EE7150 Theory and Application of Digital Signal Processing

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Midterm Examination, Spring of 2005

Time: 3:45 p.m. ~ 5:00 p.m., Wednesday, April 6 of 2005

Please ALWAYS work on the FRONT SIDE of each page. No answer on the BACK SIDE of any page will be GRADED! Ask for additional blank papers if necessary!

You may check any textbook, classnote or other references alone during the test. However, NO CHEATING or COLLABORATION is allowed and such violation of the university regulations will be reported.

Please write down you name and social security number here:

Full Name: ________________SOLUTION________________

Social Security Number: ____

Partition	Score
Question 1	
Question 2	
Question 3	
Total	

<u>Question 1</u>: (30%)

Let $X = [\alpha, 1+j, 0, 1-j]$ be the DFT of the sequence $x = [1, 0, 0, \beta]$.

(a) Determine α and β. (10%)

(b) Determine the DFT of $[1, \beta, 0, 0]$. (10%)

(c) Determine the DFT of $[0, 0, 1, \beta]$. (10%)

<u>Answer to Question 1</u>:

(a) $\alpha = 1 + \beta$, $1 - \beta = 0 \Rightarrow \beta = 1, \alpha = 2$.

(b) $x[(-n)_4] = [1, \beta, 0, 0]$

DFT{ $x[(-n)_4]$ }= $X[(-k)_4] = [2, 1 - j, 0, 1 + j]$

(c) $x[(2-n)_4] = [0, 0, 1, \beta]$

DFT{ $x[(2-n)_4]$ }= $(-1)^k X[(-k)_4] = [2, -1 + j, 0, -1 - j]$

<u>Question 2</u>: (30%)

Let $X^{II} = [1,0,1,0]$ be the DCT of the sequence $x = [1,0,0,1]$.

(a) Determine the DCT of $y = [1,0,0,1,1,0,0,1]$ (15%)

(b) Determine the DCT of $z = [0,1,1,0]$. (15%)

<u>Answer to Question 2</u>:

(a) $Y^{II} = \left[\sqrt{2}, 0, 0, 0, \sqrt{2}, 0, 0, 0\right]$

(b) $\because [0,1,1,0] = [1,1,1,1] - [1,0,0,1]$

DCT{ $[1,1,1,1]$ }= $[2,0,0,0]$.

$\therefore Z^{II} = [2,0,0,0] - [1,0,1,0] = [1,0,-1,0]$

<u>Question 3</u>: (40%)

An analog signal can be expressed as

$$x(t) = a_1 \cos(2\pi F_1 t) + a_2 \cos(2\pi F_2 t) + a_3 \cos(2\pi F_3 t),$$

where $F_1 = 1{,}000$ Hz, $F_2 = 2{,}000$ Hz and $F_3 = 2{,}500$ Hz. We would like to remove the sinusoidal components associated with frequencies F_2 and F_3 using a digital low-pass filter after we sample this signal as a discrete-time sequence.

(a) What is the minimum sampling frequency F_s to generate a discrete-time sequence $x[n] = x(t)\big|_{t=\frac{n}{F_s}}$ without aliasing? (5%)

(b) Using the sampling frequency in (a), design a digital window-FIR lowpass filter. What is the minimum filter-order N among all possible window sequences? (10%)

(c) What is the attenuation in dB in the stopband for the chosen window sequence in (b)? (5%)

(d) What is the impulse response for the causal lowpass filter using the window sequence in (b)? (20%)

Answer to Question 3:

(a) The Nyquiste rate is $F_s = 5{,}000$ Hz.

(b) $x[n] = a_1 \cos\left(\frac{2\pi n}{5}\right) + a_2 \cos\left(\frac{4\pi n}{5}\right) + a_3 \cos(\pi n)$

We can assign the specs as follows:

$\omega_p = \frac{2\pi}{5}$, $\omega_s = \frac{4\pi}{5}$, $\omega_c = \frac{1}{2}\left(\omega_s + \omega_p\right) = \frac{3\pi}{5}$, $\Delta\omega = \frac{2\pi}{5}$.

Thus, the minimum filter-order is controlled by the transition bandwidth $\Delta\omega = \frac{2\pi}{5}$, such that $N \geq 10$ (rectangular window).

(c) -13 dB.

(d) $h[n] = \begin{cases} \sin c\left(\frac{3\pi}{5}(n-5)\right), & 0 \leq n \leq 10 \\ 0, & otherwise \end{cases}$