EE 7150: Theory and Applications of Digital Signal Processing

Final Examination, Fall of 2007

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5:30-7:30 PM, Monday, December 10, 2007

Please ALWAYS work on the FRONT SIDE of each page. No answer on the BACK SIDE of any page will be GRADED! Ask for additional blank papers if necessary!

You may check any textbook, classnote or other references alone during the test. However, NO CHEATING or COLLABORATION is allowed and such violation of the university regulations will be reported.

Full Name: ________________________________

Social Security Number: ______________________

Partitions	Scores
Question 1	
Question 2	
Question 3	
Question 4	
Overall	

1. The discrete-time sequences $x_1[n]$ and $x_2[n]$ are given by

$$x_1[n] = \{\underset{\substack{\uparrow \\ n=0}}{1}, \quad 2, \quad 3, \quad 4, \quad 5, \quad 6\},$$

and

$$x_2[n] = \{\underset{\substack{\uparrow \\ n=0}}{1}, \quad 2, \quad 3, \quad 4, \quad 5, \quad 6, \quad 6, \quad 5, \quad 4, \quad 3, \quad 2, \quad 1\}.$$

The corresponding discrete Fourier transform (DFT) sequences are $X_1[k]$, $X_2[k]$, respectively. (30%)

(a) Determine the numerical value for

$$\sum_{k=0}^{5} |X_1[k]|^2 . \qquad (15\%)$$

(b) Write the mathematical relationship between $\sum_{k=0}^{5} |X_1[k]|^2$ and $\sum_{k=0}^{11} |X_2[k]|^2$. Then determine the numerical value for $\sum_{k=0}^{11} |X_2[k]|^2$ according to the result in (a). (15%)

Solution to Problem 1:

(a)

$$\begin{aligned}\sum_{k=0}^{5}|X_1[k]|^2 &= 6\sum_{n=0}^{5}|x_1[n]|^2 \\ &= 546.\end{aligned}$$

(b)

$$\begin{aligned}\sum_{k=0}^{11}|X_1[k]|^2 &= \frac{24}{6} * \sum_{k=0}^{5}|x_1[k]|^2 \\ &= 2184.\end{aligned}$$

2. We need to design a window-FIR band-pass filter. The center frequency is 10 kHz (the passband is 8 to 12 kHz) and the sampling frequency is 40 kHz. We require the transition bandwidth to be less than 100 Hz and the attenuation is at least 100 dB. (20%)

   (a) What window function will you choose? (10%)

   (b) What is the minimum window size N? (10%)

Solution to Problem 2:

(a) Kaiser window is chosen.

(b) $\Delta\omega = \frac{2\pi\times 100}{40000} = \frac{\pi}{200}$.

$N = \frac{100-8}{2.285\Delta\omega} \geq 2564.$

3. Design a digital IIR lowpass filter based on the analog Butterworth filter prototype $H_A(s)$ with a 3dB passband frequency at 1kHz and 20dB stopband freqiency at 5kHz. The sampling frequency F_s is 50kHz. (20%)

   (a) What are the poles for the analog Butterworth prototype filter $H_A(s)$? (10%)

   (b) Derive the transfer function (Z-transform) for this digital IIR filter. (10%)

Solution to Problem 3:

(a) $|H_A(\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{2000\pi}\right)^{2N}}$.

$-20 = 20\log_1 0(\delta_s)$.

$\Rightarrow \delta_s = 10^{-1} = 0.1$.

$\Rightarrow |H_A(\Omega_s)|^2 = \frac{1}{1+(5)^{2N}} \leq 10^{-2}$.

$\Rightarrow 10^{-2} + 10^{-2} \times (25)^N \geq 1 \Rightarrow N \geq 2$.

(b) $H_A(s)H_A(-s) = \frac{1}{1+\left(\frac{s}{2000\pi}\right)^4}$.

poles: $s = \pm(1000\sqrt{2}\pi - j1000\sqrt{2}\pi)$.

$H_A(s) = \frac{1}{s^2-(2000\pi)^2}$.

If Euler Approximation is used, $H_D(z) = H_A(s)|_{s=50000(1-z^{-1})}$.

If the bilinear transformation is used, $H_D(z) = H_A(s)|_{s=100000\frac{z-1}{z+1}}$.

4. Determine the polyphase decompositions (M components) for the following FIR filters: (30%)

   (a) Z-transform $H(z) = 2 - 5z^{-1} + 2z^{-2} + 10z^{-201}$, $M = 3$. (15%)

   (b) Z-transform $H(z) = \frac{z-3}{(z+0.2)(z-0.5)}$, $M = 2$. (15%)

Solution to Problem 4:

(a) Causal: $H_0(z^3) = 2 - 10z^{-201}$, $H_1(z^3) = -5$, $H_2(z^3) = 2$.

Non-Causal: $H_0(z^3) = 2 - 10z^{-201}$, $H_{-1}(z^3) = 2z^{-3}$, $H_{-2}(z^3) = -5z^{-3}$.

(b)

$$H(z) = 30 - \frac{160}{7}\frac{z}{z+0.2} - \frac{50}{7}\frac{z}{z-0.5}.$$

Impulse response $h[n] = 30\delta[n] - \frac{160}{7}(-0.2)^n\mu[n] - \frac{50}{7}(0.5)^n\mu[n]$, where $\mu[n]$ is the unite step sequence.

$$\begin{aligned} E_0(z) &= \sum_{n=0}^{\infty} h[2n]z^{-n} \\ &= 30 - \frac{160}{7}\frac{z}{z-0.04} - \frac{50}{7}\frac{z}{z-0.25} \\ E_1(z) &= \sum_{n=0}^{\infty} h[2n+1]z^{-n} \\ &= \frac{32}{7}\frac{z}{z-0.04} - \frac{25}{7}\frac{z}{z-0.25} \end{aligned} \tag{1}$$

$$\begin{aligned} E_0(z) &= \sum_{n=0}^{\infty} h[2n]z^{-n} \\ &= 30 - \frac{160}{7}\frac{z}{z-0.04} - \frac{50}{7}\frac{z}{z-0.25} \\ E_{-1}(z) &= \sum_{n=0}^{\infty} h[2n-1]z^{-n} \\ &= \frac{32}{7}\frac{1}{z-0.04} - \frac{25}{7}\frac{1}{z-0.25} \end{aligned} \tag{2}$$