EE4150 Digital Signal Processing

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Midterm Examination, Spring of 2003

Time: 10:40 a.m. ~11:30 a.m., Monday, March 10, 2003

Please ALWAYS work on the FRONT SIDE of each page. No answer on the BACK SIDE of any page will be GRADED! Ask for additional blank papers if necessary!

You may check any textbook, classnote or other references alone during the test. However, NO CHEATING or COLLABORATION is allowed and such violation of the university regulations will be reported.

Please write down your name and social security number here:

Full Name: Solution

Social Security Number: ______________________

Partition	Score
Question 1	
Question 2	
Question 3	
Total	

Question 1 (30%)

Determine whether or not each of the following signals is periodic. In case a signal is periodic, specify its fundamental period.

(a) Discrete-time signal: $x(n) = \dfrac{\sqrt{1+\cos(\frac{\pi n}{4})}}{\sin(\frac{\pi n}{65})+1.2}$, (10%)

(b) Discrete-time signal: $x(n) = \sin(\frac{\pi n}{3}) + \cos(\frac{\pi n}{5}) + \delta(n-3)$,

where $\delta(n-3) = \begin{cases} 1, & n=3 \\ 0, & otherwise \end{cases}$, (10%)

(c) Analog signal: $x_a(t) = \cos(178t)$, (10%)

Answer to Question 1:

(a) $\frac{\pi}{4} N_1 = 2\pi m$

$N_1 = 8m/2 = 4m$

$N_1 = 4$, when $m = 1$

$\frac{\pi N_2}{65} = 2\pi m'$

$N_2 = 130 m'$

$N_2 = 130$, when $m' = 1$

$N_p = \text{lcm}(N_1, N_2)$

$= 130 \times 2 = 260 \Rightarrow x(n)$ is periodic

fundamental period is $N_p = 260$

(b) Since $\delta(n-3)$ is aperiodic,

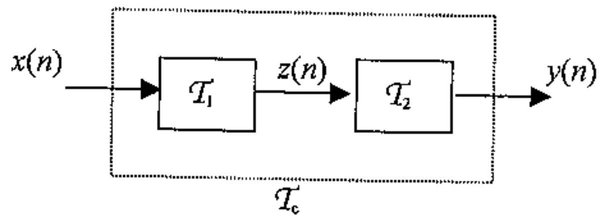
$x(n)$ is aperiodic (nonperiodic)

(c) $x_a(t)$ is periodic

$\frac{2\pi}{T_p} = 178 \Rightarrow T_p = \frac{2\pi}{178} = \frac{\pi}{89}$ seconds

Question 2 (40%)

A system $\mathcal{T}_c$ is depicted as below. It is a cascade of two LTI systems $\mathcal{T}_1$ and $\mathcal{T}_2$.



The following input-output pairs have been observed:

$$x_1(n) = \{\underset{\uparrow}{1}, 0, 3\} \overset{\mathcal{T}_1}{\leftrightarrow} z_1(n) = \{1, \underset{\uparrow}{4}, 0\}$$

$$x_2(n) = \{\underset{\uparrow}{0}, 2, 0\} \overset{\mathcal{T}_1}{\leftrightarrow} z_2(n) = \{0, 2, \underset{\uparrow}{0}\}$$

and

$$z_3(n) = \{\underset{\uparrow}{2}, 3, 0\} \overset{\mathcal{T}_2}{\leftrightarrow} y_3(n) = \{7, \underset{\uparrow}{3}, 0\}$$

$$z_4(n) = \{\underset{\uparrow}{1}, 0, 0\} \overset{\mathcal{T}_2}{\leftrightarrow} y_4(n) = \{\underset{\uparrow}{0}, 0, 1\}$$

What is the impulse response of the system $\mathcal{T}_c$?

Answer to Question 2:

$$\delta(n) = x_1(n) - \frac{3}{2} x_2(n-1)$$

$$\therefore \quad \delta(n) \xleftrightarrow{T_1} y_1(n) - \frac{3}{2} y_2(n-1)$$

$$= \{1, \underset{\uparrow}{4}, 0\} - \{0, 3, 0\}$$

$$= \{1, \underset{\uparrow}{1}, 0\}$$

$$\frac{1}{2} y_3(n) - \frac{1}{2} y_4(n-1) = \{\underset{\uparrow}{1}, 1, 0\}$$

$$\therefore \quad \{\underset{\uparrow}{1}, 1, 0\} \xleftrightarrow{T_2} \frac{1}{2} y_3(n) - \frac{1}{2} y_4(n-1)$$

$$= \{\frac{7}{2}, \underset{\uparrow}{\frac{3}{2}}, 0\} - \{\underset{\uparrow}{0}, 0, 0, 1\}$$

$$= \{\frac{7}{2}, \underset{\uparrow}{\frac{3}{2}}, 0, 0, -1\}$$

$$\therefore \quad \{1, \underset{\uparrow}{1}, 0\} \xleftrightarrow{T_2} \{\frac{7}{2}, \frac{3}{2}, \underset{\uparrow}{0}, 0, -1\}$$

$$\Rightarrow \delta(n) \xleftrightarrow{T_1} \{1, \underset{\uparrow}{1}, 0\} \xrightarrow{T_2} \{\frac{7}{2}, \frac{3}{2}, \underset{\uparrow}{0}, 0, -1\}$$

$$\Rightarrow \delta(n) \xleftrightarrow{T_c} \{\frac{7}{2}, \frac{3}{2}, \underset{\uparrow}{0}, 0, -1\}$$

Or

$$\{\underset{\uparrow}{1}, 1, 0\} = \frac{1}{3}\delta_3(n) + \frac{1}{3}\delta_4(n) \xleftrightarrow{T_2} \frac{1}{3}y_3(n) + \frac{1}{3}y_4(n)$$

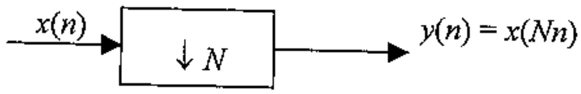
$$= \{\tfrac{7}{3}, \underset{\uparrow}{1}, 0, \tfrac{1}{3}\}$$

$$\Rightarrow \quad \delta(n) \xleftrightarrow{T_1} \{1, \underset{\uparrow}{1}, 0\} \xleftrightarrow{T_2} \{\tfrac{7}{3}, 1, \underset{\uparrow}{0}, \tfrac{1}{3}\}$$

$$\Rightarrow \quad \delta(n) \xleftrightarrow{T_c} \{\tfrac{7}{3}, 1, \underset{\uparrow}{0}, \tfrac{1}{3}\}$$

<u>Question 3</u> (30%)

A down-sampler system is depicted as below.



$x(n)$ and $y(n)$ are both discrete-time sequences. $\downarrow N$ is a down-sampler where $y(n)$ = $x(Nn)$ and N is an arbitrary integer greater than one.

(a) Is this system linear? Justify your answer through mathematics. (10%)

(b) Is this system time-invariant? Justify your answer through mathematics. (10%)

(c) Is this system BIBO stable? Justify your answer through mathematics. (10%)

Answer to Question 3:

(a) Given two arbitrary input sequences, $x_1(n)$ and $x_2(n)$, the outputs are

$$x_1(n) \longleftrightarrow y_1(n) = x_1(Nn), \quad x_2(n) \longleftrightarrow y_2(n) = x_2(Nn)$$

For any two constants α and β,

$$x_3(n) = \alpha x_1(n) + \beta x_2(n) \longleftrightarrow y_3(n) = \alpha x_1(Nn) + \beta x_2(Nn)$$
$$= \alpha y_1(n) + \beta y_2(n)$$

$\therefore$ It is linear system.

(b) Given an arbitrary input sequence $x(n)$, the output is $x(n) \longleftrightarrow y(n) = x(Nn$

For any integer M,

$$x'(n) = x(n-M) \longleftrightarrow y'(n) = x(Nn-M)$$
$$\neq y(n-M) = y(Nn-N$$

$\therefore$ It is time-varying.

(c) Given an arbitrary bounded input sequence $x(n)$, such that $|x(n)| \leq M_x$, $\forall n$

the output sequence is $x(n) \longleftrightarrow y(n) = x(Nn)$

$|y(n)| = |x(Nn)| \leq M_x$ is also bounded.

$\therefore$ It is BIBO stable