

## Types of Transfer Functions

- The time-domain classification of an LTI digital transfer function sequence is based on the length of its impulse response:
  - Finite impulse response (FIR) transfer function
  - Infinite impulse response (IIR) transfer function

1

Copyright © 2005, S. K. Mitra

## Types of Transfer Functions

- In the case of digital transfer functions with frequency-selective frequency responses, there are two types of classifications
- (1) Classification based on the shape of the magnitude function  $|H(e^{j\omega})|$
- (2) Classification based on the the form of the phase function  $\theta(\omega)$

2

Copyright © 2005, S. K. Mitra

## Classification Based on Magnitude Characteristics

- One common classification is based on an ideal magnitude response
- A digital filter designed to pass signal components of certain frequencies without distortion should have a frequency response equal to one at these frequencies, and should have a frequency response equal to zero at all other frequencies

3

Copyright © 2005, S. K. Mitra

## Ideal Filters

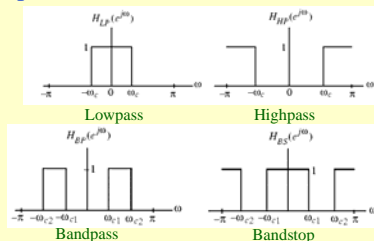
- The range of frequencies where the frequency response takes the value of one is called the **passband**
- The range of frequencies where the frequency response takes the value of zero is called the **stopband**

4

Copyright © 2005, S. K. Mitra

## Ideal Filters

- Frequency responses of the four popular types of ideal digital filters with real impulse response coefficients are shown below:



5

Copyright © 2005, S. K. Mitra

## Ideal Filters

- Lowpass filter:** Passband -  $0 \leq \omega \leq \omega_c$   
Stopband -  $\omega_c < \omega \leq \pi$
- Highpass filter:** Passband -  $\omega_c \leq \omega \leq \pi$   
Stopband -  $0 \leq \omega < \omega_c$
- Bandpass filter:** Passband -  $\omega_{c1} \leq \omega \leq \omega_{c2}$   
Stopband -  $0 \leq \omega < \omega_{c1}$  and  $\omega_{c2} < \omega \leq \pi$
- Bandstop filter:** Stopband -  $\omega_{c1} < \omega < \omega_{c2}$   
Passband -  $0 \leq \omega \leq \omega_{c1}$  and  $\omega_{c2} \leq \omega \leq \pi$

6

Copyright © 2005, S. K. Mitra

## Ideal Filters

- The frequencies  $\omega_c$ ,  $\omega_{c1}$ , and  $\omega_{c2}$  are called the **cutoff frequencies**
- An ideal filter has a magnitude response equal to one in the passband and zero in the stopband, and has a zero phase everywhere

7

Copyright © 2005, S. K. Mitra

## Ideal Filters

- Earlier in the course we derived the inverse DTFT of the frequency response  $H_{LP}(e^{j\omega})$  of the ideal lowpass filter:

$$h_{LP}[n] = \frac{\sin \omega_c n}{\pi n}, \quad -\infty < n < \infty$$

- We have also shown that the above impulse response is not absolutely summable, and hence, the corresponding transfer function is not BIBO stable

8

Copyright © 2005, S. K. Mitra

## Ideal Filters

- Also,  $h_{LP}[n]$  is not causal and is of doubly infinite length
- The remaining three ideal filters are also characterized by doubly infinite, noncausal impulse responses and are not absolutely summable
- Thus, the ideal filters with the ideal “brick wall” frequency responses cannot be realized with finite dimensional LTI filter

9

Copyright © 2005, S. K. Mitra

## Ideal Filters

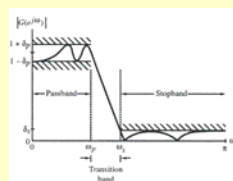
- To develop stable and realizable transfer functions, the ideal frequency response specifications are relaxed by including a **transition band** between the passband and the stopband
- This permits the magnitude response to decay slowly from its maximum value in the passband to the zero value in the stopband

10

Copyright © 2005, S. K. Mitra

## Ideal Filters

- Moreover, the magnitude response is allowed to vary by a small amount both in the passband and the stopband
- Typical magnitude response specifications of a lowpass filter are shown below



11

Copyright © 2005, S. K. Mitra

## Bounded Real Transfer Functions

- A causal stable real-coefficient transfer function  $H(z)$  is defined as a **bounded real (BR) transfer function** if

$$|H(e^{j\omega})| \leq 1 \quad \text{for all values of } \omega$$

- Let  $x[n]$  and  $y[n]$  denote, respectively, the input and output of a digital filter characterized by a BR transfer function  $H(z)$  with  $X(e^{j\omega})$  and  $Y(e^{j\omega})$  denoting their DTFTs

12

Copyright © 2005, S. K. Mitra

## Bounded Real Transfer Functions

- Then the condition  $|H(e^{j\omega})| \leq 1$  implies that

$$|Y(e^{j\omega})|^2 \leq |X(e^{j\omega})|^2$$

- Integrating the above from  $-\pi$  to  $\pi$ , and applying Parseval's relation we get

$$\sum_{n=-\infty}^{\infty} |y[n]|^2 \leq \sum_{n=-\infty}^{\infty} |x[n]|^2$$

13

Copyright © 2005, S. K. Mitra

## Bounded Real Transfer Functions

- Thus, for all finite-energy inputs, the output energy is less than or equal to the input energy implying that a digital filter characterized by a BR transfer function can be viewed as a **passive structure**
- If  $|H(e^{j\omega})| = 1$ , then the output energy is equal to the input energy, and such a digital filter is therefore a **lossless system**

14

Copyright © 2005, S. K. Mitra

## Bounded Real Transfer Functions

- A causal stable real-coefficient transfer function  $H(z)$  with  $|H(e^{j\omega})| = 1$  is thus called a **lossless bounded real (LBR) transfer function**
- The BR and LBR transfer functions are the keys to the realization of digital filters with low coefficient sensitivity

15

Copyright © 2005, S. K. Mitra

## Bounded Real Transfer Functions

- Example – Consider the causal stable IIR transfer function

$$H(z) = \frac{K}{1 - \alpha z^{-1}}, \quad 0 < |\alpha| < 1$$

where  $K$  is a real constant

- Its square-magnitude function is given by

$$|H(e^{j\omega})|^2 = H(z)H(z^{-1}) \Big|_{z=e^{j\omega}} = \frac{K^2}{(1 + \alpha)^2 - 2\alpha \cos \omega}$$

16

Copyright © 2005, S. K. Mitra

## Bounded Real Transfer Functions

- The maximum value of  $|H(e^{j\omega})|^2$  is obtained when  $2\alpha \cos \omega$  in the denominator is a maximum and the minimum value is obtained when  $2\alpha \cos \omega$  is a minimum
- For  $\alpha > 0$ , maximum value of  $2\alpha \cos \omega$  is equal to  $2\alpha$  at  $\omega = 0$ , and minimum value is  $-2\alpha$  at  $\omega = \pi$

17

Copyright © 2005, S. K. Mitra

## Bounded Real Transfer Functions

- Thus, the maximum value of  $|H(e^{j\omega})|^2$  is equal to  $K^2/(1 - \alpha)^2$  at  $\omega = 0$
  - The maximum value can be made equal to 1 by choosing  $K = 1 - \alpha$
  - Hence,
- $$H(z) = \frac{1 - \alpha}{1 - \alpha z^{-1}}, \quad 0 < \alpha < 1$$
- is a BR function for  $\alpha > 1$

18

Copyright © 2005, S. K. Mitra

## Allpass Transfer Function

### Definition

- An IIR transfer function  $A(z)$  with unity magnitude response for all frequencies, i.e.,

$$|A(e^{j\omega})|^2 = 1, \quad \text{for all } \omega$$

is called an **allpass transfer function**

- An  $M$ -th order causal real-coefficient allpass transfer function is of the form

$$A_M(z) = \pm \frac{d_M + d_{M-1}z^{-1} + \dots + d_1z^{-M+1} + z^{-M}}{1 + d_1z^{-1} + \dots + d_{M-1}z^{-M+1} + d_Mz^{-M}}$$

19

Copyright © 2005, S. K. Mitra

## Allpass Transfer Function

- If we denote the denominator polynomials of  $A_M(z)$  as  $D_M(z)$ :

$$D_M(z) = 1 + d_1z^{-1} + \dots + d_{M-1}z^{-M+1} + d_Mz^{-M}$$

then it follows that  $A_M(z)$  can be written as:

$$A_M(z) = \pm \frac{z^{-M} D_M(z^{-1})}{D_M(z)}$$

- Note from the above that if  $z = re^{j\phi}$  is a pole of a real coefficient allpass transfer function, then it has a zero at  $z = \frac{1}{r}e^{-j\phi}$

20

Copyright © 2005, S. K. Mitra

## Allpass Transfer Function

- The numerator of a real-coefficient allpass transfer function is said to be the **mirror-image polynomial** of the denominator, and vice versa
- We shall use the notation  $\tilde{D}_M(z)$  to denote the mirror-image polynomial of a degree- $M$  polynomial  $D_M(z)$ , i.e.,

$$\tilde{D}_M(z) = z^{-M} D_M(z^{-1})$$

21

Copyright © 2005, S. K. Mitra

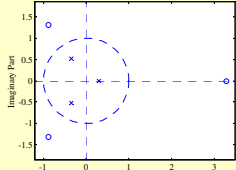
## Allpass Transfer Function

- The expression

$$A_M(z) = \pm \frac{z^{-M} D_M(z^{-1})}{D_M(z)}$$

implies that the poles and zeros of a real-coefficient allpass function exhibit **mirror-image symmetry** in the  $z$ -plane

$$A_3(z) = \frac{-0.2 + 0.18z^{-1} + 0.4z^{-2} + z^{-3}}{1 + 0.4z^{-1} + 0.18z^{-2} - 0.2z^{-3}}$$



22

Copyright © 2005, S. K. Mitra

## Allpass Transfer Function

- To show that  $|A_M(e^{j\omega})| = 1$  we observe that

$$A_M(z^{-1}) = \pm \frac{z^M D_M(z)}{D_M(z^{-1})}$$

- Therefore

$$A_M(z) A_M(z^{-1}) = \frac{z^{-M} D_M(z^{-1})}{D_M(z)} \frac{z^M D_M(z)}{D_M(z^{-1})}$$

- Hence

$$|A_M(e^{j\omega})|^2 = A_M(z) A_M(z^{-1}) \Big|_{z=e^{j\omega}} = 1$$

23

Copyright © 2005, S. K. Mitra

## Allpass Transfer Function

- Now, the poles of a causal stable transfer function must lie inside the unit circle in the  $z$ -plane
- Hence, all zeros of a causal stable allpass transfer function must lie outside the unit circle in a **mirror-image symmetry** with its poles situated inside the unit circle

24

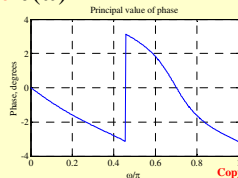
Copyright © 2005, S. K. Mitra

## Allpass Transfer Function

- Figure below shows the principal value of the phase of the 3rd-order allpass function

$$A_3(z) = \frac{-0.2 + 0.18z^{-1} + 0.4z^{-2} + z^{-3}}{1 + 0.4z^{-1} + 0.18z^{-2} - 0.2z^{-3}}$$

- Note the discontinuity by the amount of  $2\pi$  in the phase  $\theta(\omega)$

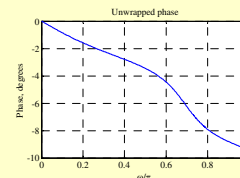


25

Copyright © 2005, S. K. Mitra

## Allpass Transfer Function

- If we unwrap the phase by removing the discontinuity, we arrive at the unwrapped phase function  $\theta_c(\omega)$  indicated below
- Note: The unwrapped phase function is a continuous function of  $\omega$



26

Copyright © 2005, S. K. Mitra

## Allpass Transfer Function

- The unwrapped phase function of any arbitrary causal stable allpass function is a continuous function of  $\omega$

### Properties

- (1) A causal stable real-coefficient allpass transfer function is a lossless bounded real (LBR) function or, equivalently, a causal stable allpass filter is a lossless structure

27

Copyright © 2005, S. K. Mitra

## Allpass Transfer Function

- (2) The magnitude function of a stable allpass function  $A(z)$  satisfies:

$$|A(z)| = \begin{cases} < 1, & \text{for } |z| > 1 \\ 1, & \text{for } |z| = 1 \\ > 1, & \text{for } |z| < 1 \end{cases}$$

- (3) Let  $\tau(\omega)$  denote the group delay function of an allpass filter  $A(z)$ , i.e.,

$$\tau(\omega) = -\frac{d}{d\omega}[\theta_c(\omega)]$$

28

Copyright © 2005, S. K. Mitra

## Allpass Transfer Function

- The unwrapped phase function  $\theta_c(\omega)$  of a stable allpass function is a monotonically decreasing function of  $\omega$  so that  $\tau(\omega)$  is everywhere positive in the range  $0 < \omega < \pi$
- The group delay of an  $M$ -th order stable real-coefficient allpass transfer function satisfies:

$$\int_0^\pi \tau(\omega) d\omega = M\pi$$

29

Copyright © 2005, S. K. Mitra

## Allpass Transfer Function

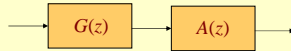
### A Simple Application

- A simple but often used application of an allpass filter is as a **delay equalizer**
- Let  $G(z)$  be the transfer function of a digital filter designed to meet a prescribed magnitude response
- The nonlinear phase response of  $G(z)$  can be corrected by cascading it with an allpass filter  $A(z)$  so that the overall cascade has a constant group delay in the band of interest

30

Copyright © 2005, S. K. Mitra

## Allpass Transfer Function



- Since  $|A(e^{j\omega})| = 1$ , we have  

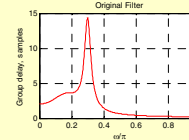
$$|G(e^{j\omega})A(e^{j\omega})| = |G(e^{j\omega})|$$
- Overall group delay is given by the sum of the group delays of  $G(z)$  and  $A(z)$

31

Copyright © 2005, S. K. Mitra

## Allpass Transfer Function

- Example – Figure below shows the group delay of a 4<sup>th</sup> order elliptic filter with the following specifications:  $\omega_p = 0.3\pi$ ,  $\delta_p = 1$  dB,  $\delta_s = 35$  dB

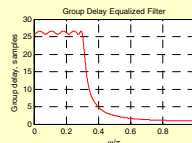


32

Copyright © 2005, S. K. Mitra

## Allpass Transfer Function

- Figure below shows the group delay of the original elliptic filter cascaded with an 8<sup>th</sup> order allpass section designed to equalize the group delay in the passband



33

Copyright © 2005, S. K. Mitra

## Classification Based on Phase Characteristics

- A second classification of a transfer function is with respect to its phase characteristics
- In many applications, it is necessary that the digital filter designed does not distort the phase of the input signal components with frequencies in the passband

34

Copyright © 2005, S. K. Mitra

## Zero-Phase Transfer Function

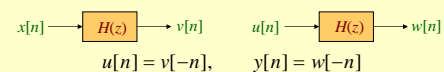
- One way to avoid any phase distortion is to make the frequency response of the filter real and nonnegative, i.e., to design the filter with a **zero phase characteristic**
- However, it is not possible to design a causal digital filter with a zero phase

35

Copyright © 2005, S. K. Mitra

## Zero-Phase Transfer Function

- For non-real-time processing of real-valued input signals of finite length, zero-phase filtering can be very simply implemented by relaxing the causality requirement
- One zero-phase filtering scheme is sketched below



36

Copyright © 2005, S. K. Mitra

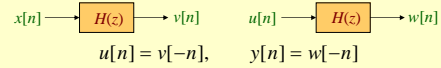
## Zero-Phase Transfer Function

- It is easy to verify the above scheme in the frequency domain
- Let  $X(e^{j\omega})$ ,  $V(e^{j\omega})$ ,  $U(e^{j\omega})$ ,  $W(e^{j\omega})$ , and  $Y(e^{j\omega})$  denote the DTFTs of  $x[n]$ ,  $v[n]$ ,  $u[n]$ ,  $w[n]$ , and  $y[n]$ , respectively
- From the figure shown earlier and making use of the symmetry relations we arrive at the relations between various DTFTs as given on the next slide

37

Copyright © 2005, S. K. Mitra

## Zero-Phase Transfer Function



$$\begin{aligned}
 V(e^{j\omega}) &= H(e^{j\omega})X(e^{j\omega}), & W(e^{j\omega}) &= H(e^{j\omega})U(e^{j\omega}) \\
 U(e^{j\omega}) &= V^*(e^{j\omega}), & Y(e^{j\omega}) &= W^*(e^{j\omega})
 \end{aligned}$$

- Combining the above equations we get
 
$$\begin{aligned}
 Y(e^{j\omega}) &= W^*(e^{j\omega}) = H^*(e^{j\omega})U^*(e^{j\omega}) \\
 &= H^*(e^{j\omega})V(e^{j\omega}) = H^*(e^{j\omega})H(e^{j\omega})X(e^{j\omega}) \\
 &= |H(e^{j\omega})|^2 X(e^{j\omega})
 \end{aligned}$$

38

Copyright © 2005, S. K. Mitra

## Zero-Phase Transfer Function

- The function `filtfilt` implements the above zero-phase filtering scheme
- In the case of a causal transfer function with a nonzero phase response, the phase distortion can be avoided by ensuring that the transfer function has a unity magnitude and a **linear-phase** characteristic in the frequency band of interest

39

Copyright © 2005, S. K. Mitra

## Zero-Phase Transfer Function

- The most general type of a filter with a linear phase has a frequency response given by

$$H(e^{j\omega}) = e^{-j\omega D}$$

which has a linear phase from  $\omega = 0$  to  $\omega = 2\pi$

- Note also  $|H(e^{j\omega})| = 1$   
 $\tau(\omega) = D$

40

Copyright © 2005, S. K. Mitra

## Linear-Phase Transfer Function

- The output  $y[n]$  of this filter to an input  $x[n] = Ae^{j\omega n}$  is then given by
 
$$y[n] = Ae^{-j\omega D}e^{j\omega n} = Ae^{j\omega(n-D)}$$
- If  $x_a(t)$  and  $y_a(t)$  represent the continuous-time signals whose sampled versions, sampled at  $t = nT$ , are  $x[n]$  and  $y[n]$  given above, then the delay between  $x_a(t)$  and  $y_a(t)$  is precisely the group delay of amount  $D$

41

Copyright © 2005, S. K. Mitra

## Linear-Phase Transfer Function

- If  $D$  is an integer, then  $y[n]$  is identical to  $x[n]$ , but delayed by  $D$  samples
- If  $D$  is not an integer,  $y[n]$ , being delayed by a fractional part, is not identical to  $x[n]$
- In the latter case, the waveform of the underlying continuous-time output is identical to the waveform of the underlying continuous-time input and delayed  $D$  units of time

42

Copyright © 2005, S. K. Mitra

## Linear-Phase Transfer Function

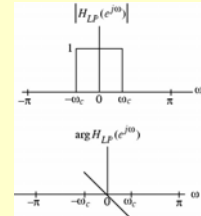
- If it is desired to pass input signal components in a certain frequency range undistorted in both magnitude and phase, then the transfer function should exhibit a unity magnitude response and a linear-phase response in the band of interest

43

Copyright © 2005, S. K. Mitra

## Linear-Phase Transfer Function

- Figure below shows the frequency response of a lowpass filter with a linear-phase characteristic in the passband



44

Copyright © 2005, S. K. Mitra

## Linear-Phase Transfer Function

- Since the signal components in the stopband are blocked, the phase response in the stopband can be of any shape
- Example** - Determine the impulse response of an ideal lowpass filter with a linear phase response:

$$H_{LP}(e^{j\omega}) = \begin{cases} e^{-j\omega n_o}, & 0 < |\omega| < \omega_c \\ 0, & \omega_c \leq |\omega| \leq \pi \end{cases}$$

45

Copyright © 2005, S. K. Mitra

## Linear-Phase Transfer Function

- Applying the frequency-shifting property of the DTFT to the impulse response of an ideal zero-phase lowpass filter we arrive at

$$h_{LP}[n] = \frac{\sin \omega_c(n - n_o)}{\pi(n - n_o)}, \quad -\infty < n < \infty$$

- As before, the above filter is noncausal and of doubly infinite length, and hence, unrealizable

46

Copyright © 2005, S. K. Mitra

## Linear-Phase Transfer Function

- By truncating the impulse response to a finite number of terms, a realizable FIR approximation to the ideal lowpass filter can be developed
- The truncated approximation may or may not exhibit linear phase, depending on the value of  $n_o$  chosen

47

Copyright © 2005, S. K. Mitra

## Linear-Phase Transfer Function

- If we choose  $n_o = N/2$  with  $N$  a positive integer, the truncated and shifted approximation

$$\hat{h}_{LP}[n] = \frac{\sin \omega_c(n - N/2)}{\pi(n - N/2)}, \quad 0 \leq n \leq N$$

will be a length  $N+1$  causal linear-phase FIR filter

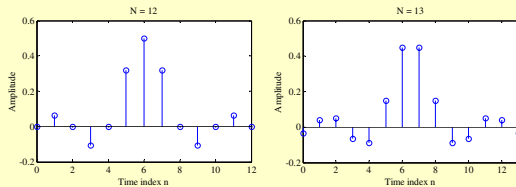
48

Copyright © 2005, S. K. Mitra



## Linear-Phase Transfer Function

- Figure below shows the filter coefficients obtained using the function `sinc` for two different values of  $N$



49

Copyright © 2005, S. K. Mitra

## Zero-Phase Response

- Because of the symmetry of the impulse response coefficients as indicated in the two figures, the frequency response of the truncated approximation can be expressed as:

$$\hat{H}_{LP}(e^{j\omega}) = \sum_{n=0}^N \hat{h}_{LP}[n] e^{-j\omega n} = e^{-j\omega N/2} \tilde{H}_{LP}(\omega)$$

where  $\tilde{H}_{LP}(\omega)$ , called the **zero-phase response** or **amplitude response**, is a real function of  $\omega$

50

Copyright © 2005, S. K. Mitra

## Minimum-Phase and Maximum-Phase Transfer Functions

- Consider the two 1st-order transfer functions:

$$H_1(z) = \frac{z+b}{z+a}, \quad H_2(z) = \frac{bz+1}{z+a}, \quad |a| < 1, \quad |b| < 1$$

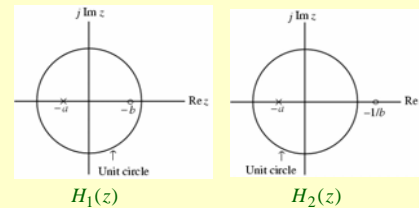
- Both transfer functions have a pole inside the unit circle at the same location  $z = -a$  and are stable
- But the zero of  $H_1(z)$  is inside the unit circle at  $z = -b$ , whereas, the zero of  $H_2(z)$  is at  $z = -1/b$  situated in a mirror-image symmetry

51

Copyright © 2005, S. K. Mitra

## Minimum-Phase and Maximum-Phase Transfer Functions

- Figure below shows the pole-zero plots of the two transfer functions



52

Copyright © 2005, S. K. Mitra

## Minimum-Phase and Maximum-Phase Transfer Functions

- However, both transfer functions have an identical magnitude function as

$$H_1(z)H_1(z^{-1}) = H_2(z)H_2(z^{-1})$$

- The corresponding phase functions are

$$\arg[H_1(e^{j\omega})] = \tan^{-1} \frac{\sin \omega}{b + \cos \omega} - \tan^{-1} \frac{\sin \omega}{a + \cos \omega}$$

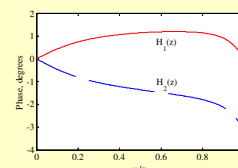
$$\arg[H_2(e^{j\omega})] = \tan^{-1} \frac{b \sin \omega}{1 + b \cos \omega} - \tan^{-1} \frac{\sin \omega}{a + \cos \omega}$$

53

Copyright © 2005, S. K. Mitra

## Minimum-Phase and Maximum-Phase Transfer Functions

- Figure below shows the unwrapped phase responses of the two transfer functions for  $a = 0.8$  and  $b = -0.5$



54

Copyright © 2005, S. K. Mitra

## Minimum-Phase and Maximum-Phase Transfer Functions

- From this figure it follows that  $H_2(z)$  has an excess phase lag with respect to  $H_1(z)$
- The excess phase lag property of  $H_2(z)$  with respect to  $H_1(z)$  can also be explained by observing that we can write

$$H_2(z) = \frac{bz+1}{z+a} = \underbrace{\left(\frac{z+b}{z+a}\right)}_{H_1(z)} \underbrace{\left(\frac{bz+1}{z+b}\right)}_{A(z)}$$

55

Copyright © 2005, S. K. Mitra

## Minimum-Phase and Maximum-Phase Transfer Functions

where  $A(z) = (bz+1)/(z+b)$  is a stable allpass function

- The phase functions of  $H_1(z)$  and  $H_2(z)$  are thus related through
- $$\arg[H_2(e^{j\omega})] = \arg[H_1(e^{j\omega})] + \arg[A(e^{j\omega})]$$
- As the unwrapped phase function of a stable first-order allpass function is a negative function of  $\omega$ , it follows from the above that  $H_2(z)$  has indeed an excess phase lag with respect to  $H_1(z)$

56

Copyright © 2005, S. K. Mitra

## Minimum-Phase and Maximum-Phase Transfer Functions

- Generalizing the above result, let  $H_m(z)$  be a causal stable transfer function with all zeros inside the unit circle and let  $H(z)$  be another causal stable transfer function satisfying  $|H(e^{j\omega})| = |H_m(e^{j\omega})|$
- These two transfer functions are then related through  $H(z) = H_m(z)A(z)$  where  $A(z)$  is a causal stable allpass function

57

Copyright © 2005, S. K. Mitra

## Minimum-Phase and Maximum-Phase Transfer Functions

- The unwrapped phase functions of  $H_m(z)$  and  $H(z)$  are thus related through
- $$\arg[H(e^{j\omega})] = \arg[H_m(e^{j\omega})] + \arg[A(e^{j\omega})]$$
- $H(z)$  has an excess phase lag with respect to  $H_m(z)$
  - A causal stable transfer function with all zeros inside the unit circle is called a **minimum-phase transfer function**

58

Copyright © 2005, S. K. Mitra

## Minimum-Phase and Maximum-Phase Transfer Functions

- A causal stable transfer function with all zeros outside the unit circle is called a **maximum-phase transfer function**
- A causal stable transfer function with zeros inside and outside the unit circle is called a **mixed-phase transfer function**

59

Copyright © 2005, S. K. Mitra

## Minimum-Phase and Maximum-Phase Transfer Functions

- Example – Consider the mixed-phase transfer function

$$H(z) = \frac{2(1+0.3z^{-1})(0.4-z^{-1})}{(1-0.2z^{-1})(1+0.5z^{-1})}$$

- We can rewrite  $H(z)$  as

$$H(z) = \underbrace{\left[ \frac{2(1+0.3z^{-1})(1-0.4z^{-1})}{(1-0.2z^{-1})(1+0.5z^{-1})} \right]}_{\text{Minimum-phase function}} \underbrace{\left[ \frac{0.4-z^{-1}}{1-0.4z^{-1}} \right]}_{\text{Allpass function}}$$

60

Copyright © 2005, S. K. Mitra