EE4150 Digital Signal Processing

Dr. Hsiao-Chun Wu

Final Examination, fall of 2005

Time: 12:30 ~2:30 p.m., Tuesday, December 13, 2005

Please ALWAYS work on the FRONT SIDE of each page. No answer on the BACK SIDE of any page will be GRADED! Ask for additional blank papers if necessary!

You may check any textbook, classnote or other references alone during the test. However, NO CHEATING or COLLABORATION is allowed and such violation of the university regulations will be reported.

Please write down your name and social security number here:

Full Name: ________________SOLUTION____________________

Social Security Number: ____

Partition	Score
Question 1	
Question 2	
Question 3	
Question 4	
Total	

Question 1 (20%)

Consider a real sequence $h[n] = h_{ev}[n] + h_{od}[n], \quad -\infty < n < \infty$. Prove that

$$\sum_{n=-\infty}^{\infty}\left(h[n]\otimes h[n]\right) = \sum_{n=-\infty}^{\infty}\left(h_{ev}[n]\otimes h_{ev}[n]\right) + \sum_{n=-\infty}^{\infty}\left(h_{od}[n]\otimes h_{od}[n]\right).$$

Answer to Question 1:

$$\sum_{n=-\infty}^{\infty}\left(h[n]\otimes h[n]\right) = \sum_{n=-\infty}^{\infty}\left(h_{ev}[n]\otimes h_{ev}[n]\right) + \sum_{n=-\infty}^{\infty}\left(h_{od}[n]\otimes h_{od}[n]\right) + 2\sum_{n=-\infty}^{\infty}\left(h_{ev}[n]\otimes h_{od}[n]\right)$$

Since $h_1[n] = h_{ev}[n]\otimes h_{od}[n]$ is odd, $h_1[n] = -h_1[-n]$ for n=1, 2, ….

$$\begin{aligned}
h_1[0] &= \sum_{m=-\infty}^{\infty} h_{ev}[m]h_{od}[n-m] = \frac{1}{4}\sum_{m=-\infty}^{\infty}\{h[m]+h[-m]\}\{h[n-m]-h[-n+m]\} \\
&= \frac{1}{4}\sum_{m=-\infty}^{\infty}\{h[m]+h[-m]\}\{h[-m]-h[m]\} \\
&= \frac{1}{4}\left\{-\sum_{m=-\infty}^{\infty}h^2[m] + \sum_{m=-\infty}^{\infty}h^2[-m]\right\} = 0
\end{aligned}$$

$$\begin{aligned}
\sum_{n=-\infty}^{\infty}\left(h_{ev}[n]\otimes h_{od}[n]\right) &= \sum_{n=-\infty}^{\infty}h_1[n] = h_1[0] + \sum_{n=-\infty}^{-1}h_1[n] + \sum_{n=1}^{\infty}h_1[n] = h_1[0] - \sum_{n=1}^{\infty}h_1[n] + \sum_{n=1}^{\infty}h_1[n] \\
&= 0
\end{aligned}$$

Thus,

$$\sum_{n=-\infty}^{\infty}\left(h[n]\otimes h[n]\right) = \sum_{n=-\infty}^{\infty}\left(h_{ev}[n]\otimes h_{ev}[n]\right) + \sum_{n=-\infty}^{\infty}\left(h_{od}[n]\otimes h_{od}[n]\right).$$

Question 2 (30%)

(a) A two-sided real sequence is given by $x[n]=\alpha^{n}$, $-\infty<n<\infty$, where α is an arbitrary real value. Show the condition for α to make the Z-transform $X(z)=Z\{x[n]\}$ exist. (15%)

(b) A two-sided real sequence is given by $y[n]=\alpha^{|n|}$, $-\infty<n<\infty$, where α is an arbitrary real value. Show the condition for α to make the Z-transform $Y(z)=Z\{y[n]\}$ exist. (15%)

Answer to Question 2:

(a)

$X(z)=Z\{x[n]\}=\sum_{n=-\infty}^{\infty}\alpha^{n}z^{-n}=\sum_{n=0}^{\infty}\alpha^{n}z^{-n}+\sum_{n=-\infty}^{-1}\alpha^{n}z^{-n}$, The ROC for $\sum_{n=0}^{\infty}\alpha^{n}z^{-n}$ is $|z|>|\alpha|$ and the ROC for $\sum_{n=-\infty}^{-1}\alpha^{n}z^{-n}$ is $|z|<|\alpha|$. There is no intersection and hence the $X(z)$ does not exist for any α.

(b)

$Y(z)=Z\{y[n]\}=\sum_{n=-\infty}^{\infty}\alpha^{|n|}z^{-n}=\sum_{n=0}^{\infty}\alpha^{n}z^{-n}+\sum_{n=-\infty}^{-1}\alpha^{-n}z^{-n}$ The ROC for $\sum_{n=0}^{\infty}\alpha^{n}z^{-n}$ is $|z|>|\alpha|$ and the ROC for $\sum_{n=-\infty}^{-1}\alpha^{n}z^{-n}$ is $|z|<\frac{1}{|\alpha|}$. The intersection exists when $|\alpha|<1$. Thus $Y(z)$ exists when $|\alpha|<1$.

Question 3 (30%)

The input signal of a filter can be described as

$$x[n] = 2\sin(an) + 3\cos(bn - c) - 2\cos(dn + e)\ .$$

The impulse response of such a filter is given by

$$h[n] = \frac{\sin((n-f)g)}{(n-f)}.$$

(a) If $\pi > a > b > g > d > 0$, what is the output $y[n] = x[n] \otimes h[n]$? (15%)

(b) If $\pi > g > a > b > d > 0$, what is the output $y[n] = x[n] \otimes h[n]$? (15%)

Answer to Question 3:

$$H(e^{j\omega}) = H_{LP}(e^{j\omega})e^{-jf\omega}, \text{ where } H_{LP}(e^{j\omega}) = \begin{cases} \pi, & 0 \le |\omega| \le g \\ 0, & g \le |\omega| \le \pi \end{cases}.$$

(a)

$$y[n] = -2\pi\cos(d(n-f) + e) = -2\pi\cos(dn - df + e)$$

(b)

$$y[n] = 2\pi\sin(an - af) + 3\pi\cos(bn - bf - c) - 2\pi\cos(dn - df + e)$$

<u>Question 4</u> (20%)

An analog signal is described as $x_a(t) = \cos(100\pi t) + \dfrac{\sin(1000\pi t)}{\pi t}$. What is the smallest sampling frequency $F_s = \dfrac{1}{T}$ in Hz for $x_a(t)$ to be uniquely determined by its samples $x_a(nT), \quad -\infty < n < \infty$?

<u>Answer to Question 4:</u>

$$\frac{1}{2\pi}\int_{-\Omega_c}^{\Omega_c} e^{j\Omega t} d\Omega = \frac{1}{2\pi}\frac{2j\sin(\Omega_c t)}{jt} = \frac{\sin(\Omega_c t)}{\pi t} \Rightarrow \int_{-\infty}^{\infty} \frac{\sin(\Omega_c t)}{\pi t} e^{-j\Omega t} dt = \begin{cases} 1, & 0 \le |\Omega| \le \Omega_c \\ 0, & |\Omega| > \Omega \end{cases}$$

Hence, the bandwidth Ω_m for $X_a(j\Omega)$ is 1000π and the minimum sampling frequency $F_s = \dfrac{2\Omega_m}{2\pi} = 1000$ Hz.