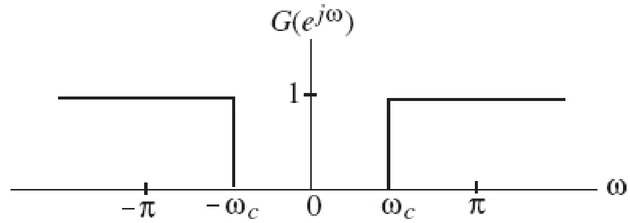
3.13 $Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \alpha^{|n|} e^{-j\omega n}$ with $|\alpha| < 1$. Rewriting we get

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{-1} \alpha^{-n} e^{-j\omega n} + \sum_{n=0}^{\infty} \alpha^{n} e^{-j\omega n} = \sum_{n=1}^{\infty} (\alpha e^{j\omega}) + \sum_{n=0}^{\infty} (\alpha e^{-j\omega})$$

$$= \frac{\alpha e^{j\omega}}{1-\alpha e^{j\omega}} + \frac{1}{1-\alpha e^{-j\omega}} = \frac{1-\alpha^2}{1-2\alpha\cos\omega+\alpha^2}.$$

3.14 $G(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\delta[n] - \frac{\sin(\omega_c n)}{\pi n} \right) e^{-j\omega n} = 1 - H_{LP}(e^{j\omega}).$



3.15 $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega.$ Hence, $x^*[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X^*(e^{j\omega}) e^{-j\omega n} d\omega.$

(a) Since $x[n]$ is real and even, we have $X(e^{j\omega}) = X^*(e^{j\omega})$. Thus

$x[-n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{-j\omega n} d\omega.$ Therefore,

$x[n] = \frac{1}{2}(x[n] + x[-n]) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \cos(\omega n) d\omega.$ As $x[n]$ is even, $X(e^{j\omega}) = X(e^{-j\omega})$.

As a result, the term $X(e^{j\omega}) \cos n(\omega n)$ inside the above integral is even, and hence

$x[n] = \frac{1}{\pi} \int_{0}^{\pi} X(e^{j\omega}) \cos(\omega n) d\omega.$

(b) Since $x[n]$ is real and odd, we have $x[n] = -x[-n]$ and $X(e^{j\omega}) = -X(e^{-j\omega})$. Thus,

$x[n] = \frac{1}{2}(x[n] - x[-n]) = \frac{j}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) \sin(\omega n) d\omega.$ As a result, the term $X(e^{j\omega}) \sin(\omega n)$

inside the above integral is even, and hence $x[n] = \frac{j}{2\pi} \int_{0}^{\pi} X(e^{j\omega}) \sin(\omega n) d\omega.$

3.23 **(a)** $H_1(e^{j\omega}) = 1 + 2\cos(\omega) + 3\cos(2\omega) = 1 + 2\left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right) + 3\left(\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right)$

$= 1 + e^{j\omega} + e^{-j\omega} + 1.5e^{j2\omega} + 1.5e^{-j2\omega}$. Therefore,
$\{h_1[n]\} = \{1.5, \quad 1, \quad 1, \quad 1, \quad 1.5\}, -2 \le n \le 2.$

(b) $H_2(e^{j\omega}) = (3 + 2\cos(\omega) + 4\cos(2\omega))\cos\left(\frac{\omega}{2}\right)e^{-j\omega/2}$

$= \left[3 + 2\left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right) + 4\left(\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right)\right]\left(\frac{e^{j\omega/2} + e^{-j\omega/2}}{2}\right)e^{-j\omega/2}$

$= \frac{1}{2}\left(3 + e^{j\omega} + e^{-j\omega} + 2e^{j2\omega} + 2e^{-j2\omega}\right)\left(1 + e^{-j\omega}\right)$

$= 2 + 1.5e^{j\omega} + 2e^{-j\omega} + e^{j2\omega} + 1.5e^{-j2\omega} + e^{-j3\omega}$. Hence,
$\{h_2[n]\} = \{1, \quad 1.5, \quad 2, \quad 2, \quad 1.5, \quad 1\}, -2 \le n \le 3.$

(c) $H_3(e^{j\omega}) = j[3 + 4\cos(\omega) + 2\cos(2\omega)]\sin(\omega)$

$= j\left[3 + 4\left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right) + 2\left(\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right)\right]\left(\frac{e^{j\omega} - e^{-j\omega}}{2j}\right)$

$= \frac{1}{2}\left(3 + 2e^{j\omega} + 2e^{-j\omega} + e^{j2\omega} + e^{-j2\omega}\right)\left(e^{j\omega} - e^{-j\omega}\right)$

$= 3 + 2e^{j\omega} + 2e^{-j\omega} + 0.5e^{j2\omega} + 0.5e^{-j2\omega}$. Hence,
$\{h_c[n]\} = \{0.5, \quad 2, \quad 3, \quad 2, \quad 0.5\}, -2 \le n \le 2.$

(d) $H_4(e^{j\omega}) = j[4 + 2\cos(\omega) + 3\cos(2\omega)]\sin(\omega/2)e^{j\omega/2}$

$= j\left[4 + 2\left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right) + 3\left(\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right)\right]\left(\frac{e^{j\omega/2} - e^{-j\omega/2}}{2j}\right)e^{-j\omega/2}$

$= \frac{1}{2}\left(4 + e^{j\omega} + e^{-j\omega} + 1.5e^{j2\omega} + 1.5e^{-j2\omega}\right)\left(1 - e^{-j\omega}\right)$

$= 1.5 - 0.25e^{j\omega} - 1.5e^{-j\omega} + 1.5e^{j2\omega} - 0.5e^{-j2\omega} - 0.75e^{-j3\omega}$. Hence,
$\{h_4[n]\} = \{-1.5, \quad -0.5, \quad -3, \quad 3, \quad -0.5, \quad -3\}, -3 \le n \le 2.$

3.27 $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$. Therefore, $X(e^{j\omega/2}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j(\omega/2)n}$ and

$X(-e^{j\omega/2}) = \sum_{n=-\infty}^{\infty} x[n](-1)^n e^{-j(\omega/2)n}$. Thus, we can write

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y[n]e^{-j\omega n} = \frac{1}{2}\left\{X(e^{j\omega/2}) + X(-e^{j\omega/2})\right\} = \frac{1}{2}\sum_{n=-\infty}^{\infty}\left(x[n] + x[n](-1)^n\right)e^{-j\omega n}.$$

Hence, $y[n] = \frac{1}{2}\left(x[n] + x[n](-1)^n\right) = \begin{cases} x[n], & \text{for } n \text{ even,} \\ 0, & \text{for } n \text{ odd.} \end{cases}$

3.33 **(a)** Since $H_1(e^{j\omega})$ is a real-valued function of ω, its inverse is an even sequence.

(b) Since $H_2(e^{j\omega})$ is a real-valued function of ω, its inverse is an even sequence.

3.63 $G(e^{j\omega}) = \frac{1}{1 - \alpha e^{-jL\omega}}$, $|\alpha| < 1$. Thus, we can write $G(e^{j\omega}) = X(e^{jL\omega})$, where

$X(e^{j\omega}) = \frac{1}{1 - \alpha e^{-j\omega}}$. From Table 3.3, the inverse DTFT of $X(e^{j\omega})$ is $x[n] = \alpha^n \mu[n]$.

Hence, from the results of Problem 3.62, it follows that

$$g[n] = \begin{cases} x[n/L], & n = 0, \pm L, \pm 2L, \pm 3L, \ldots \\ 0, & \text{otherwise.} \end{cases}$$

3.72 **(a)** $H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega}$

$= h[0] + h[1]e^{-j\omega} + h[1]e^{-j2\omega} + h[0]e^{-j3\omega} = e^{-j3\omega/2}\left(2h[0]\cos(3\omega/2) + 2h[1]\cos(\omega/2)\right).$

The two conditions to be satisfied by the filter are:

$\left|H(e^{j0.2\pi})\right| = 2h[0]\cos(0.3\pi) + 2h[1]\cos(0.1\pi) = 0.8,$

$\left|H(e^{j0.5\pi})\right| = 2h[0]\cos(0.75\pi) + 2h[1]\cos(0.25\pi) = 0.5.$ Solving these two equations we get $h[0] = 0.0414$ and $h[1] = 0.395$.

(b) $H(e^{j\omega}) = 0.0414 + 0.395e^{-j\omega} + 0.395\, e^{-j2\omega} + 0.0414e^{-j3\omega}$.