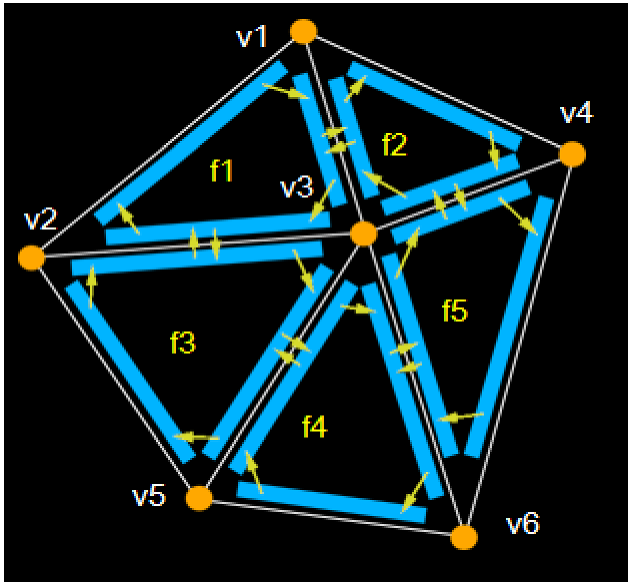




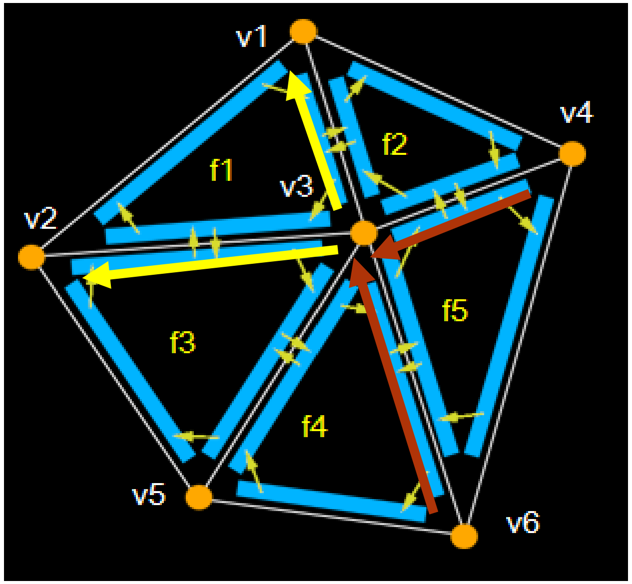
Using Half-Edge Data Structure

1. How to check whether a vertex/edge/face is on the boundary?
2. How to track the boundary?
3. How to find your one-ring neighbor?
4. How to do subdivision/simplification…?





Using Half-Edge Data Structure – Local Rotations



❑ Rotation Operations defined on halfedge :

1. clw_rotate_about_target(): e.g. [v4, v3] to [v6, v3]
   he→next()→twin()
2. clw_rotate_about_source(): e.g. [v3, v2] to [v3, v1]
   he → twin() → next() ?
3. ccw_rotate_about_target(): e.g. [v6, v3] to [v4,v3]
   he → twin() → prev() ?
4. ccw_rotate_about_source(): e.g. [v3, v1] to [v3, v2]
   he→prev()→twin()

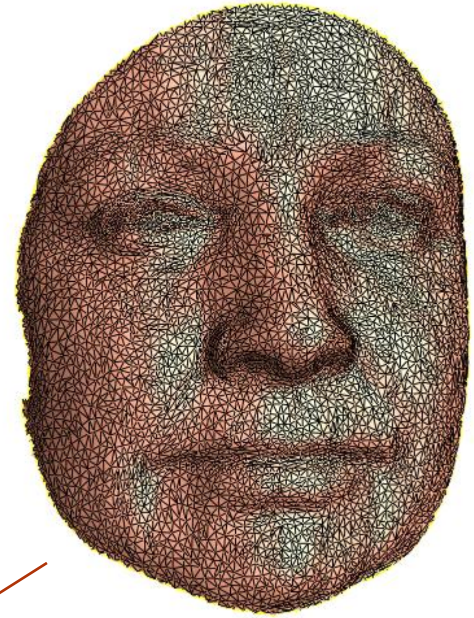
❑ Rotation Operations defined on **boundary** vertices:

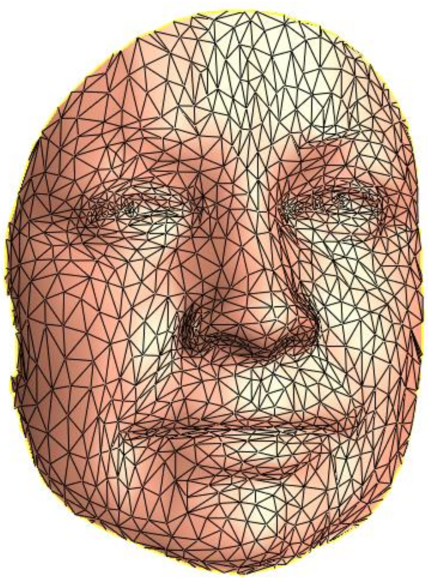
1. most_clw_in_halfedge(): e.g. for v4, it is [v2, v4]
   Let he = v →he(), then keep doing clw rotation about its target
2. most_ccw_in_halfedge(): e.g. for v4, it is [v6, v4]
   Let he = v →he(), then keep doing ccw rotation about its target
3. most_clw_out_halfedge(): e.g. for v4, it is [v4, v1]
4. most_ccw_out_halfedge(): e.g. for v4, it is [v4, v1]

➢ What about interior vertices?
  → Not well defined. Return any in/out halfedge

Using Half-Edge Data Structure

Mesh subdivision/simplification…?



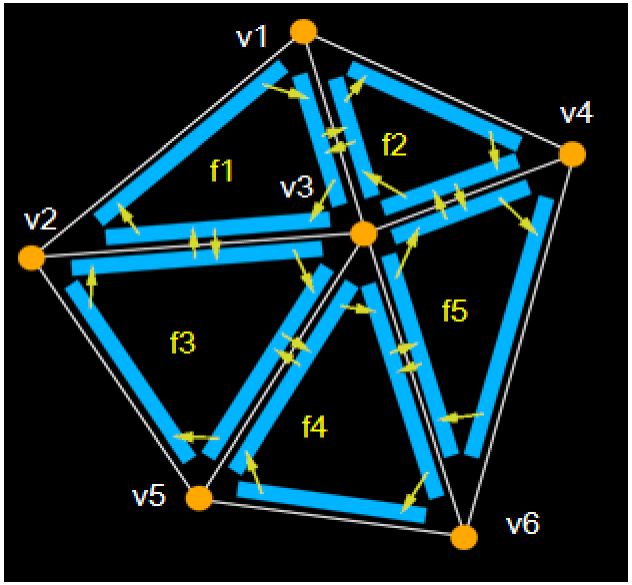


Subdivision (refining)

Simplification (coarsening)

Half-Edge Data Structure (cont.)

1) Read “iterators.h” to see how you can do local traversal
2) Read “mesh.h”, to see how you can get access to primitives
3) *Go through “mesh→read()” method, to see how the halfedge data structure is constructed from indexed vertex-face table.



Some 3D Models in Polygonal Meshes

- “m” format
  - Some models (with “.mesh” or “.m” as extension)
  - A small openGL viewer “MViewer.exe” (you can drag your downloaded “.m” file into it directly)
- “obj” format
  - Two models with .obj extension
  - You will be asked to write a program similar to “Mviewer” in homework 1 and 2

- Many 3D shapes/data available online (but in various formats):
  - Stanford 3D Scanning Repository: http://graphics.stanford.edu/data/3Dscanrep/
  - Aim@Shape Repository: http://shapes.aim-at-shape.net/index.php
  - Google 3D warehouse