EE7000-6 Adaptive Filter Theory

Quiz 2 Solution

12:40~1:30 p.m., September 29, 2004

The autocorrelation function $r(m)$ of a stationary real-valued input signal in an M^{th}-order adaptive filter, can be formulated as

$$r(m) = \frac{1}{2}\left(\left(-\frac{1}{2}\right)^m + \left(\frac{1}{2}\right)^m\right),\ m = 0,1,2,\cdots.$$

Assume that the desired signal and filter coefficients are also real-valued. The cross-correlation function $p(-k)$ between the input and desired signals can be formulated as

$$p(-k) = \frac{1}{2}\left(\left(-\frac{1}{2}\right)^k + \left(\frac{1}{2}\right)^k\right),\ \ k = 1,2,\cdots.$$

(a) Use the recursive formula, to compute the inverse correlation matrix R_3^{-1} (M=3) form the inverse correlation matrix R_2^{-1} (M=2) numerically. (30%)

(b) Compute the numerical minimum mean-square errors for M=1, 2, 3. (45%)

(c) What is the minimum model order to achieve the optimum mean-square error? Justify your answer with reasoning. (25%)

(a) $\tilde{R}_{M+1}^{-1} = \begin{bmatrix} 0 & \vec{0}^T \\ \vec{0} & \tilde{R}_M^{-1} \end{bmatrix} + \frac{1}{r(0) - \vec{r}^{\,H} \tilde{R}_M^{-1} \vec{r}} \begin{bmatrix} 1 \\ -\tilde{R}_M^{-1} \vec{r} \end{bmatrix} \begin{bmatrix} 1 & -\vec{r}^{\,H} \tilde{R}_M^{-1} \end{bmatrix}.$

$\tilde{R}_2^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \ \vec{r} = \begin{bmatrix} 0 & \frac{1}{4} \end{bmatrix}^T$

$$\tilde{R}_3^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{1}{1 - \frac{1}{16}} \begin{bmatrix} 1 \\ 0 \\ -\frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 & 0 & -\frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{16}{15} & 0 & -\frac{4}{15} \\ 0 & 1 & 0 \\ -\frac{4}{15} & 0 & \frac{16}{15} \end{bmatrix}$$

$$= \begin{bmatrix} 1.0667 & 0 & -0.2667 \\ 0 & 1 & 0 \\ -0.2667 & 0 & 1.0667 \end{bmatrix}$$

(b) Assume $\sigma_d^2 = E\{d^2(n)\}$. Then

MSE for the M^{th}-order adaptive filter is

M=1, $\vec{P} = 1, J_{\min} = \sigma_d^2 - 1$

M=2, $\vec{P} = \begin{bmatrix} 1 & 0 \end{bmatrix}^T, J_{\min} = \sigma_d^2 - 1$

M=3, $\vec{P} = \begin{bmatrix} 1 & 0 & \frac{1}{4} \end{bmatrix}^T, J_{\min} = \sigma_d^2 - \begin{bmatrix} 1 & 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} \frac{16}{15} & 0 & -\frac{4}{15} \\ 0 & 1 & 0 \\ -\frac{4}{15} & 0 & \frac{16}{15} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \frac{1}{4} \end{bmatrix} = \sigma_d^2 - 1$

(c) Among M=1, 2, 3, M=1 is optimal.