EE4150 Digital Signal Processing

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Midterm Examination, fall of 2006

Time: 10:40 ~11:30 a.m., Monday, October 16, 2006

Question 1 (30%)

Determine whether or not each of the following signals is periodic. In case a signal is periodic, specify its fundamental period.

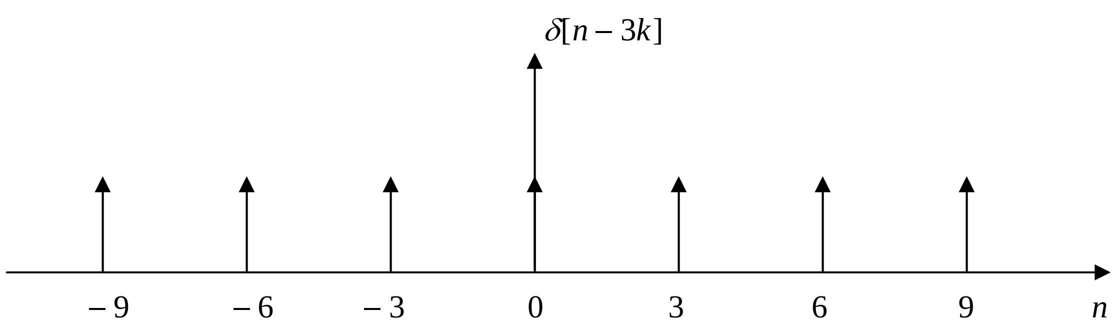
(a) Discrete-time signal: $x_1[n] = \sum_{k=-\infty}^{\infty} \delta[n-3k]$, where $\delta[n]$ is the unit sample sequence. (10%)

(b) Discrete-time signal: $x_2[n] = \sum_{k=1}^{5} \cos(0.4\pi\, n\, k)$. (10%)

(c) Discrete-time signal: $x_3[n] = \sin(0.187967\pi\, n)\mu[n]$, where $\mu[n]$ is the unit step sequence. (10%)

Answer to Question 1:

(a)



From the figure we can see $x_1[n] = \sum_{k=-\infty}^{\infty} \delta[n-3k]$ is periodic with the period $N = 3$.

(b) $x_2[n] = \cos(0.4\pi\, n) + \cos(0.8\pi\, n) + \cos(1.2\pi\, n) + \cos(1.6\pi\, n) + \cos(2.0\pi\, n)$

Since $T = \frac{2\pi}{\omega}$,

$$\Rightarrow T1 = \frac{2\pi}{\omega} = \frac{2\pi}{0.4\pi} = \frac{5}{1}$$

$$T2 = \frac{2\pi}{0.8\pi} = \frac{5}{2}$$

$$T3 = \frac{2\pi}{1.2\pi} = \frac{5}{3}$$

$$T4 = \frac{2\pi}{1.6\pi} = \frac{5}{4}$$

$$T5 = \frac{2\pi}{2\pi} = \frac{5}{5}$$

So $x_2[n]$ is periodic with period $N = 5$.

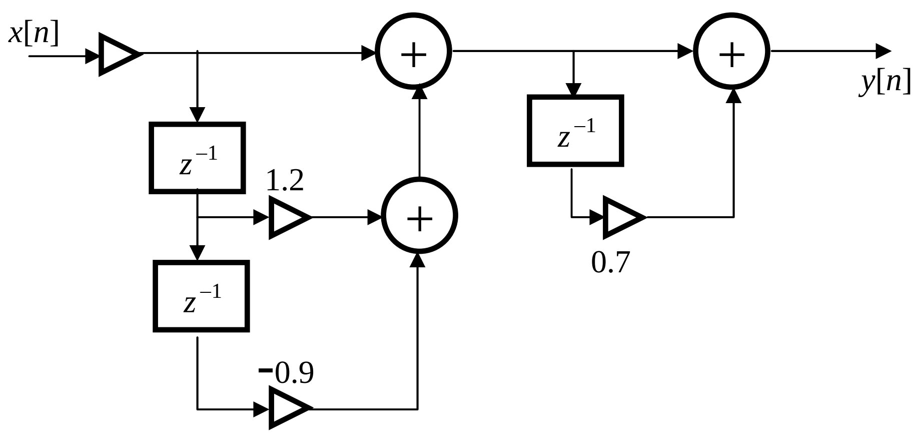
(c)

$x_3[n] = \sin(0.187967\pi n)\mu[n]$ is not periodic because $\mu[n]$ is not a periodic function.

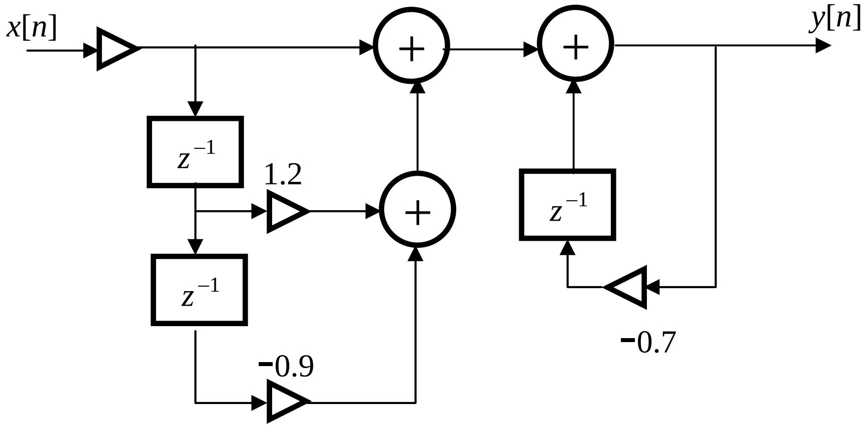
Question 2 (30%)

Write the difference equations for the following block diagrams and specify if they are recursive or non-recursive systems: (15% each)

(a)

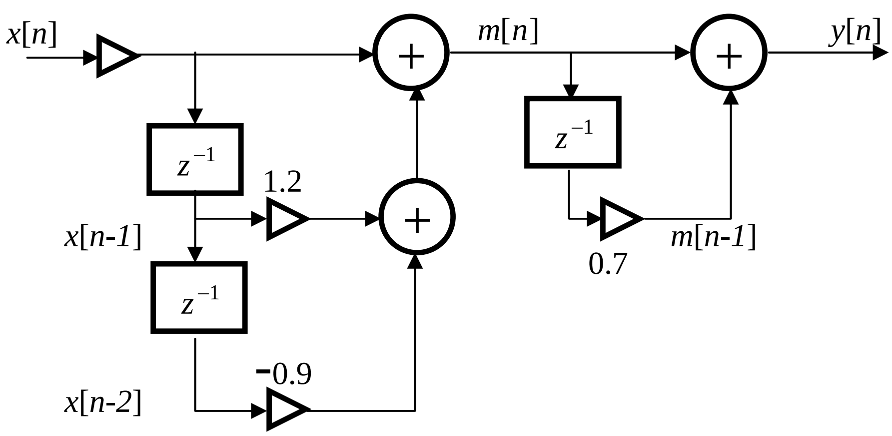


(b)



Answer to Question 2:

(a)



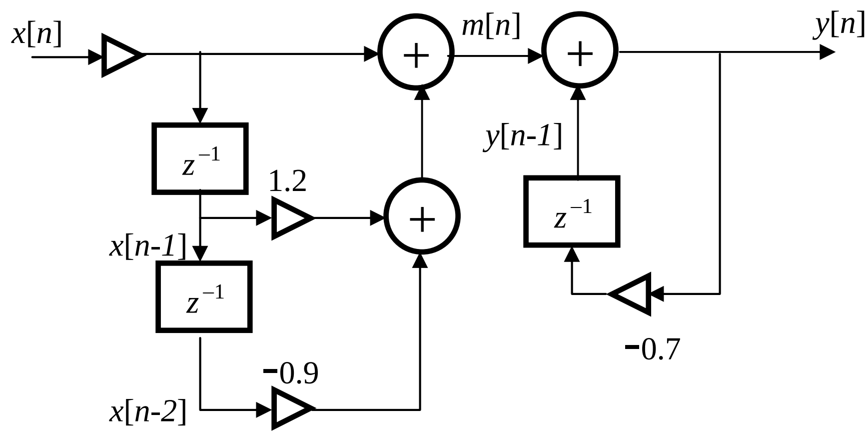
From the diagram we can get

$$\begin{cases} m[n] = x[n] + 1.2x[n-1] - 0.9x[n-2] \\ y[n] = m[n] + 0.7m[n-1] \end{cases}$$

$$\begin{aligned} \Rightarrow y[n] &= x[n] + 1.2x[n-1] - 0.9x[n-2] \\ &+0.7(x[n-1] + 1.2x[n-2] - 0.9x[n-3]) \\ &= x[n] + 1.2x[n-1] - 0.9x[n-2] \\ &+0.7x[n-1] + 0.84x[n-2] - 0.63x[n-3] \\ &= x[n] + 1.9x[n-1] - 0.06x[n-2] - 0.63x[n-3] \end{aligned}$$

This system is non-recursive because the system output only depends on the input signals.

(b)



$$\begin{cases} m[n] = x[n] + 1.2x[n-1] - 0.9x[n-2] \\ y[n] = m[n] - 0.7y[n-1] \end{cases}$$

$$\Rightarrow y[n] + 0.7y[n-1] = x[n] + 1.2x[n-1] - 0.9x[n-2]$$

This system is recursive because the system output depends on the input signals and past outputs.

<u>Question 3</u> (40%)

(a) $x[n] = \left\{ 1 \quad \underset{n=0}{\underset{\uparrow}{2}} \quad 3 \quad 2 \right\}$ and $h[n] = \left\{ \underset{n=0}{\underset{\uparrow}{1}} \quad -1 \quad 1 \right\}$. Compute $y[n] = x[n] \otimes h[n]$, where $\otimes$ is linear convolution operation . (20%)

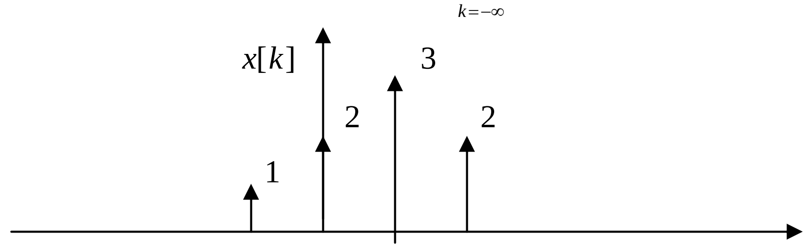
(b) Depict the block diagram for the difference equation: (20%)

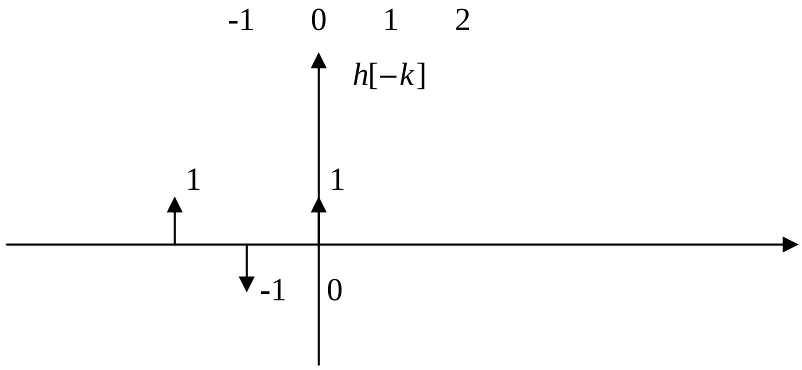
$$y[n] = 0.5y[n-1] - 0.3y[n-2] + x[n-1] - 0.5x[n-3].$$

<u>Answer to Question 3:</u>

(a)

$$y[n] = x[n] \otimes h[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k],$$





We have $y[n] = x[n] \otimes h[n]$ with the length $4+3-1=6$ and

$$y[-1] = 1\times 1 = 1$$
$$y[0] = 1\times 2 + (-1)\times 1 = 1$$
$$y[1] = 1\times 3 + (-1)\times 2 + 1\times 1 = 2$$
$$y[2] = 1\times 2 + (-1)\times 3 + 1\times 2 = 1$$
$$y[3] = (-1)\times 2 + 1\times 3 = 1$$
$$y[4] = 1\times 2 = 2$$

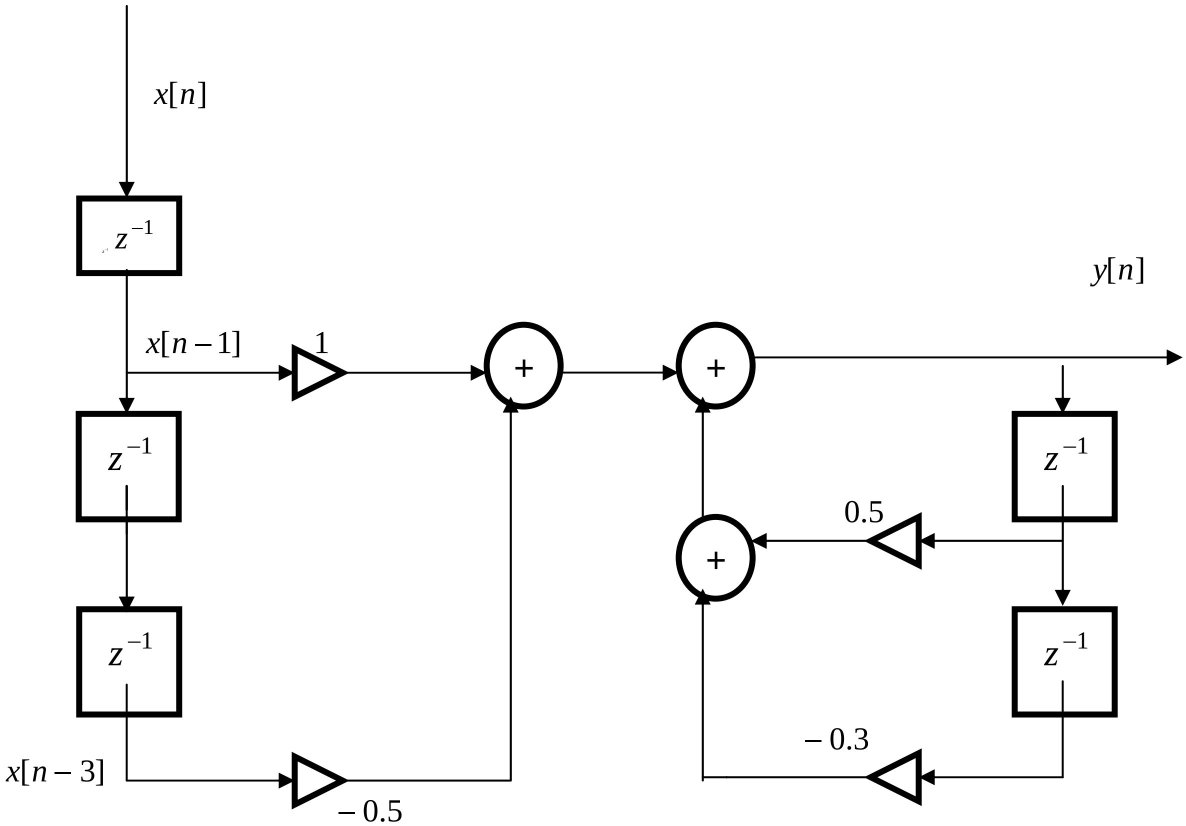
Therefore, $y[n] = \{1,1,2,1,1,2\}, -1 \le n \le 4$.

(b)

$$y[n] = 0.5y[n-1] - 0.3y[n-2] + x[n-1] - 0.5x[n-3]$$

$$\Rightarrow y[n] - 0.5y[n-1] + 0.3y[n-2] = x[n-1] - 0.5x[n-3].$$

The block diagram is:



x[n]
z^{-1}
y[n]
x[n − 1]
1
+
+
z^{-1}
z^{-1}
0.5
+
z^{-1}
z^{-1}
− 0.3
x[n − 3]
− 0.5