EE7150 Theory and Application of Digital Signal Processing

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Final Examination, Spring of 2005

Time: 10:00 a.m. ~ noon, Thursday, May 12 of 2005

Please ALWAYS work on the FRONT SIDE of each page. No answer on the BACK SIDE of any page will be GRADED! Ask for additional blank papers if necessary!

You may check any textbook, classnote or other references alone during the test. However, NO CHEATING or COLLABORATION is allowed and such violation of the university regulations will be reported.

Please write down you name and social security number here:

Full Name: __________SOLUTION___________________

Social Security Number: ____

Partition	Score
Question 1	
Question 2	
Question 3	
Question 4	
Total	

<u>Question 1</u>: (20%)

Two discrete-time sequence vectors are $\vec{x} = [1, 2, 3, 4]$, where $x[n] = \left\{ \underset{\substack{\uparrow \\ n=0}}{1}, 2, 3, 4 \right\}$ and $\vec{h} = [2, 2, 1, 1]$ where $h[n] = \left\{ \underset{\substack{\uparrow \\ n=0}}{2}, 2, 1, 1 \right\}$. We would like to determine the discrete-time sequence vector $\vec{y}$ where $y[n] = x[n] \otimes_C h[n]$.

(a) If we write the circulation convolution $y[n] = x[n] \otimes_C h[n]$ such that

$$\vec{y} = \tilde{X}\, \vec{h} = \tilde{H}\, \vec{x},$$

specify the matrices $\tilde{X}$ and $\tilde{H}$. (10%)

(b) Calculate the vector $\vec{y}$. (10%)

Answer to Question 1:

(a) $\tilde{X} = \begin{bmatrix} 1 & 4 & 3 & 2 \\ 2 & 1 & 4 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$, $\tilde{H} = \begin{bmatrix} 2 & 1 & 1 & 2 \\ 2 & 2 & 1 & 1 \\ 1 & 2 & 2 & 1 \\ 1 & 1 & 2 & 2 \end{bmatrix}$

(b) $\vec{y} = \tilde{H}\ \vec{x} = \begin{bmatrix} 15 \\ 13 \\ 15 \\ 17 \end{bmatrix}$

<u>Question 2</u>: (30%)

We would like to design a digital highpass filter using the analog lowpass prototype Butterworth filter for the following specs:

- Cuff-off frequency: 14 kHz
- Stopband frequency: 0~4 kHz
- Stopband attenuation: 40 dB
- Sampling frequency: F_S = 32 kHz

(a) Determine the minimum order N for this lowpass analog prototype Butterworth filer. (10%)

(b) Determine the Laplace transform for this minimum-order lowpass analog prototype Butterworth filter. (10%)

(c) According to (b), determine the Laplace transform for this minimum-order highpass analog Butterworth filter. (10%)

Answer to Question 2:

(a) For the lowpass prototype lowpass Butterworth filter, $\Omega_c = 28{,}000\pi$, $\Omega_p = 8{,}000\pi$,

$\Omega_{sampling} = 32{,}000\pi$

$$|H(\Omega_c)|^2 = \frac{1}{1+\left(\Omega_c / \Omega_p\right)^{2N}} \le 10^{-4} \Rightarrow N \ge \log(9999) \big/ 2\log(7/2)$$

$\therefore N \ge 3.68$ and $N_{\min} = 4$.

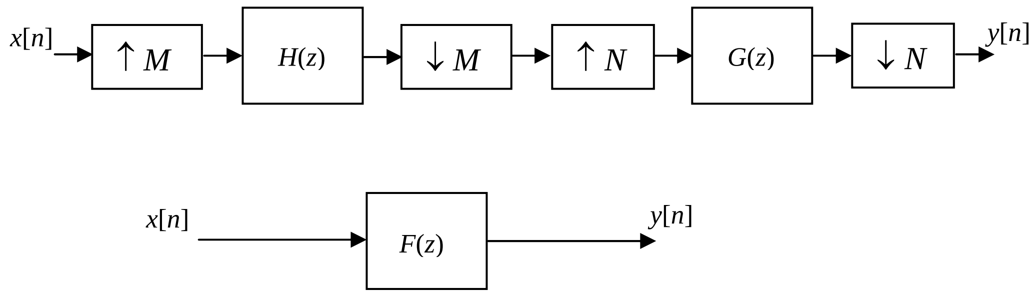
(b) $$|H(\Omega)|^2 = \frac{1}{1+\left(\Omega / \Omega_c\right)^{8}} \Rightarrow H(s)H(-s) = \frac{1}{1+(-s^2/\Omega_c^2)^4}$$

The poles p_k, $k = 0,1,2,3$, in $H(s)$ should be $j\Omega_c e^{j\frac{(2k+1)\pi}{8}}$, $k = 0,1,2,3$

$$H_{LP}(s) = \frac{1}{\prod_{k=0}^{3}(s - p_k)}$$

(c) $$H_{HP}(s) = H_{LP}(s')\Big|_{s' = \frac{\Omega_c^2}{s}}$$

<u>Question 3</u>: (20%)



The figure above depicts two equivalent linear time-invariant systems, i.e., both systems will generate identical output sequences when the input sequences are the same. $H(z)$, $G(z)$ and $F(z)$ are all causal.

(a) If $F(z) = 0$, for arbitrary positive integers M and N, what kind of conclusions on $H(z)$ and $G(z)$ can you make? (10%)

(b) If M=2, N=3, $H(z) = \dfrac{z}{(z-0.5)(z-0.2)}$ and $G(z) = 0.7 + 0.2z^{-1} + 0.8z^{-3} + 0.9z^{-6}$, determine $Y(z)$ in terms of $X(z)$. (10%)

<u>Answer to Question 3</u>:

(a) $F(z) = H_0(z)G_0(z)$,

where $H(z) = Z\{h[n]\}, G(z) = Z\{g[n]\}$, $H_0(z) = \sum_{n=0}^{\infty} h[Mn]z^{-n}$, $G_0(z) = \sum_{n=0}^{\infty} g[Nn]z^{-n}$

$F(z) = 0 \Rightarrow H_0(z) = 0$ or $G_0(z) = 0$

(b) M=2, N=3,

$$H(z) = \frac{10}{3}\frac{z}{z-0.5} - \frac{10}{3}\frac{z}{z-0.2} \Rightarrow H_0(z) = \frac{10}{3}\frac{z}{z-0.25} - \frac{10}{3}\frac{z}{z-0.04}$$

$$G_0(z) = 0.7 + 0.8z^{-1} + 0.9z^{-2}$$

$$Y(z) = \left(\frac{10}{3}\frac{z}{z-0.25} - \frac{10}{3}\frac{z}{z-0.04}\right)\left(0.7 + 0.8z^{-1} + 0.9z^{-2}\right)X(z)$$

<u>Question 4</u>: (30%)

Design an efficient DFT filter bank to decompose the signal into M=3 frequency components. The prototype filter should be an M-band, window-based FIR filter with the transition bandwidth $\Delta\omega \leq 0.1\pi$. We require the stopband attenuation is at least 40 dB.

(a) What is the appropriate window for us to comply with the specs with minimum filter order N? (10%)

(b) According to (a), what is the minimum order N? (10%)

(c) If we have to design a causal DFT filter bank to comply with the specs, what is the minimum order N? (10%)

Answer to Question 4:

(a) Hamming window.

(b) $\frac{8\pi}{N} \leq 0.1\pi \Rightarrow N \geq 80 \Rightarrow N_{\min} = 80$

(c) $\sin\left(\omega_c\left(3k - \frac{N}{2}\right)\right) = 0 \Rightarrow \frac{\pi}{3}\left(3k - \frac{N}{2}\right) = l\pi, l \in Z^+, k \in Z$

$N = 6(k - l) \Rightarrow N = 84$