Chapter 2

2.1 **(a)** $\|x_1\|_1 = 22.85$, $\|x_1\|_2 = 9.1396$, $\|x_1\|_\infty = 4.81$,

(b) $\|x_2\|_1 = 18.68$, $\|x_2\|_2 = 7.1944$, $\|x_2\|_\infty = 3.48$.

2.2 $\mu[n] = \begin{cases} 1, & n \ge 0, \\ 0, & n < 0. \end{cases}$ Hence, $\mu[-n-1] = \begin{cases} 1, & n < 0, \\ 0, & n \ge 0. \end{cases}$ Thus, $x[n] = \mu[n] + \mu[-n-1]$.

2.3 **(a)** Consider the sequence defined by $x[n] = \sum_{k=-\infty}^{n} \delta[k]$. If $n < 0$, then $k = 0$ is not included in the sum and hence, $x[n] = 0$ for $n < 0$. On the other hand, for $n \ge 0$, $k = 0$ is included in the sum, and as a result, $x[n] = 1$ for $n \ge 0$. Therefore,

$$x[n] = \sum_{k=-\infty}^{n} \delta[k] = \begin{cases} 1, & n \ge 0, \\ 0, & n < 0, \end{cases} = \mu[n].$$

(b) Since $\mu[n] = \begin{cases} 1, & n \ge 0, \\ 0, & n < 0, \end{cases}$ it follows that $\mu[n-1] = \begin{cases} 1, & n \ge 1, \\ 0, & n < 1. \end{cases}$ Hence,

$$\mu[n] - \mu[n-1] = \begin{cases} 1, & n = 0, \\ 0, & n \ne 0, \end{cases} = \delta[n].$$

2.4 Recall $\mu[n] - \mu[n-1] = \delta[n]$. Hence,

$$x[n] = \delta[n] + 3\delta[n-1] - 2\delta[n-2] + 4\delta[n-3]$$
$$= (\mu[n] - \mu[n-1]) + 3(\mu[n-1] - \mu[n-2]) - 2(\mu[n-2] - \mu[n-3]) + 4(\mu[n-3] - \mu[n-4])$$
$$= \mu[n] + 2\mu[n-1] - 5\mu[n-2] + 6\mu[n-3] - 4\mu[n-4].$$

2.5 **(a)** $c[n] = x[-n+2] = \{2 \;\; \underset{\uparrow}{0} \;\; -3 \;\; -2 \;\; 1 \;\; 5 \;\; -4\}$,

(b) $d[n] = y[-n-3] = \{-2 \;\; 7 \;\; 8 \;\; 0 \;\; -1 \;\; -3 \;\; 6 \;\; 0 \;\; \underset{\uparrow}{0}\}$,

(c) $e[n] = w[-n] = \{5 \;\; -2 \;\; 0 \;\; -1 \;\; 2 \;\; 2 \;\; 3 \;\; 0 \;\; \underset{\uparrow}{0}\}$,

(d) $u[n] = x[n] + y[n-2] = \{-4 \;\; 5 \;\; 1 \;\; \underset{\uparrow}{-2} \;\; 3 \;\; -3 \;\; 1 \;\; 0 \;\; 8 \;\; 7 \;\; -2\}$,

(e) $v[n] = x[n] \cdot w[n+4] = \{0 \;\; 15 \;\; 2 \;\; \underset{\uparrow}{-4} \;\; 3 \;\; 0 \;\; -4 \;\; 0\}$,

(f) $s[n] = y[n] - w[n+4] = \{-3 \;\; 4 \;\; \underset{\uparrow}{-5} \;\; 0 \;\; 0 \;\; 10 \;\; 2 \;\; -2\}$,

(g) $r[n] = 3.5\,y[n] = \{21 \;\; \underset{\uparrow}{-10.5} \;\; -3.5 \;\; 0 \;\; 2.8 \;\; 24.5 \;\; -7\}$.

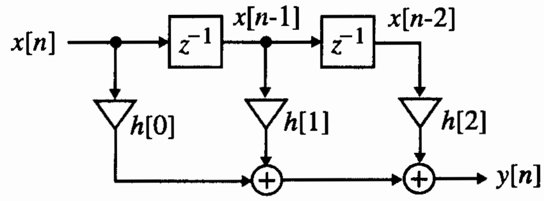
2.6 **(a)** $x[n] = -4\delta[n+3] + 5\delta[n+2] + \delta[n+1] - 2\delta[n] - 3\delta[n-1] + 2\delta[n-3]$,
$y[n] = 6\delta[n+1] - 3\delta[n] - \delta[n-1] + 8\delta[n-3] + 7\delta[n-4] - 2\delta[n-5]$,
$w[n] = 3\delta[n-2] + 2\delta[n-3] + 2\delta[n-4] - \delta[n-5] - 2\delta[n-7] + 5\delta[n-8]$,

(b) Recall $\delta[n] = \mu[n] - \mu[n-1]$. Hence,

$$x[n] = -4(\mu[n+3] - \mu[n+2]) + 5(\mu[n+2] - \mu[n+1]) + (\mu[n+1] - \mu[n])$$
$$-2(\mu[n] - \mu[n-1]) - 3(\mu[n-1] - \mu[n-2]) + 2(\mu[n-3] - \mu[n-4])$$

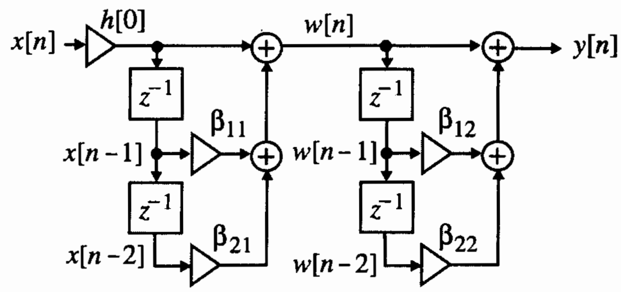
$= -4\mu[n+3]+9\mu[n+2]-4\mu[n+1]-3\mu[n]-\mu[n-1]+3\mu[n-2]+2\mu[n-3]-2\mu[n-4],$

2.7 **(a)**



From the above figure it follows that $y[n]=h[0]x[n]+h[1]x[n-1]+h[2]x[n-2]$.

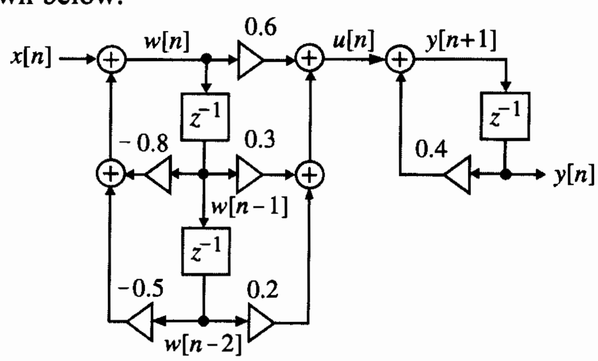
(b)



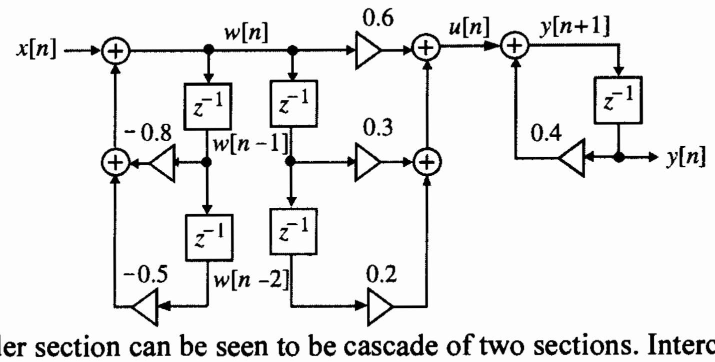
From the above figure we get $w[n]=h[0](x[n]+\beta_{11}x[n-1]+\beta_{21}x[n-2])$ and $y[n]=w[n]+\beta_{12}w[n-1]+\beta_{22}w[n-2]$. Making use of the first equation in the second we arrive at

$$\begin{aligned} y[n] &= h[0](x[n]+\beta_{11}x[n-1]+\beta_{21}x[n-2]) \\ &\quad +\beta_{12}h[0](x[n-1]+\beta_{11}x[n-2]+\beta_{21}x[n-3]) \\ &\quad +\beta_{22}h[0](x[n-2]+\beta_{11}x[n-3]+\beta_{21}x[n-4]) \\ &= h[0]\big(x[n]+(\beta_{11}+\beta_{12})x[n-1]+(\beta_{21}+\beta_{12}\beta_{11}+\beta_{22})x[n-2] \\ &\quad +(\beta_{12}\beta_{21}+\beta_{22}\beta_{11})x[n-3]+\beta_{22}\beta_{21}x[n-4]\big). \end{aligned}$$

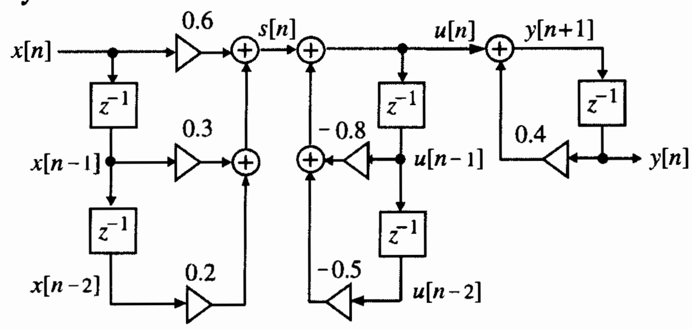
(c) Figure P2.1(c) is a cascade of a first-order section and a second-order section. The input-output relation remains unchanged if the ordering of the two sections is interchanged as shown below.



The second-order section can be redrawn as shown below without changing its input-output relation.



The second-order section can be seen to be cascade of two sections. Interchanging their ordering we finally arrive at the structure shown below:



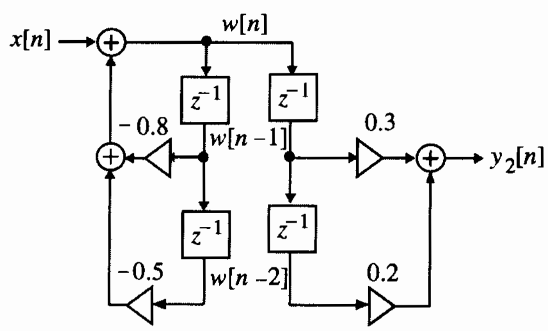
Analyzing the above structure we arrive at

$$s[n] = 0.6x[n] + 0.3x[n-1] + 0.2x[n-2],$$
$$u[n] = s[n] - 0.8u[n-1] - 0.5u[n-2],$$
$$y[n+1] = u[n] + 0.4y[n].$$

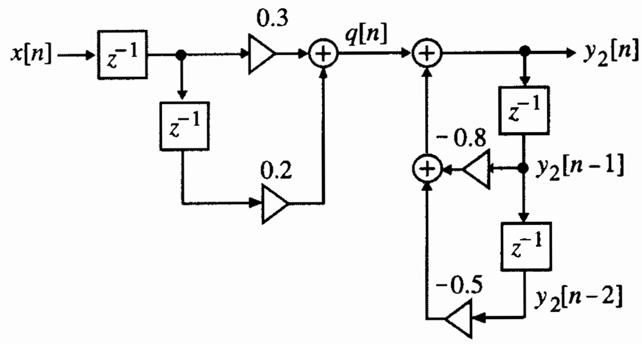
From $u[n] = y[n+1] - 0.4y[n]$. Substituting this in the second equation we get after some algebra $y[n+1] = s[n] - 0.4y[n] - 0.18y[n-1] + 0.8y[n-2]$. Making use of the first equation in this equation we finally arrive at the desired input-output relation

$$y[n] + 0.4y[n-1] + 0.18y[n-2] - 0.2y[n-3] = 0.6x[n-1] + 0.3x[n-2] + 0.2x[n-3].$$

(d) Figure P2.19(d) is a parallel connection of a first-order section and a second-order section. The second-order section can be redrawn as a cascade of two sections as indicated below:



Interchanging the order of the two sections we arrive at an equivalent structure shown below:



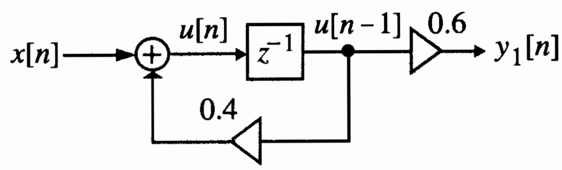
Analyzing the above structure we get

$$q[n] = 0.3x[n-1] + 0.2x[n-2],$$
$$y_2[n] = q[n] - 0.8y_2[n-1] - 0.5y_2[n-2].$$

Substituting the first equation in the second we have

$$y_2[n] + 0.8y_2[n-1] + 0.5y_2[n-2] = 0.3x[n-1] + 0.2x[n-2]. \quad (2\text{-}1)$$

Analyzing the first-order section of Figure P2.1(d) given below



we get

$$u[n] = x[n] + 0.4u[n-1],$$
$$y_1[n] = 0.6u[n-1].$$

Solving the above two equations we have

$$y_1[n] - 0.4y_1[n-1] = 0.6x[n-1]. \quad (2\text{-}2)$$

The output $y[n]$ of the structure of Figure P2.19(d) is given by

$$y[n] = y_1[n] + y_2[n]. \quad (2\text{-}3)$$

From Eq. (2-2) we get $0.8y_1[n-1] - 0.32y_1[n-2] = 0.48x[n-2]$ and $0.5y_1[n-2] - 0.2y_1[n-3] = 0.3x[n-3]$. Adding the last two equations to Eq. (2-2) we arrive at $y_1[n] + 0.4y_1[n-1] + 0.18y_1[n-2] - 0.2y_1[n-3]$

$$= 0.6x[n-1] + 0.48x[n-2] + 0.3x[n-3]. \quad (2\text{-}4)$$

Similarly, from Eq. (2-1) we get $-0.4y_2[n-1] - 0.32y_2[n-2] - 0.2y_2[n-3] = -0.12x[n-2] - 0.08x[n-3]$. Adding this equation to Eq. (2-1) we arrive at

$$y_2[n] + 0.4y_2[n-1] + 0.18y_2[n-2] - 0.2y_2[n-3]$$
$$= 0.3x[n-1] + 0.08x[n-2] - 0.08x[n-3]. \quad (2\text{-}5)$$

Adding Eqs. (2-4) and (2-5), and making use of Eq. (2-3) we finally arrive at the input-output relation of Figure P2.1(d) as:

$$y[n] + 0.4y[n-1] + 0.18y[n-2] - 0.2y[n-3] = 0.9x[n-1] + 0.56x[n-2] + 0.22x[n-3].$$

2.8 (a) $x_1^*[n] = \{1 - j4 \quad -2 - j5 \quad \underset{\uparrow}{3 + j2} \quad -7 - j3 \quad -1 - j\}$,

$x_1^*[-n] = \{-1 - j \quad -7 - j3 \quad \underset{\uparrow}{3 + j2} \quad -2 - j5 \quad 1 - j4\}$. Therefore

$x_{1,cs}[n] = \frac{1}{2}\left(x_1^*[n] + x_1^*[-n]\right) = \{j1.5 \quad -4.5 + j \quad \underset{\uparrow}{3} \quad -4.5 - j \quad -j1.5\}$,

$x_{1,ca}[n] = \frac{1}{2}\left(x_1^*[n] - x_1^*[-n]\right) = \{1 + j2.5 \quad 2.5 + j4 \quad \underset{\uparrow}{-j2} \quad -2.5 + j4 \quad -1 + j2.5\}$.

(b) $x_2[n] = e^{j\pi n/3}$. Hence, $x_2^*[n] = e^{-j\pi n/3}$ and thus, $x_2^*[-n] = e^{j\pi n/3} = x_2[n]$. Therefore, $x_{2,cs}[n] = \frac{1}{2}\left(x_2^*[n] + x_2^*[-n]\right) = e^{j2\pi n/3} = x_2[n]$, and

$x_{2,ca}[n] = \frac{1}{2}\left(x_2^*[n] - x_2^*[-n]\right) = 0.$

(c) $x_3[n] = je^{-j\pi n/5}$. Hence, $x_3^*[n] = -je^{j\pi n/5}$ and thus,

$x_3^*[-n] = -je^{-j\pi n/5} = -x_3[n]$. Therefore, $x_{3,cs}[n] = \frac{1}{2}\left(x_3^*[n] + x_3^*[-n]\right) = 0$, and

$x_{3,ca}[n] = \frac{1}{2}\left(x_3^*[n] - x_3^*[-n]\right) = x_3[n] = je^{-j\pi n/5}$.

2.9 (a) $x[n] = \{-4 \quad 5 \quad 1 \quad \underset{\uparrow}{-2} \quad -3 \quad 0 \quad 2\}$. Hence, $x[-n] = \{2 \quad 0 \quad -3 \quad \underset{\uparrow}{-2} \quad 1 \quad 5 \quad -4\}$.

Therefore, $x_{ev}[n] = \frac{1}{2}(x[n] + x[-n]) = \frac{1}{2}\{-2 \quad 5 \quad -2 \quad \underset{\uparrow}{-4} \quad -2 \quad 5 \quad -2\}$

$= \{-1 \quad 2.5 \quad -1 \quad \underset{\uparrow}{-2} \quad -1 \quad 2.5 \quad -1\}$

and $x_{od}[n] = \frac{1}{2}(x[n] - x[-n]) = \frac{1}{2}\{-6 \quad 5 \quad 4 \quad \underset{\uparrow}{0} \quad -4 \quad -5 \quad 6\}$

$= \{-3 \quad 2.5 \quad 2 \quad \underset{\uparrow}{0} \quad -2 \quad -2.5 \quad 3\}$.

(b) $y[n] = \{0 \quad 0 \quad 0 \quad 0 \quad 6 \quad \underset{\uparrow}{-3} \quad -1 \quad 0 \quad 8 \quad 7 \quad -2\}$. Hence,

$y[-n] = \{-2 \quad 7 \quad 8 \quad 0 \quad -1 \quad \underset{\uparrow}{-3} \quad 6 \quad 0 \quad 0 \quad 0 \quad 0\}$.

Therefore, $y_{ev}[n] = \frac{1}{2}(y[n] + y[-n]) = \{-1 \quad 3.5 \quad 4 \quad 0 \quad 2.5 \quad \underset{\uparrow}{-3} \quad 2.5 \quad 0 \quad 4 \quad 3.5 \quad -1\}$

and $y_{od}[n] = \frac{1}{2}(y[n] - y[-n]) = \{1 \quad -3.5 \quad -4 \quad 0 \quad 3.5 \quad \underset{\uparrow}{0} \quad -3.5 \quad 0 \quad 4 \quad 3.5 \quad -1\}$.

(c) $w[n] = \{0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \underset{\uparrow}{0} \quad 0 \quad 3 \quad 2 \quad 2 \quad -1 \quad 0 \quad -2 \quad 5\}$. Hence,

$w[-n] = \{5 \quad -2 \quad 0 \quad -1 \quad 2 \quad 2 \quad 3 \quad 0 \quad \underset{\uparrow}{0} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\}$. Therefore

$w_{ev}[n] = \frac{1}{2}(w[n] + w[-n])$

$= \{2.5 \quad -1 \quad 0 \quad -0.5 \quad 1 \quad 1 \quad 1.5 \quad 0 \quad \underset{\uparrow}{0} \quad 0 \quad 1.5 \quad 1 \quad 1 \quad -0.5 \quad 0 \quad -1 \quad 2.5\}$ and

$$w_{od}[n] = \frac{1}{2}(w[n] - w[-n])$$
$$= \{-2.5 \quad 1 \quad 0 \quad 0.5 \quad -1 \quad -1 \quad -1.5 \quad 0 \quad \underset{\uparrow}{0} \quad 0 \quad 1.5 \quad 1 \quad 1 \quad -0.5 \quad 0 \quad -1 \quad 2.5\}.$$

2.10 **(a)** $x_1[n] = \mu[n+2]$. Hence, $x_1[-n] = \mu[-n+2]$. Therefore,

$$x_{1,ev}[n] = \frac{1}{2}(\mu[n+2] + \mu[-n+2]) = \begin{cases} 1/2, & n \geq 3, \\ 1, & -2 \leq n \leq 2, \\ 1/2, & -3 \leq n, \end{cases}$$ and

$$x_{1,od}[n] = \frac{1}{2}(\mu[n+2] - \mu[-n+2]) = \begin{cases} 1/2, & n \geq 3, \\ 0, & -2 \leq n \leq 2, \\ -1/2, & -3 \leq n. \end{cases}$$

(b) $x_2[n] = \alpha^n \mu[n-3]$. Hence, $x_2[-n] = \alpha^{-n}\mu[-n-3]$. Therefore,

$$x_{2,ev}[n] = \frac{1}{2}\left(\alpha^n \mu[n-3] + \alpha^{-n}\mu[-n-3]\right) = \begin{cases} \frac{1}{2}\alpha^n, & n \geq 3, \\ 0, & -2 \leq n \leq 2, \\ \frac{1}{2}\alpha^{-n}, & -3 \leq n, \end{cases}$$ and

$$x_{2,od}[n] = \frac{1}{2}\left(\alpha^n \mu[n-3] - \alpha^{-n}\mu[-n-3]\right) = \begin{cases} \frac{1}{2}\alpha^n, & n \geq 3, \\ 0, & -2 \leq n \leq 2, \\ -\frac{1}{2}\alpha^{-n}, & -3 \leq n. \end{cases}$$

(c) $x_3[n] = n\alpha^n \mu[n]$. Hence, $x_3[-n] = -n\alpha^{-n}\mu[-n]$. Therefore,

$$x_{3,ev}[n] = \frac{1}{2}\left(n\alpha^n \mu[n] + (-n)\alpha^{-n}\mu[-n]\right) = \frac{1}{2}|n|\alpha^{|n|}$$ and

$$x_{3,od}[n] = \frac{1}{2}\left(n\alpha^n \mu[n] - (-n)\alpha^{-n}\mu[-n]\right) = \frac{1}{2}n\alpha^{|n|}.$$

(d) $x_4[n] = \alpha^{|n|}$. Hence, $x_4[-n] = \alpha^{|-n|} = \alpha^{|n|} = x_4[n]$. Therefore,

$$x_{4,ev}[n] = \frac{1}{2}(x_4[n] + x_4[-n]) = \frac{1}{2}(x_4[n] + x_4[n]) = x_4[n] = \alpha^{|n|}$$ and

$$x_{4,od}[n] = \frac{1}{2}(x_4[n] - x_4[-n]) = \frac{1}{2}(x_4[n] - x_4[n]) = 0.$$

2.11 $x_{ev}[n] = \frac{1}{2}\left(x[n] + x[-n]\right)$. Thus, $x_{ev}[-n] = \frac{1}{2}\left(x[-n] + x[n]\right) = x_{ev}[n]$. Hence, $x_{ev}[n]$ is an even sequence. Likewise, $x_{od}[n] = \frac{1}{2}\left(x[n] - x[-n]\right)$. Thus, $x_{od}[-n] = \frac{1}{2}\left(x[-n] - x[n]\right) = -x_{od}[n]$. Hence, $x_{od}[n]$ is an odd sequence.

2.12 (a) $g[n] = x_{ev}[n]x_{ev}[n]$. Thus, $g[-n] = x_{ev}[-n]x_{ev}[-n] = x_{ev}[n]x_{ev}[n] = g[n]$. Hence, $g[n]$ is an even sequence.

(b) $u[n] = x_{ev}[n]x_{od}[n]$. Thus, $u[-n] = x_{ev}[-n]x_{od}[-n] = x_{ev}[n](-x_{od}[n]) = -u[n]$. Hence, $u[n]$ is an odd sequence.

(c) $v[n] = x_{od}[n]x_{od}[n]$. Thus, $v[-n] = x_{od}[-n]x_{od}[-n] = (-x_{od}[n])(-x_{od}[n])$ $= x_{od}[n]x_{od}[n] = v[n]$. Hence, $v[n]$ is an even sequence.

2.13 (a) Since $x[n]$ is causal, $x[n] = 0,\ n < 0$. Also, $x[-n] = 0,\ n > 0$. Now, $x_{ev}[n] = \frac{1}{2}(x[n] + x[-n])$. Hence, $x_{ev}[0] = \frac{1}{2}(x[0] + x[0]) = x[0]$ and $x_{ev}[n] = \frac{1}{2}x[n], n > 0$. Combining the two equations we get $x[n] = \begin{cases} 2x_{ev}[n], & n > 0, \\ x_{ev}[n], & n = 0, \\ 0, & n < 0. \end{cases}$

Likewise, $x_{od}[n] = \frac{1}{2}(x[n] - x[-n])$. Hence, $x_{od}[0] = \frac{1}{2}(x[0] - x[0]) = 0$ and $x_{od}[n] = \frac{1}{2}x[n], n > 0$. Combining the two equations we get $x[n] = \begin{cases} 2x_{ev}[n], & n > 0, \\ 0, & n \le 0. \end{cases}$

(b) Since $y[n]$ is causal, $y[n] = 0,\ n < 0$. Also, $y[-n] = 0,\ n > 0$. Let $y[n] = y_{re}[n] + jy_{im}[n]$, where $y_{re}[n]$ and $y_{im}[n]$ are real causal sequences. Now, $y_{ca}[n] = \frac{1}{2}(y[n] - y^*[-n])$ Hence, $y_{ca}[0] = \frac{1}{2}(y[0] - y^*[0]) = jy_{im}[0]$ and $y_{ca}[n] = \frac{1}{2}y[n], n > 0$. Since $y_{re}[0]$ is not known, $y[n]$ cannot be fully recovered from $y_{ca}[n]$.

Likewise, $y_{cs}[n] = \frac{1}{2}(y[n] + y^*[-n])$ Hence, $y_{cs}[0] = \frac{1}{2}(y[0] + y^*[0]) = y_{re}[0]$ and $y_{cs}[n] = \frac{1}{2}y[n], n > 0$. Since $y_{im}[0]$ is not known, $y[n]$ cannot be fully recovered from $y_{cs}[n]$.

2.14 Since $x[n]$ is causal, $x[n] = 0,\ n < 0$. From the solution of Problem 2.13 we have

$$x[n] = \begin{cases} 2x_{ev}[n], & n > 0, \\ x_{ev}[n], & n = 0, \\ 0, & n < 0, \end{cases} = \begin{cases} 2\cos(\omega_o n), & n > 0, \\ 1 & n = 0, \\ 0, & n < 0, \end{cases} = 2\cos(\omega_o n)\mu[n] - \delta[n].$$

2.15 (a) $\{x[n]\} = \{A\alpha^n\}$ where A and α are complex numbers with $|\alpha| < 1$. Since for $n < 0$, $|\alpha|^n$ can become arbitrarily large, $\{x[n]\}$ is not a bounded sequence.

$=\{-2 \;\; 5 \;\; 3 \;\; 2 \;\; 2 \;\; -1 \;\; 0\}, 0 \le n \le 6$. Hence, one period of $\tilde{w}_p[n]$ is given by $\{-2 \;\; 5 \;\; 3 \;\; 2 \;\; 2 \;\; -1 \;\; 0\}, 0 \le n \le 6$.

2.29 $\tilde{x}[n] = A\cos(\omega_o n + \phi)$.

(a) $\tilde{x}[n] = \{1 \;\; -1 \;\; -1 \;\; 1 \;\; 1 \;\; -1 \;\; -1 \;\; 1\}$. Hence $A = \sqrt{2}, \omega_o = \pi/2, \phi = \pi/4$.

(b) $\tilde{x}[n] = \{0 \;\; -\sqrt{3} \;\; 0 \;\; \sqrt{3} \;\; 0 \;\; -\sqrt{3} \;\; 0 \;\; \sqrt{3}\}$. Hence $A = \sqrt{3},\ \omega_o = \pi/2$, $\phi = \pi/2$.

(c) $\tilde{x}[n] = \{1 \;\; -0.366 \;\; -1.366 \;\; -1 \;\; 0.366 \;\; 1.366\}$. Hence $A = \sqrt{2},\ \omega_o = \pi/3$, $\phi = \pi/4$.

(d) $\tilde{x}[n] = \{2 \;\; 0 \;\; -2 \;\; 0 \;\; 2 \;\; 0 \;\; -2 \;\; 0\}$. Hence $A = 2, \omega_o = \pi/2, \phi = 0$.

2.30 The fundamental period N of a periodic sequence with an angular frequency ω_o satisfies Eq. (2.47a) with the smallest value of N and r.

(a) $\omega_o = 0.5\pi$. Here Eq. (2.47a) reduces to $0.5\pi N = 2\pi r$ which is satisfied with $N = 4, r = 1$.

(b) $\omega_o = 0.8\pi$. Here Eq. (2.47a) reduces to $0.8\pi N = 2\pi r$ which is satisfied with $N = 5, r = 2$.

(c) We first determine the fundamental period N_1 of $\text{Re}\{e^{j\pi n/5}\} = \cos(0.2\pi n)$. In this case, Eq. (2.47a) reduces to $0.2\pi N_1 = 2\pi r_1$ which is satisfied with $N_1 = 10, r_1 = 1$. We next determine the fundamental period N_2 of $\text{Im}\{e^{j\pi n/10} = j\sin(0.1\pi n)$. In this case, Eq. (2.47a) reduces to $0.1\pi N_2 = 2\pi r_2$ which is satisfied with $N_2 = 20, r_2 = 1$. Hence the fundamental period N of $\tilde{x}_c[n]$ is given by $LCM(N_1, N_2) = LCM(10,20) = 20$.

(d) We first determine the fundamental period N_1 of $3\cos(1.3\pi n)$. In this case, Eq. (2.47a) reduces to $1.3\pi N_1 = 2\pi r_1$ which is satisfied with $N_1 = 20, r_1 = 13$. We next determine the fundamental period N_2 of $4\sin(0.5\pi n + 0.5\pi)$. In this case, Eq. (2.47a) reduces to $0.5\pi N_2 = 2\pi r_2$ which is satisfied with $N_2 = 4, r_2 = 1$. Hence the fundamental period N of $\tilde{x}_4[n]$ is given by $LCM(N_1, N_2) = LCM(20,4) = 20$.

(e) We first determine the fundamental period N_1 of $5\cos(1.5\pi n + 0.75\pi)$. In this case, Eq. (2.47a) reduces to $1.5\pi N_1 = 2\pi r_1$ which is satisfied with $N_1 = 4, r_1 = 3$. We next determine the fundamental period N_2 of $4\cos(0.6\pi n)$. In this case, Eq. (2.47a) reduces to $0.6\pi N_2 = 2\pi r_2$ which is satisfied with $N_2 = 10, r_2 = 3$. We finally determine the fundamental period N_3 of $\sin(0.5\pi n)$. In this case, Eq. (2.47a) reduces to $0.5\pi N_3 = 2\pi r_3$ which is satisfied with $N_3 = 4, r_3 = 1$. Hence the fundamental period N of $\tilde{x}_5[n]$ is given by $LCM(N_1, N_2, N_3) = LCM(4,10,4) = 20$.

2.31 The fundamental period N of a periodic sequence with an angular frequency ω_o satisfies Eq. (2.47a) with the smallest value of N and r.

(a) $\omega_o = 0.6\pi$. Here Eq. (2.47a) reduces to $0.6\pi N = 2\pi r$ which is satisfied with $N = 10, r = 3$.

(b) $\omega_o = 0.28\pi$. Here Eq. (2.47a) reduces to $0.28\pi N = 2\pi r$ which is satisfied with $N = 50, r = 7$.

(c) $\omega_o = 0.45\pi$. Here Eq. (2.47a) reduces to $0.45\pi N = 2\pi r$ which is satisfied with $N = 40, r = 9$.

(d) $\omega_o = 0.55\pi$. Here Eq. (2.47a) reduces to $0.55\pi N = 2\pi r$ which is satisfied with $N = 40, r = 11$.

(e) $\omega_o = 0.65\pi$. Here Eq. (2.47a) reduces to $0.65\pi N = 2\pi r$ which is satisfied with $N = 40, r = 13$.

2.32 $\omega_o = 0.08\pi$. Here Eq. (2.47a) reduces to $0.08\pi N = 2\pi r$ which is satisfied with $N = 25, r = 1$. For a sequence $\tilde{x}_2[n] = \sin(\omega_2 n)$ with a fundamental period of $N = 25$, Eq. (2.47a) reduces to $25\omega_2 = 2\pi r$. For example, for $r = 2$ we have $\omega_2 = 4\pi/25 = 0.16\pi$. Another sequence with the same fundamental period is obtained by setting $r = 3$ which leads to $\omega_3 = 6\pi/25 = 0.24\pi$. The corresponding periodic sequences are therefore $\tilde{x}_2[n] = \sin(0.16\pi n)$ and $\tilde{x}_3[n] = \sin(0.24\pi n)$.

2.33 The three parameters A, Ω_o, and ϕ of the continuous-time signal $x_a(t)$ can be determined from $x[n] = x_a(nT) = A\cos(\Omega_o nT + \phi)$ by setting 3 distinct values of n. For example

$$x[0] = A\cos\phi = \alpha,$$
$$x[-1] = A\cos(-\Omega_o T + \phi) = A\cos(\Omega_o T)\cos\phi + A\sin(\Omega_o T)\sin\phi = \beta,,$$
$$x[1] = A\cos(\Omega_o T + \phi) = A\cos(\Omega_o T)\cos\phi - A\sin(\Omega_o T)\sin\phi = \gamma.$$

Substituting the first equation into the last two equations and then adding them we get $\cos(\Omega_o T) = \dfrac{\beta + \gamma}{2\alpha}$ which can be solved to determine Ω_o. Next, from the second equation we have $A\sin\phi = \beta - A\cos(\Omega_o T)\cos\phi = \beta - \alpha\cos(\Omega_o T)$. Dividing this equation by the last equation on the previous page we arrive at $\tan\phi = \dfrac{\beta - \alpha\cos(\Omega_o T)}{\alpha\sin(\Omega_o T}$ which can be solved to determine ϕ. Finally, the parameter is determined from the first equation of the last page.

2.46 (a) $\alpha^n\mu[n]\circledast\mu[n]=\sum_{k=-\infty}^{\infty}\alpha^k\mu[k]\mu[n-k]=\sum_{k=0}^{\infty}\alpha^k\mu[n-k]=\begin{cases}\sum_{k=0}^{n}\alpha^k, & n\geq 0,\\ 0, & n<0.\end{cases}$

(b) $n\alpha^n\mu[n]\circledast\mu[n]=\sum_{k=-\infty}^{\infty}k\alpha^k\mu[k]\mu[n-k]=\sum_{k=0}^{\infty}k\alpha^k\mu[n-k]=\begin{cases}\sum_{k=0}^{n}k\alpha^k, & n>0,\\ 0, & n\leq 0.\end{cases}$

2.47 Now from Eq. (2.72) an arbitrary input $x[n]$ can be expressed as

$x[n]=\sum_{k=-\infty}^{\infty}x[k]\delta[n-k]$ which can be rewritten using Eq. (2.41b) as

$$x[n]=\sum_{k=-\infty}^{\infty}x[k]\big(\mu[n-k]-\mu[n-k-1]\big)=\sum_{k=-\infty}^{\infty}x[k]\mu[n-k]-\sum_{k=-\infty}^{\infty}x[k]\mu[n-k-1].$$

Since $s[n]$ is the response of an LTI system for an input $\mu[n]$, $s[n-k]$ is the response for an input $\mu[n-k]$ and $s[n-k-1]$ is the response for an input $\mu[n-k-1]$. Hence, the output for an input $\sum_{k=-\infty}^{\infty}x[k]\mu[n-k]-\sum_{k=-\infty}^{\infty}x[k]\mu[n-k-1]$ is given by

$$y[n]=\sum_{k=-\infty}^{\infty}x[k]s[n-k]-\sum_{k=-\infty}^{\infty}x[k]s[n-k-1]=x[n]\circledast s[n]-x[n-1]\circledast s[n-1].$$

2.48 $y[n]=\sum_{m=-\infty}^{\infty}h[m]\tilde{x}[n-m]$. Hence,

$y[n+kN]=\sum_{m=-\infty}^{\infty}h[m]\tilde{x}[n+kN-m]=\sum_{m=-\infty}^{\infty}h[m]x[n-m]=y[n]$. Thus, $y[n]$ is also a periodic sequence with a period N.

2.49 In this problem we make use of the identity $\delta[n-m]\circledast\delta[n-r]=\delta[n-m-r]$.

(a) $y_1[n]=x_1[n]\circledast h_1[n]=\big(3\delta[n-2]-2\delta[n+1]\big)\circledast\big(-\delta[n+2]+4\delta[n]+2\delta[n-1]\big)$

$=-3\delta[n-2]\circledast\delta[n+2]+12\delta[n-2]\circledast\delta[n]-6\delta[n-2]\circledast\delta[n-1]+2\delta[n+1]\circledast\delta[n+2]$

$-8\delta[n+1]\circledast\delta[n]+4\delta[n+1]\circledast\delta[n-1]$. Hence

$y_1[n]=-3\delta[n]+12\delta[n-2]-6\delta[n-3]+2\delta[n+3]-8\delta[n+1]+4\delta[n]$
$=2\delta[n+3]-8\delta[n+1]+\delta[n]+12\delta[n-2]-6\delta[n-3].$

(b) $y_2[n]=x_2[n]\circledast h_2[n]=\big(5\delta[n-3]+2\delta[n+1]\big)\circledast\big(3\delta[n-4]+1.5\delta[n-2]-\delta[n+1]\big)$

$=15\delta[n-3]\circledast\delta[n-4]+7.5\delta[n-3]\circledast\delta[n-2]-5\delta[n-3]\circledast\delta[n+1]+6\delta[n+1]\circledast\delta[n-4]$

$+3\delta[n+1]\circledast\delta[n-2]-2\delta[n+1]\circledast\delta[n+1]=15\delta[n-7]+7.5\delta[n-5]-5\delta[n-2]$

$+6\delta[n-3]+3\delta[n-1]-2\delta[n+2].$

(c) $y_3[n] = x_1[n] \circledast h_2[n] = \left(-3\delta[n-2] - 2\delta[n+1]\right) \circledast \left(3\delta[n-4] + 1.5\delta[n-2] - \delta[n+1]\right)$

$= 9\delta[n-2] \circledast \delta[n-4] + 4.5\delta[n-2] \circledast \delta[n-2] - 3\delta[n-2] \circledast \delta[n+1] - 6\delta[n+1] \circledast \delta[n-4]$

$-3\delta[n+1] \circledast \delta[n-2] + 2\delta[n+1] \circledast \delta[n+1] = 9\delta[n-6] + 4.5\delta[n-4] - 3\delta[n-1]$

$-6\delta[n-3] - 3\delta[n-1] - 3\delta[n-1] + 2\delta[n+2] = 2\delta[n+2] - 6\delta[n-1] - 6\delta[n-3]$
$+4.5\delta[n-4] + 9\delta[n-6]$.

(d) $y_4[n] = x_2[n] \circledast h_1[n] = \left(5\delta[n-3] + 2\delta[n+1]\right) \circledast \left(-\delta[n+2] + 4\delta[n] - 2\delta[n-1]\right)$

$= -5\delta[n-3] \circledast \delta[n+2] + 20\delta[n-3] \circledast \delta[n] - 10\delta[n-3] \circledast \delta[n-1] - 2\delta[n+1] \circledast \delta[n+2]$

$+8\delta[n+1] \circledast \delta[n] - 4\delta[n+1] \circledast \delta[n-1] = -5\delta[n-1] + 20\delta[n-3] - 10\delta[n-4] - 2\delta[n+3]$

$+8\delta[n+1] - 4\delta[n] = -2\delta[n+3] + 8\delta[n+1] - 4\delta[n] - 5\delta[n-1] + 20\delta[n-3] - 10\delta[n-4]$.

2.50 (a) $u[n] = x[n] \circledast y[n]$

$= \{-24,\ 42,\ -5,\ -20,\ -45,\ 23,\ 66,\ -25,\ -42,\ -17,\ 22,\ 14,\ -4\},\ -4 \le n \le 8$.

(b) $v[n] = x[n] \circledast w[n]$

$= \{-12,\ 7,\ 5,\ 10,\ -16,\ -3,\ -28,\ 30,\ 13,\ -6,\ -15,\ -4,\ 10\},\ -1 \le n \le 11$.

(c) $g[n] = w[n] \circledast y[n]$

$= \{18,\ 3,\ 3,\ -14,\ 25,\ 26,\ 60,\ -11,\ -16,\ -14,\ 26,\ 39,\ -10\},\ 1 \le n \le 13$.

2.51 $y[n] = \sum_{m=N_1}^{N_2} g[m]h[n-m]$. Now, $h[n-m]$ is defined for $M_1 \le n-m \le M_2$. Thus, for $m = N_1$, $h[n-m]$ is defined for $M_1 \le n - N_1 \le M_2$, or equivalently, for $M_1 + N_1 \le n \le M_2 + N_1$. Likewise, for $m = N_2$, $h[n-m]$ is defined for $M_1 \le n - N_2 \le M_2$, or equivalently, for $M_1 + N_2 \le n \le M_2 + N_2$. For the specified sequences $N_1 = -3, N_2 = 4, M_1 = 2, M_2 = 6$. **(a)** The length of $y[n]$ is $M_2 + N_2 - M_1 - N_1 + 1 = 6 + 4 - 2 - (-3) + 1 = 12$. **(b)** The range of n for $y[n] \ne 0$ is $\min(M_1 + N_1, M_2 + N_2) \le n \le \max(M_1 + N_1, M_2 + N_2)$, i.e., $M_1 + N_1 \le n \le M_2 + N_2$. For the specified sequences the range of n is $-1 \le n \le 10$.

2.60 (a) $y[n] = g_{ev}[n] \circledast h_{ev}[n] = \sum_{k=-\infty}^{\infty} h_{ev}[n-k] g_{ev}[k]$. Now,

$y[-n] = \sum_{k=-\infty}^{\infty} h_{ev}[-n-k] g_{ev}[k]$. Replace k by $-k$. Then the summation on the left becomes $y[-n] = \sum_{k=-\infty}^{\infty} h_{ev}[-n+k] g_{ev}[-k] = \sum_{k=-\infty}^{\infty} h_{ev}[-(n-k)] g_{ev}[-k]$ $= y[n]$. Hence $g_{ev}[n] \circledast h_{ev}[n]$ is an even sequence.

(b) $y[n] = g_{ev}[n] \circledast h_{od}[n] = \sum_{k=-\infty}^{\infty} h_{od}[n-k] g_{ev}[k]$. Now,

$y[-n] = \sum_{k=-\infty}^{\infty} h_{od}[-n-k] g_{ev}[k] = \sum_{k=-\infty}^{\infty} h_{od}[-n+k] g_{ev}[-k]$
$= \sum_{k=-\infty}^{\infty} h_{od}[-(n-k)] g_{ev}[-k] = -\sum_{k=-\infty}^{\infty} h_{od}[n-k] g_{ev}[k] = -y[n]$.
Hence $g_{ev}[n] \circledast h_{od}[n]$ is an odd sequence.

(c) $y[n] = g_{od}[n] \circledast h_{od}[n] = \sum_{k=-\infty}^{\infty} h_{od}[n-k] g_{od}[k]$. Now,

$y[-n] = \sum_{k=-\infty}^{\infty} h_{od}[-n-k] g_{od}[k] = \sum_{k=-\infty}^{\infty} h_{od}[-n+k] g_{od}[-k]$
$= \sum_{k=-\infty}^{\infty} h_{od}[-(n-k)] g_{od}[-k] = \sum_{k=-\infty}^{\infty} h_{od}[n-k] g_{od}[k] = y[n]$.
Hence $g_{od}[n] \circledast h_{od}[n]$ is an even sequence.

2.61 The impulse response of the cascade is given by $h[n] = h_1[n] \circledast h_2[n]$ where

$h_1[n] = \alpha^n \mu[n]$ and $h_2[n] = \beta^n \mu[n]$. Hence, $h[n] = \left(\sum_{k=0}^{n} \alpha^k \beta^{n-k}\right)\mu[n]$.

2.62 Now $h[n] = \alpha^n \mu[n]$. Therefore $y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k] = \sum_{k=0}^{\infty} \alpha^k x[n-k]$
$= x[n] + \sum_{k=1}^{\infty} \alpha^k x[n-k] = x[n] + \alpha \sum_{k=0}^{\infty} \alpha^k x[n-1-k] = x[n] + \alpha\, y[n-1]$.
Hence, $x[n] = y[n] - \alpha y[n-1]$. Thus the inverse system is given by
$y[n] = x[n] - \alpha x[n-1]$. The impulse response of the inverse system is given by
$h[n] = \{1,\ \alpha\},\ 0 \le n \le 1$.

2.63 From the results of Problem 2.62 we have $h[n] = \left(\sum_{k=0}^{n} \alpha^k \beta^{n-k}\right)\mu[n]$. Now,

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k] h[k] = \sum_{k=-\infty}^{\infty} \left(\sum_{m=0}^{k} \alpha^m \beta^{k-m} \right) \mu[m] x[n-k] = \sum_{k=0}^{\infty} \left(\sum_{m=0}^{k} \alpha^m \beta^{k-m} \right) x[n-k]$$

$= x[n] + \sum_{k=1}^{\infty} \left(\sum_{m=0}^{k} \alpha^m \beta^{k-m} \right) x[n-k]$. Substituting $r = k-1$ in the last expression we get

$$y[n] = x[n] + \sum_{r=0}^{\infty} \left(\sum_{m=0}^{r+1} \alpha^m \beta^{r+1-m} \right) x[n-r-1] = x[n] + \sum_{r=0}^{\infty} \left(\sum_{m=0}^{r} \alpha^m \beta^{r+1-m} + \alpha^{r+1} \right) x[n-r-1]$$

$$= x[n] + \beta \sum_{r=0}^{\infty} \left(\sum_{m=0}^{r} \alpha^m \beta^{r-m} \right) x[n-r-1] + \sum_{r=0}^{\infty} \alpha^{r+1} x[n-r-1]$$

$= x[n] + \beta y[n-1] + \alpha x[n-1] + \alpha^2 x[n-2] + \alpha^3 x[n-3] + \ldots$. The inverse system is therefore given by $x[n] = y[n] - (\alpha+\beta) y[n-1] + \alpha\beta y[n-2]$.

2.64 (a) $h[n] = h_1[n] \circledast h_2[n] \circledast h_3[n] \circledast h_3[n] + h_1[n] \circledast h_2[n] + h_3[n] \circledast h_4[n]$.

(b) $h[n] = h_4[n] + \dfrac{h_1[n] \circledast h_2[n] \circledast h_3[n]}{1 - h_1[n] \circledast h_2[n] \circledast h_5[n]}$.

2.65 $h[n] = h_1[n] \circledast h_2[n] + h_3[n]$. Now

$h_1[n] \circledast h_2[n] = (2\delta[n-2] - 3\delta[n+1]) \circledast (\delta[n-1] + 2\delta[n+2])$

$= 2\delta[n-2] \circledast \delta[n-1] - 3\delta[n+12] \circledast \delta[n-1] + 2\delta[n-2] \circledast \delta[n+2] - 3\delta[n+1] \circledast \delta[n+2]$

$= 2\delta[n-3] + \delta[n] - 6\delta[n-3]$. Therefore,
$h[n] = 2\delta[n-3] + \delta[n] - 6\delta[n-3] + 5\delta[n-5] + 7\delta[n-3] + 2\delta[n-1] - \delta[n] + 3\delta[n+1]$.

2.66 (a) The length of $x[n]$ is $8-4+1=5$. Using $x[n] = \dfrac{1}{h[0]}\left\{ y[n] - \sum_{k=0}^{3} h[k] x[n-k] \right\}$ we arrive at $\{x[n]\} = \{3,\ -2,\ 0,\ 1,\ 2\},\ 0 \le n \le 4$.

(b) The length of $x[n]$ is $7-4+1=4$. Using $x[n] = \dfrac{1}{h[0]}\left\{ y[n] - \sum_{k=0}^{3} h[k] x[n-k] \right\}$ we arrive at $\{x[n]\} = \{1,\ 2,\ 3,\ 4\},\ 0 \le n \le 3$.

(c) The length of $x[n]$ is $8-5+1=4$. Using $x[n] = \dfrac{1}{h[0]}\left\{ y[n] - \sum_{k=0}^{4} h[k] x[n-k] \right\}$ we arrive at $\{x[n]\} = \{1,\ -2,\ 3,\ -1\},\ 0 \le n \le 3$.

2.67 $y[n] = ay[n-1] + bx[n]$. Hence, $y[0] = ay[-1] + bx[0]$. Next, $y[1] = ay[0] + bx[1]$ $= a(ay[-1] + bx[0]) + bx[1] = a^2 y[-1] + abx[0] + bx[1]$. Continuing further in a similar way we obtained $y[n] = a^{n+1} y[-1] + \sum_{k=0}^{n} a^{n-k} bx[k]$.

(a) Let $y_1[n]$ be the output due to an input $x_1[n]$. Then
$y_1[n] = a^{n+1} y[-1] + \sum_{k=0}^{n} a^{n-k} bx_1[k]$. If $x_1[n] = x[n-n_o]$, then
$y_1[n] = a^{n+1} y[-1] + \sum_{k=0}^{n} a^{n-k} bx[k-n_o] = a^{n+1} y[-1] + \sum_{r=0}^{n-n_o} a^{n-n_o-r} bx[r]$.
However,
$y[n-n_o] = a^{n+1} y[-1] + \sum_{k=0}^{n} a^{n-k} bx[k-n_o] = a^{n-n_o+1} y[-1] + \sum_{r=0}^{n-n_o} a^{n-n_o-r} bx[r]$.
Hence $y_1[n] \ne y[n-n_o]$ if $y[-1] \ne 0$, i.e., the system is time-variant. The system is time-invariant if and only if $y[-1] = 0$, as then $y_1[n] = y[n-n_o]$.