EE7600-2 Adaptive Filter Theory

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Midterm Examination, Fall of 2005

Time: 1:40 p.m. ~ 2:30 p.m., Wednesday, October 19 of 2005

Please ALWAYS work on the FRONT SIDE of each page. No answer on the BACK SIDE of any page will be GRADED! Ask for additional blank papers if necessary!

You may check any textbook, classnote or other references alone during the test. However, NO CHEATING or COLLABORATION is allowed and such violation of the university regulations will be reported.

Please write down you name and social security number here:

Full Name: ________________**SOLUTION**__________________

Social Security Number: ____

Partition	Score
Question 1	
Question 2	
Total	

<u>Question 1</u>: (45%)

A stationary real-valued signal $u(n)$ is generated by an ARMA process such that

$$u(n) = -0.2u(n-1) + 0.5v(n) + v(n-1),$$

where $v(n)$ is a real-valued, zero-mean white process with the variance σ_v^2. Determine $r(k) = E\{u(n)u(n-k)\}$, $k = 0,1,2$.

Answer to Question 1:

$$r(0) = -0.2r(1) + 1.25\sigma_v^2 - 0.1\sigma_v^2$$

$$r(0) = 0.04r(0) + 1.25\sigma_v^2 - 0.2\sigma_v^2 \Rightarrow r(0) = \frac{35}{32}\sigma_v^2 = 1.0938\sigma_v^2$$

$$r(1) = -0.2r(0) + 0.5\sigma_v^2 \Rightarrow r(1) = \frac{9}{32}\sigma_v^2 = 0.2812\sigma_v^2$$

$$r(2) = -0.2r(1) = -\frac{9}{160}\sigma_v^2 = -0.0563\sigma_v^2$$

<u>Question 2</u>: (55%)

A stationary real-valued signal $u(n)$ is generated by an ARMA process such that

$$u(n) = -0.2u(n-1) + 0.5v(n) + v(n-1) \, ,$$

where $v(n)$ is a real-valued, zero-mean white process with the variance σ_v^2. Design an adaptive filter to minimize the mean square error such that

$$y(n) = \sum_{k=0}^{M-1} w_{o,k} \, v(n-k) = \vec{w}_o^T \vec{v}(n)$$ and the desired signal is $u(n)$,

where $\vec{w}_o = \begin{bmatrix} w_{o,0} & w_{o,1} & \cdots & w_{o,M-1} \end{bmatrix}$ and $\vec{v}(n) = \begin{bmatrix} v(n) & v(n-1) & \cdots & v(n-M+1) \end{bmatrix}^T$.

(a) Determine the minimum mean square error in terms of σ_v^2 for M=1. (15%)

(b) Determine the minimum mean square error in terms of σ_v^2 for M=2? (20%)

(c) What is the minimum model order M to reach the optimum mean square error? (20%)

<u>Answer to Question 2</u>:

(a) M=1, $r_v(0)=\sigma_v^2$, $p(0)=0.5\sigma_v^2$, $J_{\min}=\frac{35}{32}\sigma_v^2-0.5\sigma_v^2\frac{0.5\sigma_v^2}{\sigma_v^2}=\frac{27}{32}\sigma_v^2=0.8438\sigma_v^2$

(b) M=2, $\bar{w}_o=\begin{bmatrix}\sigma_v^2 & 0\\ 0 & \sigma_v^2\end{bmatrix}^{-1}\begin{bmatrix}0.5\sigma_v^2\\ 0.9\sigma_v^2\end{bmatrix}=\begin{bmatrix}0.5\\ 0.9\end{bmatrix}$,

$$J_{\min}=\frac{35}{32}\sigma_v^2-\begin{bmatrix}0.5\sigma_v^2\\ 0.9\sigma_v^2\end{bmatrix}^T\begin{bmatrix}0.5\\ 0.9\end{bmatrix}=0.0337\sigma_v^2$$

(c) Since the input $u(n)$ is an ARMA process,

$$U(z)=\frac{0.5+z^{-1}}{1+0.2z^{-1}}V(z)+I(z),$$

where $U(z)=Z\{u(n)\}$, $V(z)=Z\{v(n)\}$ and $I(z)$ is the response due to the initial conditions. Then, $u(n)=\sum_{k=0}^{\infty}0.5(-0.2)^k v(n-k)+\sum_{k=1}^{\infty}(-0.2)^k v(n-k)+i(n)$, where $i(n)=Z^{-1}\{I(z)\}$. Hence the mean-square error will keep decreasing as the model M increases. There is no minimum model order to achieve the optimality.