<u>EE7000-6 Adaptive Filter Theory</u>

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Midterm Examination, Fall of 2004

Time: 12:40 p.m. ~ 1:30 p.m., Friday, November 5 of 2004

Please ALWAYS work on the FRONT SIDE of each page. No answer on the BACK SIDE of any page will be GRADED! Ask for additional blank papers if necessary!

You may check any textbook, classnote or other references alone during the test. However, NO CHEATING or COLLABORATION is allowed and such violation of the university regulations will be reported.

Please write down you name and social security number here:

Full Name: ______________SOLUTION_______________

Social Security Number: ____

Partition	Score
Question 1	
Question 2	
Total	

Question 1: (50%)

The autocorrelation function $r(m)$ of a stationary real-valued signal $u(n)$ can be formulated as

$$r(m) = \frac{1}{2}\left(\left(-\frac{1}{2}\right)^m + \left(\frac{1}{2}\right)^m\right),\ m = 0,1,2,\cdots.$$

An m^{th}-order, L-step linear prediction-error filter is to be established such that

$$f_{m,L}(n) = u(n) + \sum_{k=L}^{m+L-1} a_{m,k}^{*} u(n-k).$$

(a) Write the Levinson-Durbin algorithm (without the matrix inversion) for this L-step linear prediction-error filter. Hint: Define the L-step linear prediction-error filter coefficients as a vector: $\vec{a}_m \equiv \begin{bmatrix} 1 & \underbrace{0\cdots0}_{(L-1)\ zeros} & a_{m,L} & a_{m,L+1} & \cdots & a_{m,m+L-1} \end{bmatrix}^T$. (20%)

(b) Using the Levinson-Durbin algorithm, compute the numerical value of the reflection coefficient k_3 and the numerical value of the minimum mean-squared prediction error P_3 when L=3. (20%)

(c) What is the minimum order m to achieve the optimum mean-squared prediction error when L=3? (10%)

Answer to Question 1:

(a) Define the $\vec{a}_m \equiv \begin{bmatrix} 1 & \underbrace{0\cdots0}_{(L-1)\ zeros} & a_{m,L} & a_{m,L+1} & \cdots & a_{m,m+L-1} \end{bmatrix}^T$

and $\vec{r}_m \equiv \begin{bmatrix} r^*(1) & \cdots & r^*(m) \end{bmatrix}^T$.

(i) initialization: $\vec{a}_0 = \begin{bmatrix} 1 & \underbrace{0\cdots0}_{(L-1)\ zeros} \end{bmatrix}^T$, $P_0 = r(0)$.

(ii) m=1, 2, …

$$\Delta_{m-1} = \vec{r}_{m+L-1}^{\,BT} \vec{a}_{m-1}$$

$$k_m = -\frac{\Delta_{m-1}}{P_{m-1}}$$

$$P_m = P_{m-1} + k_m \Delta_{m-1}^*$$

$$a_{m,l} = \begin{cases} a_{m-1,l} + k_m a_{m-1,m+L-l}^*, & l = 0, L, L+1, \cdots, m+L-2 \\ 0, & l = 1, \cdots, L-1 \end{cases}$$

$$a_{m,m+L-1} = k_m$$

(b) $P_0 = r(0) = 1$; $\vec{a}_0 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$

m=1,

$$\Delta_0 = \vec{r}_3^{\,BT} \vec{a}_0 = r(3) = 0.$$

$$k_1 = -\frac{\Delta_0}{P_0} = 0$$

$$P_1 = P_0 + k_1 \Delta_0^* = 1$$

$$a_{1,3} = k_1 = 0$$

$$\vec{a}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$$

m=2,

$$\Delta_1 = \vec{r}_4^{\,BT} \vec{a}_1 = r(4) = \frac{1}{16} = 0.0625$$

$$k_2 = -\frac{\Delta_1}{P_1} = -\frac{1}{16} = -0.0625$$

$$P_2 = P_1 + k_2\Delta_1^* = 1 - \frac{1}{256} = 0.9961$$

$$a_{2,3} = a_{1,3} + k_2 a_{1,2}^* = 0$$

$$a_{2,4} = k_2 = -0.0625$$

$$\vec{a}_2 = \begin{bmatrix}1 & 0 & 0 & 0 & -0.0625\end{bmatrix}^T$$

m=3,

$$\Delta_2 = \vec{r}_5^{\,BT}\,\vec{a}_2 = r(5) - \frac{1}{256} r(1) = 0$$

$$k_3 = -\frac{\Delta_2}{P_2} = 0$$

$$P_3 = P_2 + k_3\Delta_2^* = P_2 = 0.9961$$

$$a_{3,3} = a_{2,3} + k_3 a_{2,3}^* = 0$$

$$a_{3,4} = a_{2,4} + k_3 a_{2,2}^* = -0.0625$$

$$a_{3,5} = k_3 = 0$$

$$\vec{a}_3 = \begin{bmatrix}1 & 0 & 0 & 0 & -0.0625 & 0\end{bmatrix}^T$$

$$\therefore k_3 = 0, \quad P_3 = \frac{255}{256} = 0.9961$$

(c) m=2 is the minimum order to achieve the optimum mean-squared prediction error.

Question 2: (50%)

A stationary real-valued signal $u(n)$ is generated by an ARMA process such that

$$u(n) = -0.2u(n-1) + 0.15u(n-2) + 0.5v(n) + v(n-1),$$

where $v(n)$ is a real-valued, zero-mean white process with the variance σ_v^2. An M^{th}-order one-step linear forward prediction-error filter is established.

(a) Determine the minimum model order to achieve the optimal prediction error and the corresponding prediction-error filter coefficients. (30%)

(b) What is the minimum mean-squared prediction error in terms of σ_v^2? (20%)

Answer to Question 2:

(a) $S_{ff}(\omega) = S_{uu}(\omega)\left|H_{f,M}(e^{j\omega})\right|$ needs to be whitened

where $S_{uu}(\omega) = 2\pi\sigma_v^2\left|\left[G(z)\big|_{z=e^{j\omega}}\right]\right|^2$ and $G(z) = \dfrac{0.5+z^{-1}}{1+0.2z^{-1}-0.15z^{-2}}$.

$$G(z) = \frac{0.5+z^{-1}}{1+0.2z^{-1}-0.15z^{-2}} = \frac{(0.5+z^{-1})}{(1+0.5z^{-1})}\frac{(1+0.5z^{-1})}{(1+0.5z^{-1})(1-0.3z^{-1})}$$

$$= \frac{(0.5+z^{-1})}{(1+0.5z^{-1})}\frac{1}{(1-0.3z^{-1})}$$

Since $\dfrac{(0.5+z^{-1})}{(1+0.5z^{-1})}$ is all-pass transfer function,

$$S_{uu}(\omega) = 2\pi\sigma_v^2\left|\left[\frac{1}{(1-0.3z^{-1})}\bigg|_{z=e^{j\omega}}\right]\right|^2 \Rightarrow H_{f,M}(z) = 1-0.3z^{-1}.$$

M=1, and $a_{1,0} = 1$, $a_{1,1} = -0.3$.

(b) $P_M = \int_{-\pi}^{\pi} S_{uu}(\omega)\left|H_{f,M}(e^{j\omega})\right|^2 \dfrac{d\omega}{2\pi} = \sigma_v^2$