<u>EE4150 Digital Signal Processing</u>

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Midterm Examination, Spring of 2004

Time: 11:40 a.m. ~12:30 p.m., Monday, March 8, 2004

Please ALWAYS work on the FRONT SIDE of each page. No answer on the BACK SIDE of any page will be GRADED! Ask for additional blank papers if necessary!

You may check any textbook, classnote or other references alone during the test. However, NO CHEATING or COLLABORATION is allowed and such violation of the university regulations will be reported.

Please write down your name and social security number here:

Full Name: ________SOLUTION________________

Social Security Number: ____

Partition	Score
Question 1	
Question 2	
Question 3	
Total	

<u>Question 1</u> (30%)

Determine whether or not each of the following signals is periodic. In case a signal is periodic, specify its fundamental period.

(a) Discrete-time signal: $x(n) = \sin(7n)$, (10%)

(b) Discrete-time signal: $x(n) = \sin\left(\frac{2\pi n}{3}\right) + \cos\left(\frac{\pi n}{5}\right)$, (10%)

(c) Discrete-time signal: $\left|\cos\left(\frac{\pi n}{10}\right)\right|$, (10%)

Answer to Question 1:

(a) $7N = 2\pi m \Rightarrow N = \frac{2\pi m}{7}$ is always irrational for all m. Therefore ***x*(*n*) is aperiodic.**

(b) $\frac{2\pi N_1}{3} = 2\pi m_1 \Rightarrow N_1 = 3m_1$. Therefore $N_1 = 3$.

$\frac{\pi N_2}{5} = 2\pi m_2 \Rightarrow N_2 = 10m_2$. Therefore $N_2 = 10$.

$N = lcm(N_1, N_2) = 30$. ***x*(*n*) is periodic with period 30.**

(c) $\frac{\pi N}{10} = 2\pi m \Rightarrow N = 20m$. Therefore $N = 20$. However, the absolute value will reduce the period in half and hence ***x*(*n*) is periodic with period 10.**

<u>Question 2</u> (45%)

A system is described as follows:

$$y(n) = 3x(2n-1),$$

where $x(n)$, $y(n)$ are the input and output signals respectively.

(a) Is this system linear? (15%)

(b) Is this system time-variant? (15%)

(c) Is this system BIBO stable? (15%)

Answer to Question 2:

(a) Two input sequences $x_1(n)$ and $x_2(n)$ are assumed to generate the output sequences $y_1(n) = 3x_1(2n-1)$ and $y_2(n) = 3x_2(2n-1)$. For two arbitrary constants α, β, a linear combination $\alpha x_1(n) + \beta x_2(n)$ will lead to the output $y'(n)$ such that

$$y'(n) = 3\alpha x_1(2n-1) + 3\beta x_2(2n-1) = \alpha y_1(n) + \beta y_2(n).$$

Thus, **this system is linear.**

(b) First assume the system is time-invariant. Let's take a counterexample to verify the assumption.

Given an input sequence $x(n)$ such that

$$x(n) \to y(n) = 3x(2n-1) = \left\{\cdots, \underset{\uparrow}{3x(-1)}, 3x(1), 3x(3), \cdots\right\}.$$

Another input sequence is formed by the time-delayed version of $x(n)$ such that

$$x'(n) = x(n-1) = \left\{\cdots, x(-2), \underset{\uparrow}{x(-1)}, x(0), x(1), \cdots\right\} \text{ and}$$

$$x'(n) \to y'(n) = 3x'(2n-1) = \left\{\ldots, \underset{\uparrow}{3x(-2)}, 3x(0), 3x(2), \cdots\right\}.$$

Since $y'(n) \neq y(n-1)$, the assumption doesn't exist and hence **the system is time-variant.**

(c) Given the bounded input sequence $x(n)$ such that $|x(n)| \leq M_x, \quad \forall n$, the corresponding output sequence will be $y(n)$ such that

$$|y(n)| = 3|x(2n-1)| \leq 3M_x, \quad \forall n.$$

Hence **the system is BIBO stable.**

Question 3 (25%)

Two finite-duration sequences are given as follows:

$$x(n) = \left\{ -1, \underset{\uparrow}{1}, 0, 1 \right\}$$
$$h(n) = \left\{ \underset{\uparrow}{1}, 0, 1 \right\}.$$

Determine the discrete-time convolution, $y(n) = x(n) \otimes h(n)$.

Answer to Question 3:

We need to calculate $y(n), \quad -1 \le n \le 4$.

$$y(n) = \sum_{m=0}^{2} x(n-m)h(m) = \left\{-1, \underset{\uparrow}{1}, -1, 2, 0, 1\right\}.$$