# Chapter 7. LTI Discrete-Time Systems in the Transform Domain

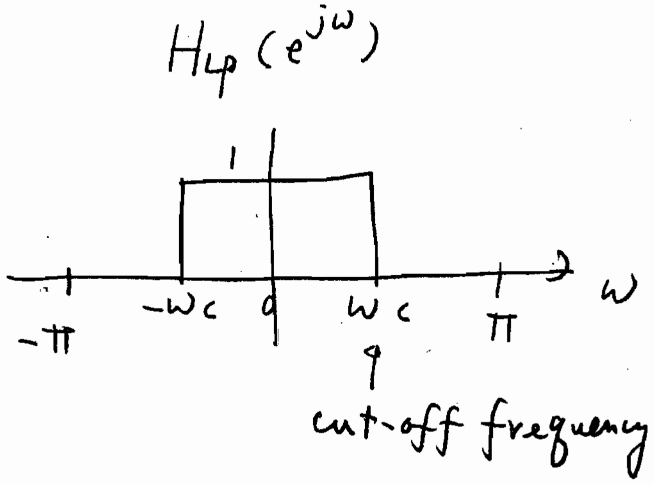
## 7.1 Transfer Function Classification Based on Magnitude Spectrum

### 7.1.1 Digital Filters with Ideal Magnitude Responses

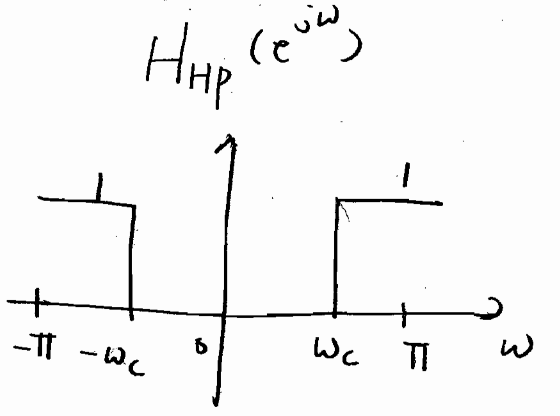
An ideal magnitude response should possess the ideal passband (magnitude 1) and ideal stopband (magnitude 0) in the primary frequency range $(-\pi \le \omega < \pi)$ of the DTFT domain.

Hence, the four types of ideal fiters can be illustrated as follows: (Fig. 7.1)

(a) ideal lowpass filter.

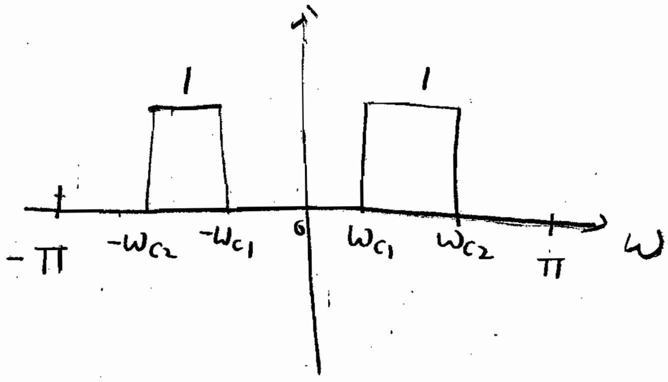



(b) ideal highpass filter



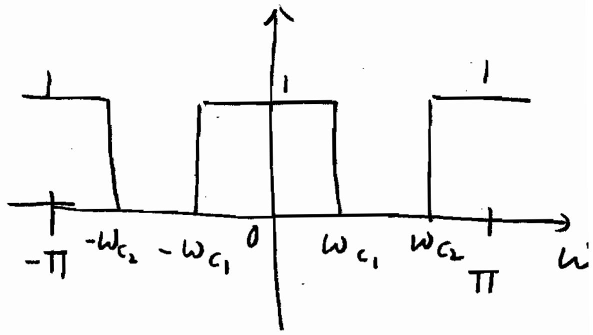
(c) ideal bandpass filter

$H_{BP}(e^{j\omega})$




(d) ideal band stop filter

$H_{BS}(e^{j\omega})$




## 7.1.3 Allpass Transfer Function

The IIR transfer function of $M^{th}$-order, $A_M(z)$ can denote an all-pass causal filter as

$$A_M(z) = \pm \prod_{i=1}^{M} \left( \frac{-\lambda_i^* + z^{-1}}{1 - \lambda_i z^{-1}} \right),$$

where $\lambda_i$ is the $i^{th}$ pole of $A_M(z)$.

Check the magnitude spectrum

$$|A_M(e^{j\omega})| = |A_M(z)|\Big|_{z=e^{j\omega}}$$

$$= \prod_{i=1}^{M} \left| \frac{-\lambda_i^* + e^{-j\omega}}{1 - \lambda_i e^{-j\omega}} \right|$$

$$= \prod_{i=1}^{M} \frac{\cancel{|e^{-j\omega}|}\;|\cancel{1 - \lambda_i^* e^{j\omega}}|}{|\cancel{1 - \lambda_i e^{-j\omega}}|} = 1$$

Since $|A_M(e^{jw})| = 1$, $A_M(z)$ is the all-pass transfer function.

## 7.2 Transfer Function Classification Based on Phase Spectrum

### 7.2.1 Zero-phase Transfer Function

The zero-phase transfer function $H_{zp}(z)$ is the transfer function such that

$$\measuredangle H_{zp}(e^{jw}) = \measuredangle H_{zp}(z)\Big|_{z=e^{jw}}$$

$$= 0, \quad \forall w.$$

The zero-phase transfer function will induce "no time-delay" for all the frequency components in the signal passed through it.

### 7.2.2 Linear-Phase Transfer Function

The linear-phase transfer function $H_{LP}(z)$ is the transfer function such that

$$\measuredangle H_{LP}(e^{j\omega}) = \measuredangle H_{LP}(z) \Big|_{z=e^{j\omega}}$$

$$= -\omega D, \quad -\pi \le \omega < \pi$$

Since $H_{LP}(e^{j\omega}) = |H_{LP}(e^{j\omega})| \, e^{-j\omega D}$,

then if the input signal $x[n] = e^{j\omega n}$,

we have the output $y[n] = e^{j\omega n} H_{LP}(e^{j\omega})$

$$= |H_{LP}(e^{j\omega})| \, e^{j\omega(n-D)}.$$

Thus, all the frequency components in the signal will experience an identical time delay D when such a signal passes through a linear-phase system.

## 7.2.3 Minimum-phase and Maximum-phase Transfer Functions.

The minimum-phase transfer function $H_{mp}(z)$ is the transfer function whose zeros and poles are all inside the unit circle. $|z|=1$.

The maximum-phase transfer function is the transfer function whose zeros and poles are all outside the unit circle $|z|=1$.