EE 7630: Detection and Estimation Theory

Final Examination, Spring of 2007

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Due at 5:45 PM, Wednesday, May 2, 2007

1. The probability density of r_i, $i = 1, 2, \ldots, N$, given a_1 and a_2 is

$$f_{R_i|a_1,a_2}(r_i|a_1, a_2) = \frac{1}{\sqrt{2\pi a_2}} \exp\left[-\frac{(r_i - a_1)^2}{2a_2}\right],$$

where a_1 is the mean and a_2 is the variance.

(a) Find the joint maximum-likelihood estimates $\hat{a}_1$, $\hat{a}_2$ of a_1 and a_2 respectively by using N independent observations.

(b) Are they biased?

(c) Another variance estimator is described as

$$\hat{a}'_2 = \frac{1}{N-1} \sum_{i=1}^{N} \left[r_i^2 - \frac{1}{N} \sum_{i=1}^{N} r_i\right]^2.$$

Is this biased?

(d) Check if the mean estimator $\hat{a}_1$ achieves the Cramer-Rao lower bound.

2. We want to transmit two parameters, a_1 and a_2. In a simple attempt to achieve a secure communication system we construct two signals to be transmitted over separate channels such that

$$\begin{aligned} s_1 &= x_{11}a_1 + x_{12}a_2, \\ s_2 &= x_{21}a_1 + x_{22}a_2, \end{aligned}$$

where x_{ij}, $i, j = 1, 2$, are known. The received variables are

$$\begin{aligned} r_1 &= s_1 + n_1, \\ r_2 &= s_2 + n_2. \end{aligned}$$

The additive noise processes n_1, n_2 are independent, identically distributed and zero-mean Gaussian, $N(0, \sigma_n)$. The parameters a_1 and a_2 are nonrandom.

(a) what are the maximum-likelihood estimates $\hat{a}_1$ and $\hat{a}_2$?

(b) Are $\hat{a}_1$ and $\hat{a}_2$ biased?

(c) Compute the variances of $E\{(\hat{a}_1 - a_1)^2\}$ and $E\{(\hat{a}_2 - a_2)^2\}$.

(d) Do the maximum-likelihood estimates $\hat{a}_1$ and $\hat{a}_2$ achieve the Cramer-Rao lower bounds?

3. The joint probability density function for the two observations r_1 and r_2 is

$$f_{R_1,R_2|\rho}(r_1, r_2|\rho) = \frac{1}{2\pi\sqrt{(1-\rho^2)}} \exp\left[-\frac{(r_1^2 - 2\rho r_1 r2 + r_2^2)}{2(1-\rho^2)}\right].$$

(a) Determine the maximum-likelihood estimate $\hat{\rho}$ in terms of r_1 and r_2.

(b) Is this $\hat{\rho}$ biased?

(c) Determine the Cramer-Rao lower bound for ρ.