EE 7630: Detection and Estimation Theory

Homework Two (& Quiz Two), Spring of 2007

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Due at 5:40 pm, Wednesday, March 28, 2007

1. Let

$$r_i = a + n_i, i = 1, 2, \ldots, N,$$

where r_i is the ith observation sample, a is the unknown nonrandom parameter to be estimated and n_i is the ith additive noise sample. We assume that n_i are each independent Gaussian variables, $N(0, \sigma_n)$.

   (a) Derive the *a priori* density function $f_{\vec{R}|A}(\vec{r}|a)$ in terms of σ_n, a and r_i.

   (b) Obtain the maximum-likelihood estimate $\hat{a}_{ml}(\vec{r})$ according to (a).

   (c) Is the estimate $\hat{a}_{ml}(\vec{r})$ in (b) biased or unbiased? Justify it.

   (d) Calculate the Cramer-Rao lower bound for all the unbiased estimates $\hat{a}(\vec{r})$.

   (e) We construct a different estimator from $\hat{a}_{ml}(\vec{r})$ in (b) such that

$$\hat{a'}(\vec{r}) \equiv r_1.$$

   First justify whether $\hat{a'}(\vec{r})$ is unbiased or not. Is this estimator more efficient than the estimator $\hat{a}_{ml}(\vec{r})$ in (b)?

2. Let

$$r_i = a + n_i, i = 1, 2, \ldots, N,$$

where r_i is the ith observation sample, a is the unknown parameter to be estimated and n_i is the ith additive noise sample. We assume that a is Gaussian, $N(0, \sigma_a)$, and that n_i are each independent Gaussian variables, $N(0, \sigma_n)$.

   (a) Derive the *a priori* density function $f_{\vec{R}|A}(\vec{r}|a)$ in terms of σ_n, σ_a, a and r_i.

   (b) Derive the *a posteriori* density function $f_{A|\vec{R}}(a|\vec{r})$ in terms of σ_n, σ_a, a and r_i.

   (c) Determine the maximum *a posteriori* estimate $\hat{a}_{MAP}(\vec{r})$.

   (d) Is the estimate $\hat{a}_{MAP}(\vec{r})$ in (c) biased or unbiased? Justify it.

   (e) Calculate the Cramer-Rao lower bound for all the unbiased estimates $\hat{a}(\vec{r})$.