Fragmented Skull Modeling Using Heat Kernels

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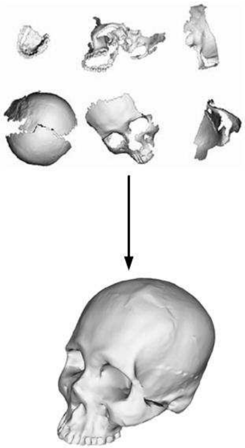
Introduction

Goals

- To explore reliable and effective algorithms to help geometric reassembly of the fragmented model.

Motivation

- Current Matching Algorithm
  - Poor effect for template based assembly problem.
  - Sensitive to local noise.
- Proposed Approach
  - Effective and robust in matching partial models to the complete model.



Related Work

- 3D Shape Descriptor:
  - Local Point Signature. [G. Barequet and M. Sharir 1994], [F.Stein and G. Medioni 1992], [Ruiz-Correa et al 2001]
  - Spin Image. [Johnson and Hebert 1999], [S. Belonie et al 2002]
  - Global Point Signature. [Raif M. Rustamov 2007], [Ovsjanikov M et al 2008]
- Fragments Assembly:
  - Assembling virtual pots from 3D measurements of their fragments. [Cooper et al 2001]
  - Bayesian Assembly of 3D Axially Symmetric Shapes from Fragments. [Willis and Cooper 2004]
  - Feature-based Part Retrieval for Interactive 3D Reassembly. [Parikh et al 2007]
  - Skull Assembly and Completion using Template-based Surface Matching. [Wei et al 2011]
  - An Automatic Assembly and Completion Framework for Fragmented Skulls [Yin et al 2011]

Backgrounds

Challenges

- Subtle geometry.
- Inconsistent scale and resolution.
- Not exactly the same between fragments and template.

Desirable Signature

- Multi-scale signature.
- Scale-invariance and resolution-invariance signature.
- Robust to noise.
- Efficient to compute.
- Easy to compare and implement.

Heat Kernel

Heat Kernel Shape Descriptor

M is a compact Riemannian manifold, and $u(x, t)$ is the amount of heat at a point $x \in M$ at time t. The heat propagation over M is governed by the *heat diffusion equation*:

$$\begin{cases} \frac{\partial u(x,t)}{\partial t} = -\Delta u(x, t) \\ u(x, 0) = f(x) \end{cases} \tag{1}$$

For any M, there exists a function $h_t(x, y)$ that

$$u(x, t) = \int_M h_t(x, y) f(y) dy. \tag{2}$$

And the heat kernel has the following eigen-decomposition:

$$h_t(x, y) = \sum_{i=0}^{\infty} e^{-\lambda_i t} \Phi_i(x) \Phi_i(y) \tag{3}$$

Heat Kernel Signature (HKS)

Heat Kernel Signature

Heat kernel signature is a powerful descriptor that characterizes local and global geometry of the surface patch centered at each point:

$$h_t(x) = \sum_{i=0}^{\infty} e^{-\lambda i_t} \Phi_i(x)^2. \tag{4}$$

In the discrete setting, heat kernel signature can be computed from the eigen-values and eigen-vectors of the mesh Laplace opertor.

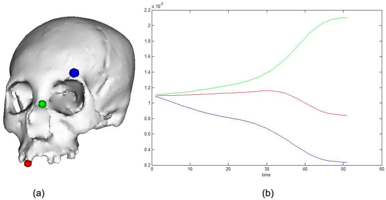


Figure : Each point has a unique heat diffusion curves. Different points have different signatures.

Properties of HKS

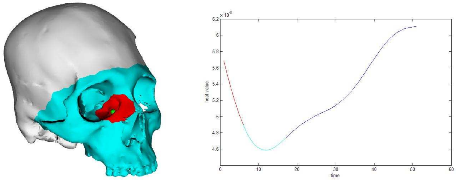
Multi-scale Property

Current state:

- Local descriptor $\longrightarrow$ can be easily affected by local noise and geometry disparity.
- Global descriptor $\longrightarrow$ could not tolerate the intrinsic difference between a *complete* template and an *incomplete* fragment.

For the function $h_t(x, y)$:

- A small t $\longrightarrow$ reflects characteristic of a small neighborhood of x
- As t increases $\longrightarrow$ its neighborhood grows to a bigger region.



(a) (b)

Figure : The green point (a), considered in the fragment (red region) and in the whole model (cyan) has the overlapped signature curves (b).

Properties of HKS

Scale-invariance Property

Current state:

- Fragments are scanned separately $\longrightarrow$ the scales of these digital models are usually inconsistent $\longrightarrow$ need to preprocess the original skulls $\longrightarrow$ tedious, error-prone, and could contaminate the original skull

A Scale-invariance descriptor

- Based on a logarithmically sampled scale-space and Fourier transform modulus (FTM), HKS can be modified to a scale invariant vision using the approach proposed in [Bronstein, M.M. CVPR 2010].

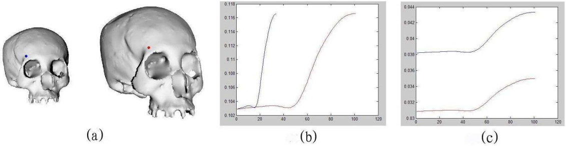


Figure : (a) shows one skull with two scaling, the right one is twice larger. (b) shows their HKS in the same coordinate, and (c) shows the result of normalization.

Properties of HKS

Robustness Property

Current state:

- Scanned separately $\longrightarrow$ different sampling and tessellations
- Occlusions and low reflectance $\longrightarrow$ Holes and local noise

A stable descriptor

- Heat kernel signature is stable against local noise (e.g. small local geometric perturbation) due to the nature of heat diffusion process on the manifold.

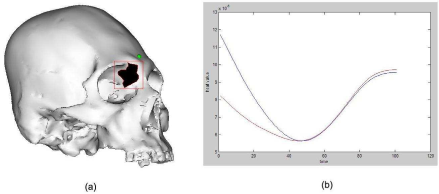
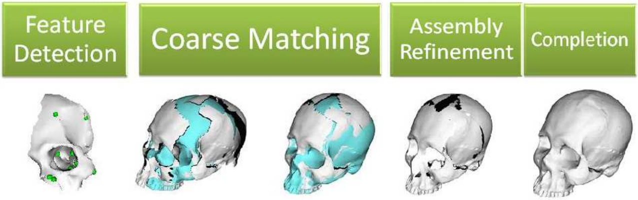


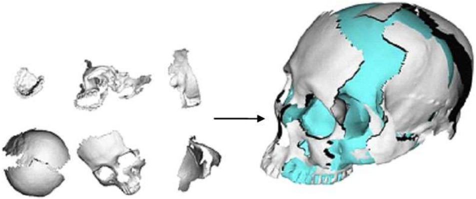
Figure : The green point on an incomplete skull (a) has a similar signature (b, the blue curve) to the signature on the completed skull (b, the red curve).

Algorithm Pipeline



Fragmented Skull Assembling

- In: A set of fragments s_i and template t.
- Out: A set of rigid transformations T_i (applied on s_i), so that the arrangement of all fragments in world coordinates well approximates t.



Step 1 - Feature Detection

- **In:** A set of fragments s_i and template t.
- **Out:** Fragments and template with feature points.

Feature Extraction

- Feature: A point with a local maximum or minimum heat kernel value.
- Step k:
  - Range from 0 - 100 (Sampled following in the log scale)
  - A small k: Mainly encode local geometry.
  - A large k: Characterize more global geometry.
  - In our experiments, we usually use k = 60.

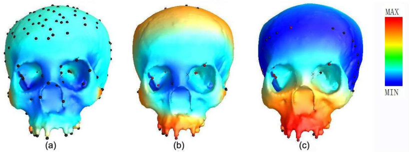


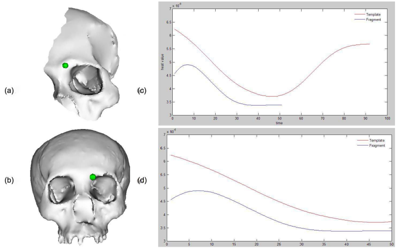
Figure : The color indicates the heat value of the point, and features are extracted in different scales. (a) k = 0, (b) k = 60, (c) k = 100.

Step 2 - Coarse Matching

- **In:** HKS on fragments and template feature points.
- **Out:** Correspondence from fragments to the template.

Sub-step 1: Initial Matching

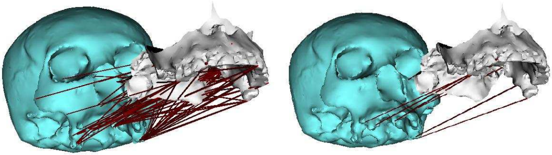
The most possible many-to-many mapping (evaluate the difference between two HKS) is the coarse correspondence.



Step 2 - Coarse Matching

Sub-step 2: Local Refinement

Given such a candidate matching graph, we need a filter to eliminate these wrong matches.
We develop such a filtering scheme based on the RANSAC strategy to exact a most isometric sub-set.



(a) (b)

Figure : (a) is the superset of matches which includes many wrong matches and (b) is the final matches sifted by the filter.

Step 2 - Coarse Matching

Sub-step 3: Local Registration

After the correct matching is computed, we can compute the rigid transformation T_i for each fragment by solving an over-determined system:

$$T_i \begin{pmatrix} p_i^1 \\ p_i^2 \\ \cdots \\ p_i^n \end{pmatrix} = \begin{pmatrix} q_i^1 \\ q_i^2 \\ \cdots \\ q_i^n \end{pmatrix}$$

Figure : p_i and q_i: the feature point on fragment and on template.
T_i: the rigid transformation on fragment.

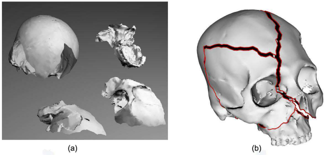


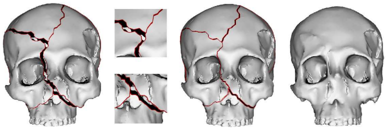
Figure : A coarse assembly example

Step 3 - Reassembly Refinement

- In: The assembled skull with damage regions, and a template.
- Out: A repaired skull.

Global Position Optimization

We further refine the reassembly through an optimization of the least square transformation error (LSTE) of break-curves.[Yin et al. ICCV 2011]



(a) (b) (c)

Figure : (a) shows the reassembled skull after rough assembly; (b) shows the result after break curve matching and assembly refinement; (c) is the final completed skull.

Step 3 - Skull Completion

Skull completion

We use a template based and a symmetry based completion to fix the damaged method.[Li et al. 2011]

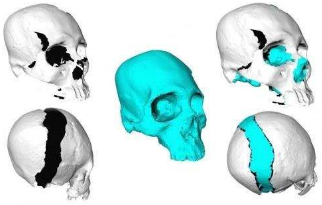


Figure : Use a non-rigid registration computation to map the template (cyan) to subject (white).

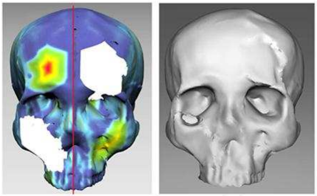
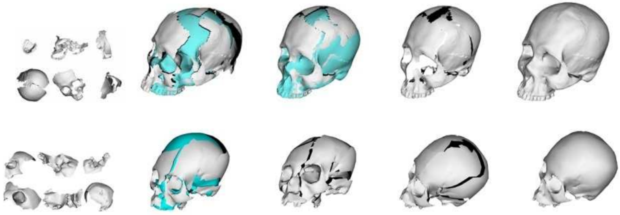


Figure : Symmetry detection on model with big missing regions (left) and a completed skull (right)

Experimental Results



NO.	#△ (K)	#F	T_{HKS}	T_{RAM}	T_{COM}
1	37.2	6	343.5	6.3	29.6
2	43.8	6	400.6	6.1	37.2

Figure : $\#\Delta$(K): the number of thousand triangles in the mesh; $\#F$: number of fragments; T_{HKS}: the time of computing HKS in seconds; T_{RAN}: the time of RANSAC process with 500 iterations. T_{Com}: the time of post-processing and skull completion. Experimental time is measured in seconds.

Comparison Experiment

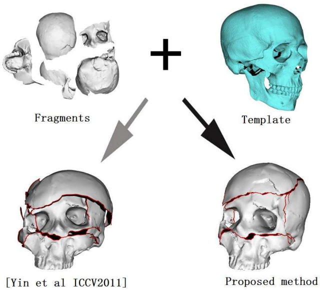


Figure : Comparison of our proposed reassembly and the algorithm of [Yin et al. ICCV 2011]. The fragments (white) are assembled using the template (cyan), bottom left is the result of our previous method and bottom right is the result of our method.

Completion in Various Cases

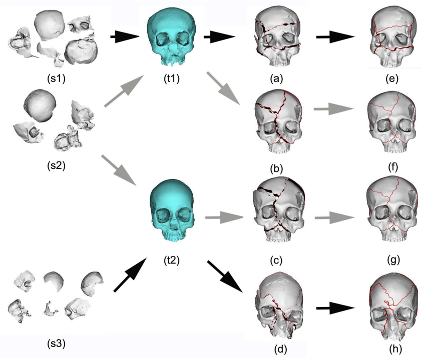


Figure : (s1 - s3) are the fragmented skulls. (t1) and (t2) are the templates. The different coarse reassembling results are shown in (a) - (d);
(e) - (h) show the results after the refinement guided by break curve matching.

Conclusion

- Shape Descriptor:
  - Use a multi-scale descriptor based on heat kernel for data reassembly.
  - Analyze its several desirable properties in geometric reassembly and in our task.
- Skull Assembling:
  - Develop a robust and efficient partial matching algorithm based on this descriptor.
  - Integrate the developed methodologies into a three-step skull reassembly pipeline.
  - The new algorithm demonstrated to have better efficacy than existing techniques.

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