EE7150 Theory and Application of Digital Signal Processing

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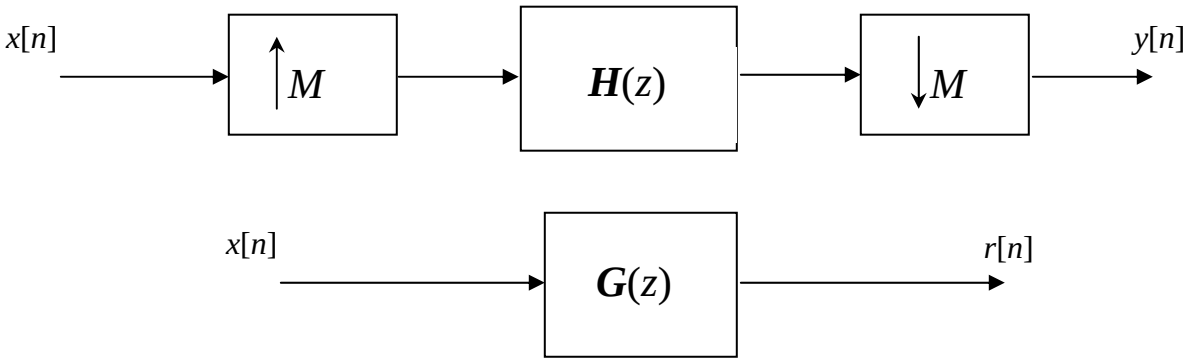
Quiz 2, Spring of 2005

SOLUTION



For the same input sequence $x[n]$, the two systems have the identical response $y[n]=r[n]$.

(a) If $H(z)=\sum_{n=0}^{\infty} h[n]z^{-n}$, write the expression for $G(z)$ in terms of $h[n]$. (30%)

(b) Given that

$$H(z)=\frac{z^2}{(z-0.5)(z-0.2)(z-0.1)},$$

derive the transfer function $G(z)$. (40%)

(c) If $M=2$, $x[n]=u[n]=\begin{cases}1, & n\geq 0\\ 0, & n<0\end{cases}$ (unit-step function), write the expression for the sequence $y[n]$. (30%)

Answer:

(a) $G(z) = \sum_{n=0}^{\infty} h[Mn] z^{-n}$.

(b) $H(z) = \frac{25}{6}\frac{z}{z-0.5} - \frac{20}{3}\frac{z}{z-0.2} + \frac{5}{2}\frac{z}{z-0.1}$

$\Rightarrow h[n] = \frac{25}{6}(0.5)^n u[n] - \frac{20}{3}(0.2)^n u[n] + \frac{5}{2}(0.1)^n u[n]$

$G(z) = \frac{25}{12}\sum_{n=0}^{\infty}(0.5)^{Mn} z^{-n} - \frac{20}{3}\sum_{n=0}^{\infty}(0.2)^{Mn} z^{-n} + \frac{5}{2}\sum_{n=0}^{\infty}(0.1)^{Mn} z^{-n}$

$= \frac{25}{6}\frac{z}{z-(0.5)^M} - \frac{20}{3}\frac{z}{z-(0.2)^M} + \frac{5}{2}\frac{z}{z-(0.1)^M}$

(c) $Z\{y[n]\} = G(z)X(z) = \left(\frac{25}{6}\frac{z}{z-0.25} - \frac{20}{3}\frac{z}{z-0.04} + \frac{5}{2}\frac{z}{z-0.01}\right)\frac{z}{z-1}$

$= \frac{25}{6}\frac{z^2}{(z-0.25)(z-1)} - \frac{20}{3}\frac{z^2}{(z-0.04)(z-1)} + \frac{5}{2}\frac{z^2}{(z-0.01)(z-1)}$

$= -\frac{25}{18}\frac{z}{z-0.25} + \frac{5}{18}\frac{z}{z-0.04} - \frac{5}{198}\frac{z}{z-0.01} + \frac{225}{198}\frac{z}{z-1}$

$= -\frac{25}{18}(0.25)^n u[n] + \frac{5}{18}(0.04)^n u[n] - \frac{5}{198}(0.01)^n u[n] + \frac{225}{198}u[n]$

$= -1.3889(0.25)^n u[n] + 0.2778(0.04)^n u[n] - 0.0253(0.01)^n u[n] + 1.1364u[n]$