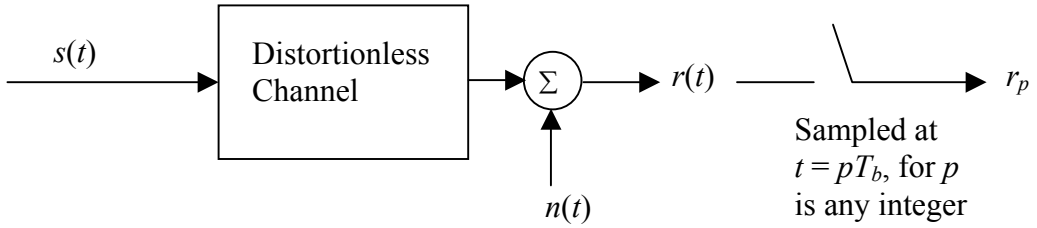
EE7000 Advanced Digital Signal Processing for Wireless Communications

Homework 1

Due on February 17, 2003, by 11:40 am. (NO LATE SUBMISSION IS ALLOWED!)

1. A baseband BPSK communication system is depicted as below. $n(t)$ is an additive



white Gaussian channel noise and the transmitted signal $s(t)$ is a BPSK rectangular pulse train such that

$$s(t) = \sum_{i=-\infty}^{\infty} m_i\, p(t - iT_b), \text{ where } m_i = \pm 1 \text{ and } p(t) = \begin{cases} 1, & 0 \le t \le T_b \\ 0, & elsewhere \end{cases}.$$

(a) If m_i is a random i. i. d. information process, prove r_p is also i. i. d.

(b) If m_i consists of 3-bit code words $[m_{3k}\ m_{3k+1}\ m_{3k+2}]$, and m_{3k+2} is the parity check, which will be modulo-2 sum of m_{3k} and m_{3k+1}, is r_p i. i. d.? Justify your conclusion.

2. If we replace the distortionless channel in Problem 1, by a linear time-invariant channel $h(t) = \delta(t) + 0.8\delta(t - 3.73T_b)$, and m_i is a random i. i. d. information process, is r_p still i. i. d.? Justify your conclusion.

3. In problem 1(a), if we make a decision according to the following decision rule

$$r_p \underset{\hat{m} = -1}{\overset{\hat{m} = 1}{\gtrless}} 0,$$

what is the error probability associated with this decision rule in terms of ϕ function?

4. In problem 2, if we make a decision according to the following decision rule

$$r_p \underset{\hat{m} = -1}{\overset{\hat{m} = 1}{\gtrless}} 0,$$

what is the error probability associated with this decision rule in terms of ϕ function?

5. A discrete-time received sequence can be described as $r_i = \sum_{k=0}^{N} a_k m_{i-k} + n_i$, where $m_i = \pm 1$ is the equally probable i.i.d. BPSK signal and n_i is the zero-mean white Gaussian noise with variance σ^2.
   (a) What is the characteristic function $\psi_{R_i}(j\omega)$ for r_i?
   (b) What is $E\{r_i^4\}$ in terms of a_k's and σ^2?

6. A continuous-time received signal can be described as $r(t) = s(t) \otimes h(t) + n(t)$, where $s(t)$ is the modulated transmitted signal, $n(t)$ is AWGN and $h(t) = \delta(t) + 0.8\delta(t - 3T_b)$.
   (a) What is the Fourier Transform of a perfect equalizer for this channel $h(t)$?
   (b) What is the power spectral density for $r(t)$ if $s(t)$ is modulated by QPSK?
   (c) What is the noise energy amplification factor due to the perfect equalizer $w(t)$ within the QPSK effective bandwidth?