EE4150 Digital Signal Processing

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Final Examination, Spring of 2004

Time: 5:30~7:30 p.m., Monday, May 10, 2004

Please ALWAYS work on the FRONT SIDE of each page. No answer on the BACK SIDE of any page will be GRADED! Ask for additional blank papers if necessary!

You may check any textbook, classnote or other references alone during the test. However, NO CHEATING or COLLABORATION is allowed and such violation of the university regulations will be reported.

Please write down your name and social security number here:

Full Name: ____________SOLUTION__________________

Social Security Number: ____

Partition	Score
Question 1	
Question 2	
Question 3	
Question 4	
Total	

<u>Question 1</u> (25%)

The Z-transform of a digital causal filter is $\dfrac{1}{z^2 - 0.95z + 0.95^2}$.

(a) Determine the characteristic of this filter (high-pass, low-pass, band-pass, notch, etc.) according to the zero-pole locations. (10%)

(b) Determine the impulse response of this filter. (15%)

Answer to Question 1:

(a) The poles are $z = 0.95e^{\frac{\pi}{3}}, 0.95e^{-\frac{\pi}{3}}$. Thus it is a band-pass filter.

(b) $$\frac{H(z)}{z} = \frac{1}{z\left(z^2 - 1.9\cos\left(\frac{\pi}{3}\right)z + 0.95^2\right)} = \frac{A_{11}}{z} + \frac{A_{21}}{\left(z - 0.95e^{j\frac{\pi}{3}}\right)} + \frac{A_{31}}{\left(z - 0.95e^{-j\frac{\pi}{3}}\right)}$$

$$A_{11} = \left.\frac{1}{\left(z^2 - 1.9\cos\left(\frac{\pi}{3}\right)z + 0.95^2\right)}\right|_{z=0} = \frac{1}{0.95^2} = 1.108$$

$$A_{21} = \left[\frac{1}{z\left(z - 0.95e^{-j\frac{\pi}{3}}\right)}\right]_{z=0.95e^{j\frac{\pi}{3}}} = \frac{1}{2j(0.95)^2 e^{j\frac{\pi}{3}}\sin\left(\frac{\pi}{3}\right)}$$

$$= \frac{-je^{-j\frac{\pi}{3}}}{(0.95)^2\sqrt{3}} = -0.5540 - 0.3199j$$

$$A_{31} = \left[\frac{1}{z\left(z - 0.95e^{j\frac{\pi}{3}}\right)}\right]_{z=0.95e^{-j\frac{\pi}{3}}} = \frac{1}{-2j(0.95)^2 e^{-j\frac{\pi}{3}}\sin\left(\frac{\pi}{3}\right)}$$

$$= \frac{je^{j\frac{\pi}{3}}}{(0.95)^2\sqrt{3}} = -0.5540 + 0.3199j$$

$$H(z) = 1.108 + \frac{(-0.5540 - 0.3199j)}{\left(z - 0.95e^{j\frac{\pi}{3}}\right)} + \frac{(-0.5540 + 0.3199j)}{\left(z - 0.95e^{-j\frac{\pi}{3}}\right)}$$

Thus, $h(n) = 1.108\delta(n) + (-0.5540 - 0.3199j)0.95^n e^{j\frac{\pi n}{3}} u(n)$

$$+ (-0.5540 + 0.3199j)0.95^n e^{-j\frac{\pi n}{3}} u(n)$$

or $h(n) = 1.108\delta(n) - 1.108(0.95)^n \cos\left(\frac{\pi n}{3}\right) u(n)$.

Question 2 (15%)

A causal LTI system has the transfer function as follows:

$$H(z) = \frac{1}{1 + 9z^{-1} + 8z^{-2} + 3z^{-3} + 2z^{-4}}.$$

Determine the BIBO stability of this system without the knowledge of the poles.

Answer to Question 2:

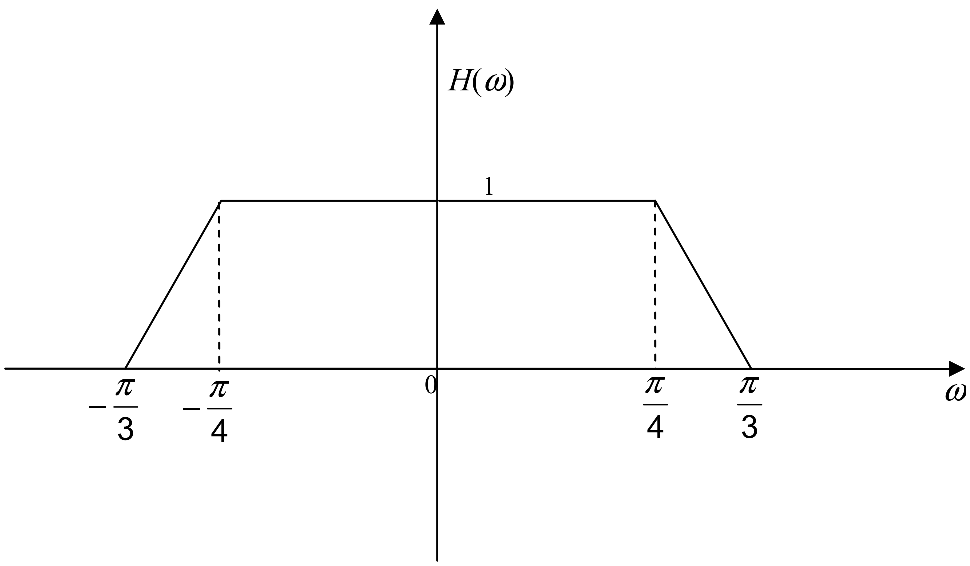
$A(z) = 1 + 9z^{-1} + 8z^{-2} + 3z^{-3} + 2z^{-4}$

$A_4(z) = 1 + 9z^{-1} + 8z^{-2} + 3z^{-3} + 2z^{-4}$, $K_4 = a_4(4) = 2 > 1$

Since $|K_4| > 1$, the system is unstable.

Question 3 (25%)

The DTFT $H(\omega)$ of a digital filter is depicted as below.



It can be expressed as

$$H(\omega) = \begin{cases} 1, & -\frac{\pi}{4} \le \omega \le \frac{\pi}{4} \\ \frac{12}{\pi}\left(\omega + \frac{\pi}{3}\right), & -\frac{\pi}{3} \le \omega \le -\frac{\pi}{4} \\ \frac{12}{\pi}\left(\frac{\pi}{3} - \omega\right), & \frac{\pi}{4} \le \omega \le \frac{\pi}{3} \\ 0, & otherwise \end{cases}.$$

(a) Determine the impulse response of this filter, which is $h(n)$ = IDTFT$\{H(\omega)\}$. (15%)

(b) Determine the 3-dB bandwidth (half-band) of this filter in terms of the sampling frequency F_s Hz. (10%)

Answer to Question 3:

(a) $h(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}H(\omega)e^{j\omega n}d\omega=\frac{6}{\pi^2}\int_{-\pi/3}^{-\pi/4}\left(\omega+\frac{\pi}{3}\right)e^{j\omega n}d\omega+\frac{1}{2\pi}\int_{-\pi/4}^{\pi/4}e^{j\omega n}d\omega$

$$+\frac{6}{\pi^2}\int_{\pi/4}^{\pi/3}\left(\frac{\pi}{3}-\omega\right)e^{j\omega n}d\omega$$

$$=\frac{6}{\pi^2}\left\{\frac{\omega}{jn}e^{j\omega n}\Big|_{-\pi/3}^{-\pi/4}+\frac{e^{j\omega n}}{n^2}\Big|_{-\pi/3}^{-\pi/4}\right\}+\frac{2}{\pi}\frac{e^{j\omega n}}{jn}\Big|_{-\pi/3}^{-\pi/4}+\frac{1}{2\pi}\frac{e^{j\omega n}}{jn}\Big|_{-\pi/4}^{\pi/4}$$

$$+\frac{2}{\pi}\frac{e^{j\omega n}}{jn}\Big|_{\pi/4}^{\pi/3}-\frac{6}{\pi^2}\left\{\frac{\omega}{jn}e^{j\omega n}\Big|_{\pi/4}^{\pi/3}+\frac{e^{j\omega n}}{n^2}\Big|_{\pi/4}^{\pi/3}\right\}$$

$$=-\frac{3}{\pi n}\sin\left(\frac{\pi n}{4}\right)+\frac{4}{\pi n}\sin\left(\frac{\pi n}{3}\right)-\frac{12j}{\pi^2n^2}\sin\left(\frac{\pi n}{4}\right)+\frac{12j}{\pi^2n^2}\sin\left(\frac{\pi n}{3}\right)+\frac{4}{\pi n}\sin\left(\frac{\pi n}{4}\right)$$

$$-\frac{1}{\pi n}\sin\left(\frac{\pi n}{4}\right)-\frac{4}{\pi n}\sin\left(\frac{\pi n}{3}\right)$$

$$=\frac{12j}{\pi^2n^2}\sin\left(\frac{\pi n}{3}\right)-\frac{12j}{\pi^2n^2}\sin\left(\frac{\pi n}{4}\right),\quad n\neq 0$$

$$h(0)=\frac{1}{2\pi}\int_{-\pi}^{\pi}H(\omega)d\omega=\frac{1}{2}\left(\frac{2\pi}{3}+\frac{\pi}{2}\right)\frac{1}{2\pi}=\frac{7}{24}.$$

(b) $\frac{12}{\pi}\left(\frac{\pi}{3}-\omega_B\right)=1/2\Rightarrow\omega_B=\frac{7\pi}{24}$

The half-band bandwidth is $\left(\frac{\omega_B}{2\pi}F_s\right)=\frac{7}{48}F_s$ Hz or $\frac{\omega_B}{2\pi}=7/48$ (both are acceptable).

<u>Question 4</u> (35%)

Three sequences $x(n)$, $h(n)$ and $g(n)$ are given as follows:

$$x(n) = \left\{ 1,\ \ 2,\ \ \underset{\uparrow}{1},\ \ 1 \right\}$$
$$h(n) = \left\{ \underset{\uparrow}{1},\ \ 1 \right\}\quad .$$
$$g(n) = \left\{ 1,\ \ \underset{\uparrow}{2} \right\}$$

(a) Extend both $x(n)$ and $h(n)$ to periodic sequences $x_p(n)$ and $h_p(n)$ respectively with a common period of four samples. Determine the periodic sequence $y_{1,p}(n)$, which results from the 4-point circular convolution of $x_p(n)$ and $h_p(n)$, without using DFT and IDFT. (10%)

(b) Using the DFT and IDFT, determine the periodic sequence $y_{1,p}(n)$, which results from the 4-point circular convolution of $x_p(n)$ and $h_p(n)$. (10%)

(c) Using the DFT and IDFT, determine the sequence $y_2(n)$, which results from the linear convolution of $g(n)$ and $h(n)$. (15%)

Answer to Question 4:

(a) $x_p(n) = \left\{ \underset{\uparrow}{1}, \ 1, \ 1, \ 2 \right\}, \quad h_p(n) = \left\{ \underset{\uparrow}{1}, \ 1, \ 0, \ 0 \right\}.$

$y_1(n) = x_p(n) \otimes_c h_p(n)$, where $\otimes_c$ denotes the circular convolution operation.

$$y_{1,p}(n) = \sum_{m=0}^{3} x_p(m) h_p(n-m) = \left\{ \underset{\uparrow}{3}, \ 2, \ 2, \ 3 \right\}.$$

(b) $X_p(k) = \sum_{n=0}^{3} x_p(n) e^{-j\frac{\pi}{2}nk} = 1 + (-j)^k + (j)^{2k} + 2(-j)^{3k}, \quad k = 0,1,2,3$

$$= \left\{ \underset{\substack{\uparrow \\ k=0}}{5}, \ j, \ -1, \ -j \right\}$$

$$H_p(k) = \sum_{n=0}^{3} h_p(n) e^{-j\frac{\pi}{2}nk} = 1 + (-j)^k, \ k = 0,1,2,3$$

$$= \left\{ \underset{\substack{\uparrow \\ k=0}}{2}, \ 1-j, \ 0, \ 1+j \right\}$$

$$Y_1(k) \equiv X_p(k) H_p(k) = \left\{ \underset{\substack{\uparrow \\ k=0}}{10}, \ 1+j, \ 0, \ 1-j \right\}.$$

$$y_{1,p}(n) = \frac{1}{4} \sum_{k=0}^{3} Y_1(k) e^{j\frac{\pi}{2}kn} = \frac{1}{4}\left[10 + (1+j)j^n + (1-j)j^{3n}\right], \ n = 0,1,2,3$$

$$= \left\{ \underset{\substack{\uparrow \\ n=0}}{3}, \ 2, \ 2, \ 3 \right\}$$

(c) $g_p(n) = \left\{ \underset{\uparrow}{2}, \ 0, \ 0, \ 1 \right\}, \quad h_p(n) = \left\{ \underset{\uparrow}{1}, \ 1, \ 0, \ 0 \right\}$

$$G_p(k) = \sum_{n=0}^{3} g_p(n) e^{-j\frac{\pi}{2}nk} = 2 + (-j)^{3k}, \quad k = 0,1,2,3$$

$$= \left\{ \underset{\substack{\uparrow \\ k=0}}{3}, \quad 2+j, \quad 1, \quad 2-j \right\}$$

$$H_p(k) = \sum_{n=0}^{3} h_p(n) e^{-j\frac{\pi}{2}nk} = 1 + (-j)^{k}, \; k = 0,1,2,3$$

$$= \left\{ \underset{\substack{\uparrow \\ k=0}}{2}, \quad 1-j, \quad 0, \quad 1+j \right\}$$

$$Y_2(k) \equiv G_p(k) H_p(k) = \left\{ \underset{\substack{\uparrow \\ k=0}}{6}, \quad 3-j, \quad 0, \quad 3+j \right\}.$$

$$y_{2,p}(n) = \frac{1}{4} \sum_{k=0}^{3} Y_2(k) e^{j\frac{\pi}{2}kn} = \frac{1}{4}\left[6 + (3-j)j^{n} + (3+j)j^{3n}\right], \; n = 0,1,2,3$$

$$= \left\{ \underset{\substack{\uparrow \\ n=0}}{3}, \quad 2, \quad 0, \quad 1 \right\}.$$

Since $y_2(n) = \begin{cases} y_{2,p}(n), & -1 \le n \le 1 \\ 0, & otherwise \end{cases}$, $y_2(n) = \left\{ 1, \quad \underset{\uparrow}{3}, \quad 2 \right\}$.