EE7000 Adaptive Filter Theory

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Quiz 2, Fall of 2003

Time: 10:40 a.m. ~ 11:30 p.m., Wednesday, November 26 of 2003

Please ALWAYS work on the FRONT SIDE of each page. No answer on the BACK SIDE of any page will be GRADED! Ask for additional blank papers if necessary!

You may check any textbook, classnote or other references alone during the test. However, NO CHEATING or COLLABORATION is allowed and such violation of the university regulations will be reported.

Please write down you name and social security number here:

Full Name: _____SOLUTION_____

Social Security Number: _____

Given the values of an autocorrelation function $r(k)$ associated with the real-valued input sequence $u(n)$, an m^{th}-order one-step adaptive linear predictor is applied. $r(0)=3$, $r(1)=2$, $r(2)=1$, $r(3)=0.5$ ……

(a) What is the minimum mean-square error in terms of numerical values for m =2? (40%)

(b) What is the asymptotical mean square error $J(\infty)$ in terms of the step size μ for m=2? (25%)

(c) What is the misadjustment in terms of step size μ for m =4? (35%)

Solution:

(a) $\sigma_d^2 = r(0) = 3$, $\widetilde{R} = \begin{bmatrix} r(0) & r(1) \\ r(1) & r(0) \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$, $\vec{P} = [r(1) \quad r(2)]^T = [2 \quad 1]^T$

$$J_{\min} = \sigma_d^2 - \vec{P}^H \widetilde{R}^{-1} \vec{P} = 3 - [2 \quad 1] \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}^{-1} [2 \quad 1]^T = 3 - [2 \quad 1] \begin{bmatrix} \frac{3}{5} & -\frac{2}{5} \\ -\frac{2}{5} & \frac{3}{5} \end{bmatrix} [2 \quad 1]^T$$

$$= 3 - \begin{bmatrix} \frac{4}{5} & -\frac{1}{5} \end{bmatrix} [2 \quad 1]^T = 3 - \frac{7}{5} = \frac{8}{5}.$$

(b) $$J(\infty) = \lim_{n \to \infty} J(n) = J_{\min} + \mu J_{\min} \sum_{k=1}^{M} \frac{\lambda_k}{2 - \mu\lambda_k} \approx J_{\min} + \frac{\mu J_{\min}}{2} \sum_{k=1}^{M} \lambda_k$$

$$= J_{\min} + \frac{\mu J_{\min}}{2} trace(\widetilde{R}) = \frac{8}{5} + \frac{24\mu}{5}$$

(c) $M = \frac{\mu}{2} trace(\widetilde{R}) = 6\mu$