EE 7630: Detection and Estimation Theory

Homework One (& Quiz One), Spring of 2007

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Due at 5:40 pm, Monday, March 12, 2007

1. Let

$$r_i = a + n_i, i = 1, 2, \ldots, N,$$

where r_i is the ith observation sample, a is the unknown parameter to be estimated and n_i is the ith additive noise sample. We assume that a is Gaussian, $N(0, \sigma_a)$, and that the n_i are each independent Gaussian variables, $N(0, \sigma_n)$. The cost function for Bayes estimation is related to the error as defined by

$$a_\epsilon(\vec{r}) \equiv \hat{a}(\vec{r}) - a.$$

   (a) Derive the *a priori* density function $f_{\vec{R}|A}(\vec{r}|a)$ in terms of σ_n, σ_a, a and r_i.

   (b) Derive the *a posteriori* density function $f_{A|\vec{R}}(a|\vec{r})$ in terms of σ_n, σ_a, a and r_i.

   (c) Determine the conditional mean $\hat{a}_{ms}(\vec{r})$.

   (d) We define a family of cost functions $C_p(a_\epsilon)$ as

$$C_p(a_\epsilon) \equiv |a_\epsilon|^p, p = 1, 2, 3, \ldots. \tag{1}$$

   According to Eq. (1), show that $C_p(a_\epsilon)$ are invariant to the Bayes estimation, $\forall p \in \mathbb{Z}^+$. Hint: apply Property 1 on pp. 60-61 in Van Trees' text.

2. Redo Problem 1 (d) using Property 2 on pp. 61 in Van Trees' text instead.