EE7000 Advanced Digital Signal Processing for Wireless Communications

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Midterm Examination 1, Fall of 2001

Time: 12:40 p.m. -- 1:30 p.m., Wednesday, October 17 of 2001

Please ALWAYS work on the FRONT SIDE of each page. No answer on the BACK SIDE of any page will be GRADED! Ask for additional blank papers if necessary!

You may check any textbook, classnote or other references alone during the test. However, NO CHEATING or COLLABORATION is allowed and such violation of the university regulations will be reported.

Please write down you name and social security number here:

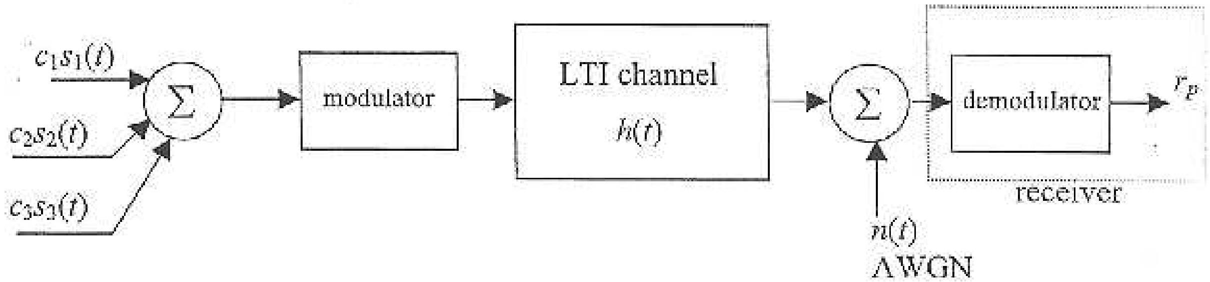
Full Name: Solution

Social Security Number: ______________________

Partition	Score
Question 1	
Question 2	
Total	

Question 1: (60%)

A Multi-access BPSK communication system is depicted as below. $n(t)$ is an additive



white Gaussian channel noise (zero mean and variance σ^2) and the transmitted signal $s_k(t)$ is a BPSK rectangular pulse train such that

$$s_k(t) = \sum_{i=-\infty}^{\infty} m_{k,i}\, p(t - iT_b), \text{ where } m_{k,i} = \pm 1 \text{ and } p(t) = \begin{cases} 1, & 0 \le t \le T_b \\ 0, & elsewhere \end{cases}.$$

k=1,2,3 is the user index. The channel is assumed to be distortionless, i.e., $h(t) = \delta(t)$.

We define two hypotheses for each user k here:

$H_{-c_k k}$: Hypothesis for the negative pulse to be sent by user k.

$H_{c_k k}$: Hypothesis for the positive pulse to be sent by user k.

(a) What are the six a priori conditional probability density functions

$f_{R|H_{-c_k k}}(r \mid H_{-c_k k})$ and $f_{R|H_{c_k k}}(r \mid H_{c_k k})$, k=1,2,3? (30%)

(b) According to a single decision rule $r \underset{\hat{m}=-1}{\overset{\hat{m}=1}{\gtrless}} 0$, if the error probability for user k is $P_{e,k}$

and the average user error probability $P_e = \frac{1}{3}\sum_{k=1}^{3} P_{e,k}$, what is P_e in terms of c_k's and

ϕ function (or Q functions)? (30%)

Answer to Question 1:

(a)

$$r_{k,p} = C_k S_{k,p} + C_i S_{i,p} + C_j S_{j,p} + n_p$$

$$f_{IR|H_{-C_k,k}}(r|H_{-C_k,k}) = \delta(r + C_k) \circledast \left[\frac{1}{2}\delta(r + C_i) + \frac{1}{2}\delta(r - C_i)\right] \circledast \left[\frac{1}{2}\delta(r + C_j) + \frac{1}{2}\delta(r - C_j)\right] \circledast \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{r^2}{2\sigma^2}}$$

$$= \frac{1}{4}\frac{1}{\sqrt{2\pi\sigma^2}}\left[e^{-\frac{(r+C_k+C_i+C_j)^2}{2\sigma^2}} + e^{-\frac{(r+C_k+C_i-C_j)^2}{2\sigma^2}} + e^{-\frac{(r+C_k-C_i+C_j)^2}{2\sigma^2}} + e^{-\frac{(r+C_k-C_i-C_j)^2}{2\sigma^2}}\right]$$

Similarly,

$$f_{IR|H_{C_k,k}}(r|H_{C_k,k}) = \delta(r - C_k) \circledast \left[\frac{1}{2}\delta(r + C_i) + \frac{1}{2}\delta(r - C_i)\right] \circledast \left[\frac{1}{2}\delta(r + C_j) + \frac{1}{2}\delta(r - C_j)\right] \circledast \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{r^2}{2\sigma^2}}$$

$$= \frac{1}{4}\frac{1}{\sqrt{2\pi\sigma^2}}\left[e^{-\frac{(r-C_k+C_i+C_j)^2}{2\sigma^2}} + e^{-\frac{(r-C_k+C_i-C_j)^2}{2\sigma^2}} + e^{-\frac{(r-C_k-C_i+C_j)^2}{2\sigma^2}} + e^{-\frac{(r-C_k-C_i-C_j)^2}{2\sigma^2}}\right]$$

$$\therefore\ f_{IR|H_{-C_1,1}}(r|H_{-C_1,1}) = \frac{1}{4}\frac{1}{\sqrt{2\pi\sigma^2}}\left[e^{-\frac{(r+C_1+C_2+C_3)^2}{2\sigma^2}} + e^{-\frac{(r+C_1+C_2-C_3)^2}{2\sigma^2}} + e^{-\frac{(r+C_1-C_2+C_3)^2}{2\sigma^2}} + e^{-\frac{(r+C_1-C_2-C_3)^2}{2\sigma^2}}\right]$$

$$f_{IR|H_{C_1,1}}(r|H_{C_1,1}) = \frac{1}{4}\frac{1}{\sqrt{2\pi\sigma^2}}\left[e^{-\frac{(r-C_1+C_2+C_3)^2}{2\sigma^2}} + e^{-\frac{(r-C_1+C_2-C_3)^2}{2\sigma^2}} + e^{-\frac{(r-C_1-C_2+C_3)^2}{2\sigma^2}} + e^{-\frac{(r-C_1-C_2-C_3)^2}{2\sigma^2}}\right]$$

$$f_{R|H_{-c_2,2}}(r \mid H_{-c_2,2}) = \frac{1}{4}\frac{1}{\sqrt{2\pi\sigma^2}}\Big[e^{-\frac{(r+c_2+c_1+c_3)^2}{2\sigma^2}} + e^{-\frac{(r+c_2+c_1-c_3)^2}{2\sigma^2}} + e^{-\frac{(r+c_2-c_1+c_3)^2}{2\sigma^2}} + e^{-\frac{(r+c_2-c_1-c_3)^2}{2\sigma^2}} \Big]$$

$$f_{R|H_{c_2,2}}(r \mid H_{c_2,2}) = \frac{1}{4}\frac{1}{\sqrt{2\pi\sigma^2}}\Big[e^{-\frac{(r-c_2+c_1+c_3)^2}{2\sigma^2}} + e^{-\frac{(r-c_2+c_1-c_3)^2}{2\sigma^2}} + e^{-\frac{(r-c_2-c_1+c_3)^2}{2\sigma^2}} + e^{-\frac{(r-c_2-c_1-c_3)^2}{2\sigma^2}} \Big]$$

$$f_{R|H_{-c_3,3}}(r \mid H_{-c_3,3}) = \frac{1}{4}\frac{1}{\sqrt{2\pi\sigma^2}}\Big[e^{-\frac{(r+c_3+c_1+c_2)^2}{2\sigma^2}} + e^{-\frac{(r+c_3+c_1-c_2)^2}{2\sigma^2}} + e^{-\frac{(r+c_3-c_1+c_2)^2}{2\sigma^2}} + e^{-\frac{(r+c_3-c_1-c_2)^2}{2\sigma^2}} \Big]$$

$$f_{R|H_{c_3,3}}(r \mid H_{c_3,3}) = \frac{1}{4}\frac{1}{\sqrt{2\pi\sigma^2}}\Big[e^{-\frac{(r-c_3+c_1+c_2)^2}{2\sigma^2}} + e^{-\frac{(r-c_3+c_1-c_2)^2}{2\sigma^2}} + e^{-\frac{(r-c_3-c_1+c_2)^2}{2\sigma^2}} + e^{-\frac{(r-c_3-c_1-c_2)^2}{2\sigma^2}} \Big]$$

(b)

$$P_{e,k} = \frac{1}{2}\int_0^{\infty} f_{R|H_{-c_k,k}}(r \mid H_{-c_k,k})\, dr + \frac{1}{2}\int_{-\infty}^{0} f_{R|H_{c_k,k}}(r \mid H_{c_k,k})\, dr$$

[illegible] $\frac{1}{4}$

$$= \frac{1}{2}\Big[\phi\Big(\frac{-c_k-c_i-c_j}{\sigma}\Big) + \phi\Big(\frac{-c_k-c_i+c_j}{\sigma}\Big) + \phi\Big(\frac{-c_k+c_i-c_j}{\sigma}\Big) + \phi\Big(\frac{-c_k+c_i+c_j}{\sigma}\Big) \Big]$$

$$+ \frac{1}{2}\Big[\phi\Big(\frac{-c_k+c_i+c_j}{\sigma}\Big) + \phi\Big(\frac{-c_k+c_i-c_j}{\sigma}\Big) + \phi\Big(\frac{-c_k-c_i+c_j}{\sigma}\Big) + \phi\Big(\frac{-c_k-c_i-c_j}{\sigma}\Big) \Big]$$

$$= \phi\left(\frac{-C_k - C_i - C_j}{\sigma}\right) + \phi\left(\frac{-C_k - C_i + C_j}{\sigma}\right) + \phi\left(\frac{-C_k + C_i - C_j}{\sigma}\right) + \phi\left(\frac{-C_k + C_i + C_j}{\sigma}\right)$$

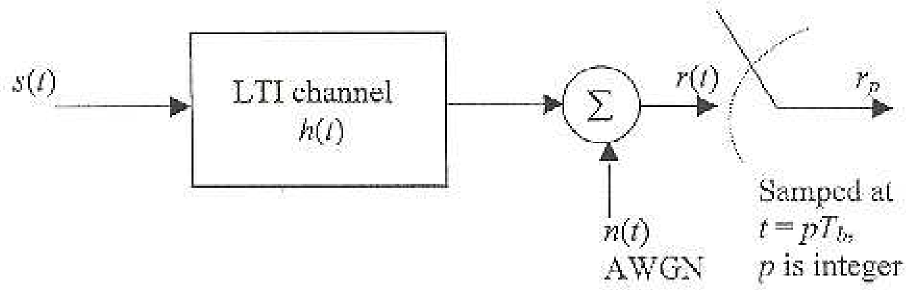
$$P_e = \frac{1}{3}\Big[3\phi\left(\frac{-C_1 - C_2 - C_3}{\sigma}\right) + 2\phi\left(\frac{-C_1 - C_2 + C_3}{\sigma}\right) + 2\phi\left(\frac{-C_1 + C_2 - C_3}{\sigma}\right) + \phi\left(\cancel{\frac{-C_1 + C_2 + C_3}{\sigma}}\right) + 2\phi\left(\frac{-C_2 + C_1 - C_3}{\sigma}\right) + \phi\left(\cancel{\frac{-C_3 + C_1 + C_2}{\sigma}}\right) + \phi\left(\cancel{\frac{-C_2 + C_1 + C_3}{\sigma}}\right)\Big]$$

$$= \frac{1}{3}\Big[3\phi\left(\frac{-C_1 - C_2 - C_3}{\sigma}\right) + 1 + \phi\left(\frac{-C_1 - C_2 + C_3}{\sigma}\right) + 1 + \phi\left(\frac{-C_1 + C_2 - C_3}{\sigma}\right) + 1 + \phi\left(\frac{-C_2 + C_1 - C_3}{\sigma}\right)\Big]$$

$$= \phi\left(\frac{-C_1 - C_2 - C_3}{\sigma}\right) + 1 + \frac{1}{3}\Big[\phi\left(\frac{-C_1 - C_2 + C_3}{\sigma}\right) + \phi\left(\frac{-C_1 + C_2 - C_3}{\sigma}\right) + \phi\left(\frac{C_1 - C_2 - C_3}{\sigma}\right)\Big]$$

Question 2: (40%)

A single-access BPSK communication system is depicted as below. $n(t)$ is an additive



white Gaussian channel noise (zero mean and variance σ^2) and the transmitted signal $s(t)$ is a BPSK rectangular pulse train such that

$$s(t) = \sum_{i=-\infty}^{\infty} m_i p(t - iT_b), \text{ where } m_i = \pm 1 \text{ and } p(t) = \begin{cases} 1, & 0 \le t \le T_b \\ 0, & elsewhere \end{cases}.$$

The channel impulse response can be modeled as

$$h(t) = \begin{cases} \delta(t - 0.3T_b) + 0.6e^{-t/T_b}, & 0 \le t \le 2T_b \\ 0, & otherwise \end{cases}.$$

We define two hypotheses for here:

H_0: Hypothesis for the negative pulse to be sent.

H_1: Hypothesis for the positive pulse to be sent.

(a) What is the priori conditional probability density functions $f_{R|H_0}(r \mid H_0)$ and $f_{R|H_1}(r \mid H_1)$? (20%)

(b) What is the error probability according to a single decision rule $r \underset{\hat{m}=-1}{\overset{\hat{m}=1}{\gtrless}} 0$? (20%)

Answer to Question 2:

(a)

$$C_\ell = \int_{\ell T_b}^{(\ell+1)T_b} h(\tau)\, d\tau$$

$$C_0 = \int_0^{T_b} h(\tau)\, d\tau$$

$$= \int_0^{T_b} \delta(t - 0.3T_b)\, dt + 0.6 \int_0^{T_b} e^{-\frac{t}{T_b}}\, dt$$

$$= 1 - 0.6\, T_b\, e^{-\frac{t}{T_b}} \Big|_0^{T_b}$$

$$= 1 + 0.6T_b - 0.6T_b/e$$

$$C_1 = \int_{T_b}^{2T_b} h(\tau)\, d\tau$$

$$= 0.6 \int_{T_b}^{2T_b} e^{-\frac{t}{T_b}}\, dt = -0.6\, T_b\, e^{-\frac{t}{T_b}} \Big|_{T_b}^{2T_b}$$

$$= 0.6T_b/e - 0.6T_b/e^2$$

$r_p = C_0 S_p + C_1 S_{p-1} =$ [illegible]

$$f_{R|H_0}(r|H_0) = \delta(r + C_0) \otimes \left[\frac{1}{2}\delta(r + C_1) + \frac{1}{2}\delta(r - C_1)\right]$$

$$\otimes \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{r^2}{2\sigma^2}}$$

$$= \frac{1}{2}\frac{1}{\sqrt{2\pi\sigma^2}}\left[e^{-\frac{(r+C_0+C_1)^2}{2\sigma^2}} + e^{-\frac{(r+C_0-C_1)^2}{2\sigma^2}}\right]$$

$=$

$$f_{R|H_1}(r|H_1) = \delta(r-c_0) \circledast \left[\frac{1}{2}\delta(r+c_1) + \frac{1}{2}\delta(r-c_1)\right]$$

$$\circledast \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{r^2}{2\sigma^2}}$$

$$= \frac{1}{2}\frac{1}{\sqrt{2\pi\sigma^2}}\left[e^{-\frac{(r-c_0+c_1)^2}{2\sigma^2}} + e^{-\frac{(r-c_0-c_1)^2}{2\sigma^2}}\right]$$

(b)

$$P_e = \frac{1}{2}\int_{-\infty}^{0} f_{R|H_1}(r|H_1)\,dr + \frac{1}{2}\int_{0}^{\infty} f_{R|H_0}(r|H_0)\,dr$$

$$= \frac{1}{2}\left[\phi\left(\frac{-c_0+c_1}{\sigma}\right) + \phi\left(\frac{-c_0-c_1}{\sigma}\right)\right]$$

$$+ \frac{1}{2}\left[\phi\left(\frac{-c_0-c_1}{\sigma}\right) + \phi\left(\frac{-c_0+c_1}{\sigma}\right)\right]$$

$$= \phi\left(\frac{-c_0+c_1}{\sigma}\right) + \phi\left(\frac{-c_0-c_1}{\sigma}\right)$$