EE7150 Theory and Application of Digital Signal Processing

Dr. Hsiao-Chun Wu

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Solution

An N-point real-valued discrete-time sequence $x[n]$ is given by $x[n] = \{x_0, x_1, \cdots, x_{N-1}\}$. The corresponding DFT is

$$X[k] = [X_0, X_1, \cdots, X_{N-1}],$$

and the corresponding DCT is

$$X^{II}[k] = \left[X_0^{II}, X_1^{II}, \cdots, X_{N-1}^{II}\right].$$

The DFT of the zero-padded sequence $\bar{x}[n] = \left\{ x_0, x_1, \cdots, x_{N-1}, \underbrace{0, \cdots, 0}_{N\ zeros} \right\}$ is

$$\bar{X}[k] = [\bar{X}_0, \bar{X}_1, \cdots, \bar{X}_{2N-1}].$$

A $2N$-point discrete-time sequence $y[n]$ is given by

$$y[n] = \begin{cases} x[n], & 0 \le n \le N-1 \\ x[2N-1-n], & N \le n \le 2N-1 \end{cases}.$$

(a) Can we express DFT of $y[n]$ in terms of $X[k]$ or $\bar{X}[k]$? Write the expression. (50%)

(b) Determine the DCT of $y[n]$ in terms of $X^{II}[k]$. (50%)

Answer:

(a) $DFT\{y[n]\} = \sum_{n=0}^{2N-1} y[n] e^{-j\frac{2\pi nk}{2N}} = \sum_{n=0}^{N-1} x[n] e^{-j\frac{\pi nk}{N}} + \sum_{n=N}^{2N-1} x[2N-1-n] e^{-j\frac{\pi nk}{N}}$

$$= \overline{X}[k] + \sum_{m=0}^{N-1} x[m] e^{j\frac{\pi mk}{N}} e^{-j\frac{2\pi Nk}{N}} e^{j\frac{\pi k}{N}} = \overline{X}[k] + e^{j\frac{\pi k}{N}} \sum_{m=0}^{N-1} x[m] e^{j\frac{\pi mk}{N}}$$

$$= \overline{X}[k] + e^{j\frac{\pi k}{N}} \overline{X}^*[k]$$

(b) $DCT\{y[n]\} = \sqrt{\frac{2}{2N}} C[k] \sum_{n=0}^{2N-1} y[n] \cos\left[\frac{\pi k(2n+1)}{4N}\right]$

$$= \sqrt{\frac{1}{N}} C[k] \sum_{n=0}^{N-1} x[n] \cos\left[\frac{\pi k(2n+1)}{4N}\right] + \sqrt{\frac{1}{N}} C[k] \sum_{n=N}^{2N-1} x[2N-1-n] \cos\left[\frac{\pi k(2n+1)}{4N}\right]$$

$$= \sqrt{\frac{1}{N}} C[k] \sum_{n=0}^{N-1} x[n] \cos\left[\frac{\pi k(2n+1)}{4N}\right] + \sqrt{\frac{1}{N}} C[k] \sum_{m=0}^{N-1} x[m] \cos\left[\frac{\pi k(4N-2m-1)}{4N}\right]$$

$$= \begin{cases} \sqrt{\frac{1}{N}} C[k] \sum_{n=0}^{N-1} x[n] \cos\left[\frac{\pi k(2n+1)}{4N}\right] - \sqrt{\frac{1}{N}} C[k] \sum_{m=0}^{N-1} x[m] \cos\left[\frac{\pi k(2m+1)}{4N}\right], & k = 2k'+1 \\ \sqrt{\frac{1}{N}} C[k'] \sum_{n=0}^{N-1} x[n] \cos\left[\frac{\pi k'(2n+1)}{2N}\right] + \sqrt{\frac{1}{N}} C[k'] \sum_{m=0}^{N-1} x[m] \cos\left[\frac{\pi k'(2m+1)}{2N}\right], & k = 2k' \end{cases},$$

for k' = 0, 1, …, N-1,

$$= \begin{cases} 0, & k = 2k'+1 \\ \sqrt{2} X[k'], & k = 2k' \end{cases}, \quad k' = 0, 1, \cdots, N-1$$