EE7000-6 Adaptive Filter Theory

Quiz 3

SOLUTION

12:40~1:30 p.m., November 21, 2004

The autocorrelation function $r(m)$ of a stationary real-valued input signal in an M^{th}-order adaptive filter, can be formulated as

$$r(m) = \frac{1}{2}\left(\left(-\frac{1}{2}\right)^m + \left(\frac{1}{2}\right)^m\right),\ m = 0,1,2,\cdots.$$

Assume that the desired signal and filter coefficients are also real-valued and $\sigma_d^2 = 1$. The cross-correlation function $p(-k)$ between the input and desired signals can be formulated as

$$p(-k) = \frac{1}{2}\left(\left(-\frac{1}{2}\right)^{k+1} + \left(\frac{1}{2}\right)^{k+1}\right),\ \ k = 0,1,2,\cdots.$$

An LMS algorithm is designed with the step size $\mu << 1$.

(a) What is the misadjustment $\mathcal{M}$ in terms of μ and M? (20%)

(b) What is the condition of μ for both $J(\infty)$ and $J_{\min}$ to reach their minima among different M=1, 2, 3,…, at the same specific model order? (40%)

(c) What is the minimum asymptotical MSE value $J(\infty)$ in terms of μ and the corresponding model order M when the condition in (b) is satisfied? (20%)

(d) If the model order M is chosen as (c), what is the asymptotic MSD value $D(\infty)$ in terms of μ? (20%)

(a) $\mathcal{M} = \frac{\mu}{2} tr(\tilde{R}) = \frac{\mu M}{2}$

(b) $J(\infty) = J_{\min} + \frac{\mu J_{\min}}{2} tr(\tilde{R})$

$$J_{\min} = \sigma_d^2 - \vec{P}^T \tilde{R}_M^{-1} \vec{P}. \quad R_1^{-1} = 1, \ R_2^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ R_3^{-1} = \begin{bmatrix} \frac{15}{16} & 0 & -\frac{4}{15} \\ 0 & 1 & 0 \\ -\frac{4}{15} & 0 & \frac{15}{16} \end{bmatrix}$$

$$M = 1, \ J_{\min} = 1; \ M = 2, \ J_{\min} = \frac{15}{16}; \ M = 3, \ J_{\min} = \frac{15}{16}$$

$$1 + \frac{\mu}{2} < \frac{15}{16} + \frac{15}{16}\mu \Rightarrow 0 < \mu < \frac{1}{7}.$$

(c)

When M=2, $J(\infty) = \frac{15}{16}(1+\mu)$.

(d) $D(\infty) = \lim_{n\to\infty} D(n) = \lim_{n\to\infty} \sum_{k=1}^{2} E\{v_k(n)\} = \sum_{k=1}^{2} \frac{\mu J_{\min}}{2} = \frac{15}{16}\mu$