EE7000 Adaptive Filter Theory

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Quiz 1, Fall of 2003

Time: 10:40 a.m. ~ 11:30 p.m., Friday, October 24 of 2003

Please ALWAYS work on the FRONT SIDE of each page. No answer on the BACK SIDE of any page will be GRADED! Ask for additional blank papers if necessary!

You may check any textbook, classnote or other references alone during the test. However, NO CHEATING or COLLABORATION is allowed and such violation of the university regulations will be reported.

Please write down you name and social security number here:

Full Name: ______________SOLUTION______________

Social Security Number: ____

Given the values of an autocorrelation function $r(k)$ associated with the real-valued input sequence $u(n)$, an m^{th}-order linear predictor is applied. $r(0)=3$, $r(1)=2$, $r(2)=1$, $r(3)=0.5$ ……

(a) What is the optimal tap weight vector $\vec{w}_b$ of a one-step backward predictor (m=2) in terms of numerical values? (25%)

(b) What is the optimal one-step forward prediction-error filter vector $\vec{a}_3$ (m=3) in terms of numerical values? (35%)

(c) What are the numerical values of the set of reflection coefficients k_1, k_2, k_3 in (b)? (40%)

Solution:

(a)

$$\tilde{R} = \begin{bmatrix} r(0) & r(1) \\ r(1) & r(0) \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}, \ \vec{r}^{\,B*} = \begin{bmatrix} r(2) \\ r(1) \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

$$\vec{w}_b = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & -\frac{2}{5} \\ -\frac{2}{5} & \frac{3}{5} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} \\ \frac{4}{5} \end{bmatrix}$$

(b)

$$\tilde{R} = \begin{bmatrix} r(0) & r(1) & r(2) \\ r(1) & r(0) & r(1) \\ r(2) & r(1) & r(0) \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \ \vec{r} = \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix},$$

$$\vec{w}_f = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix} = \begin{bmatrix} \frac{5}{8} & -\frac{1}{2} & \frac{1}{8} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ \frac{1}{8} & -\frac{1}{2} & \frac{5}{8} \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix} = \begin{bmatrix} \frac{13}{16} \\ -\frac{1}{4} \\ \frac{1}{16} \end{bmatrix}$$

$$\vec{a}_3 = \begin{bmatrix} 1 & -\frac{13}{16} & \frac{1}{4} & -\frac{1}{16} \end{bmatrix}$$

(c) From (a), $\vec{a}_2 = \begin{bmatrix} 1 & -\frac{4}{5} & \frac{1}{5} \end{bmatrix}$. In addition, $\vec{a}_1 = \begin{bmatrix} 1 & -\frac{2}{3} \end{bmatrix}$.

$$k_m = a_{m,m} \Rightarrow k_1 = -\frac{2}{3}, \quad k_2 = \frac{1}{5}, \quad k_3 = -\frac{1}{16}$$