# Traffic-Aware Joint Allocation of Uplink Control and Communication Channels in Multicast Systems

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Abstract—In multicast wireless access systems such as a trunked mobile radio system, channel allocation is performed based on multicast groups instead of radio units. Thus, in these systems, the call source units are these groups, which we also call the users. In a practical system, these users are finite in number, and each user or group may consist of many radio units. In this paper, we discuss the coupling of uplink control and communication segments (layers) of such finite source systems, in which the performance of one layer directly affects that of the other. The conventional studies model these system segments separately and, thus, are unable to capture the coupling issues in networks with the finite sources' constraint. We first propose a novel model that incorporates this coupling by jointly quantifying both the collision loss at the control layer and congestion loss at the communication layer. Under our proposed framework, we further optimize the number of uplink control and communication channels to minimize the joint total loss rate given a constraint on the total number of available channels. In addition, we demonstrate the capability of our proposed model in estimating the invisible actual traffic load and provide guidelines for developing an algorithm for the traffic-aware allocation of channels, based on the proposed model.

*Index Terms*—Coupling, finite sources, group-based calls, joint uplink-channel allocations, multicast.

# I. INTRODUCTION

# A. Motivation

In *multicast* wireless access systems, such as trunked mobile radio systems [1], [2], radio units are divided into multicast talk groups. In such systems, multicasting generated from within a talk group is a predominately primary traffic. Moreover, when a radio talks, the rest of the radios in the group listen; therefore, a group that is busy in communication does not generate a new call request. That is why, in these systems, the number of groups busy in communications affects the call arrival traffic over the uplink. The future wideband and nationwide first-responder network, i.e., the *FirstNet* [3], [4], is one of the modern exam-

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ples of such systems. In these systems, an exclusively dedicated repeater and a frequency channel, which is called the *control channel*, is allocated for the *access control* process, due to the urgent nature of public safety calls. When a radio needs to talk, it sends a call request to the *access station* (AS) over an uplink control channel (see [5, Fig. 4.1]). After successfully receiving a call request from a radio, the AS then assigns communication resources to the caller and the rest of the radios of the group, to broadcast the call throughout the group. Thus, in such systems, the channel allocation is performed on a group basis. Therefore, instead of radios, the *call source units* or the system "*users*" are the groups. Note that these groups or users are also finite in number in practical systems; therefore, we call such systems as *finite source* wireless access systems. However, each user or group may consist of many radios [6].

There are two types of frequency channels in the firstresponder networks, which are control-channel-based wireless access systems as described earlier. One of the types is the access control or simply control channel that is dedicated for the access control process, whereas the rest of them are of the type of communication channels. Therefore, we divide the system into two layers, based on both operation and resources, namely, the access layer and the communication layer. Here, the term *layer* does not correspond to the one used in the open systems interconnection model; rather, it only signifies a particular segment of the system. Note that, at the access layer, multiple users may select the same control channel at the same time to send their call requests to the access point, called the AS, which results in contention. Such contention is taken care of by a prescribed multiple access control (MAC) protocol [7] through which a subset of contending users get hold of the control channel. As a result, the rest of the failed attempts contribute to the *collision* loss. For those calls that successfully go through the control channel, there are still chances of further losses due to nonavailability of channels at the communication layer. Such losses are coined as congestion losses in this paper. The narrowband voice is one of the foremost services in the first-responder networks. In these systems, unlike the commercial applications, both the collision and congestion losses severely deteriorate the quality of voice communications among the first responders in time-critical emergency situations.

Since the bandwidth is a scarce resource, we need to allocate efficiently channels for the access control and communications to attain a balanced tradeoff between congestion and collision losses in the system. More specifically, the wireless access systems need a mechanism for an optimal channel allocation for the access control and communications, such that the total system loss is minimized, for a given total number of channels available. This is the problem under consideration in this paper. Note that, our results, presented in Section VI-E, demonstrate that the conventional system with a single control channel performs considerably worse than the system with the same total number of channels but having an optimal allocation for the access control and communications.

What makes such a tradeoff issue acute is a persistent coupling relationship between two types of losses in the networks with finite sources. This is because the number of control channels and the number of users contending for system access affect the congestion loss by affecting the number of users further demanding for communication channel access, whereas the number of communication channels and the service rate affect the collision loss at the MAC layer by affecting the number of idle and thus potential contending users, given a finite set of users in the system. The latter effect is because, in such networks, only idle users (groups) can possibly become contending ones. Consequently, it is critical to consider allocating channels to *jointly* cater the needs for both access control and communication services, in terms of minimizing both types of losses while considering the coupling, and a constraint on the total number of channels, for such networks with finite sources. The primary goal of this paper is to provide our solutions to such problems. To this end, we propose such a novel model for these systems that jointly quantifies the collision and congestion losses, and that provides a framework to jointly allocate the control and communication channels to minimize the total system loss.

Another important issue that needs to be resolved is the limitation of using the model for the channel allocation for practical systems. The limitation is that the practical multiple access systems usually only keep records of the calls that successfully get access to the system, but no record is kept for the call requests that are lost due to collision. Hence, for the practical systems, the number of calls that successfully get access and the congestion loss are visible, whereas the actual traffic load and the collision loss are *invisible*. Note that the optimal channel allocation requires the knowledge of this invisible actual traffic load. Our proposed model can help overcome this limitation as well, by providing the statistical relationships between the invisible actual traffic load and the visible system states, which can be used to estimate the invisible actual traffic load based on the known values of the visible system states. It is this capability of estimating the invisible actual traffic load, provided by our model, which brings the traffic awareness characteristic to our proposed channel allocation scheme.

Since collision loss is invisible in practical systems, a possibly misleading performance metric, which we call as the *reported system loss rate*, is often used as the system's performance metric. This reported loss is described in detail in Section V. The reported loss rate is evaluated based on the available system data of the recorded calls and does not count the collision loss. Recall that in the first-responder networks, unlike the commercial applications, both the collision and congestion losses severely deteriorate the quality of voice communications among the first responders in time-critical emergency situations. Moreover, as discussed in Section VI-D, collision loss is a significant part of the total loss. In fact, most of the time, it is far more significant than the congestion loss. Therefore, since the reported loss rate does not incorporate the information of the collision loss, it is a considerably underestimated measure of system loss. Thus, the reported loss is misleading if we try to make system design decisions on its basis. However, our model also resolves this issue by evaluating both the collision and the congestion losses that can be used to evaluate the actual total system loss.

# B. Related Works

In most existing works, collision losses in MAC layer and congestion losses in data-link layer have been treated separately, e.g., trunked radio systems [8], Long-Term Evolutionbased systems [9], ad hoc wireless networks [10], [11], and intervehicle networks [12], [13]. Note that these studies focus only on the MAC layer, without considering the communication channel allocation and the congestion loss. On the other hand, separate studies on the communication channel allocation are also available, but without considering the control channel allocation, e.g., [14]-[17]. Furthermore, for those works where the finite source systems are studied, e.g., in [8], busy sources are still considered to generate Poisson-type traffic, thereby without the coupling problem we study here. Therefore, the existing models and approaches cannot completely capture the control-communication coupling that we have addressed in this paper. For the networks with finite sources. The most relevant works to-date is [18], where a simple dynamic algorithm for the allocation of control and communication channels is proposed without offering any model or analysis for deriving this algorithm nor showing the optimality of the proposed algorithm. Moreover, note that an abridged version of this study has been presented in [19], wherein we discuss the system model to elaborate the coupling of control and communication layers, and demonstrate the concept of *channel allocation map* to represent the optimal channel allocation.

# C. Organization of this Paper

The rest of this paper is organized as follows. We present the system model in Section II. The problem formulation is discussed in Section III. A demonstration of estimating the invisible actual traffic load, along with remarks on the development of the traffic-aware channel allocation algorithm, is discussed in Section IV. We then discuss the reported loss rate in Section V and the channel allocation map, along with the rest of the numerical results, in Section VI. Finally, we conclude in Section VII.

# II. SYSTEM MODEL

We have presented all the symbols with descriptions in [5, Tab. 4.1]. We consider a single *cell* or a *site* of a multiple access communication system, with a finite number of users (groups), e.g., M, and an AS. The AS consists of N frequency channels. Assume that all N channels are of same quality. Out of these N channels, there are  $N_x$  number of control channels and  $N_c = N - N_x$  number of communication channels.

(n-1)th AP	(n-1)th FP	nth AP	nth FP	
◄	(n-1)th SP	-	nth SP	

Fig. 1. Time-horizon discretization.

We assume that the bandwidths of the control and communication channels are the same. However, in practice, these bandwidth requirements can be different. Note that our proposed model can be easily modified to incorporate this scenario, but such modifications in the model and the corresponding analysis are left as a part of future work. Moreover, the main focus of this paper is to highlight and explore the coupling of collision and congestion losses over the MAC layer, and to propose a model that can help. Therefore, we consider a perfect physical layer to avoid any unnecessary complications in the model, and we assume that a new call is blocked only due to a collision at the MAC layer or due to an unavailability of the communication channel. Assuming an error-free physical layer is a simple but reasonable assumption because we can leave the error detection and error correction to be separately treated in that layer. This is a standard approach usually adopted in the conventional MAC layer studies without considering the crosslayer issues [8]. In this paper, the system under consideration employs a synchronized random access protocol, e.g., slotted-Aloha [20], [21]. Under this protocol, in our model, the access procedure is synchronized and discrete in time. Moreover, a user only attempts for the system access at the beginning of an *access time slot*. During the same time slot, the user gets a reply from the AS regarding the success or failure of the access attempt. Note that we consider a loss system, i.e., there is no queueing buffer for the calls. This assumption helps analyze and highlight the coupling of the control and communication layers, and its impact on the allocation of channels, which is the main objective of this paper, without making the model too complicated. We model the system as a discrete-time Markov chain (DTMC), with the time horizon divided into discrete-time segments, called the system periods (SPs). We assume that the time scale of SP is such that each idle user (group) can have at the most one call arrival during an SP. Each SP is divided into two time segments as shown in Fig. 1.

The time segment, at the beginning of an SP, during which all the access process takes place, is called the *access period* (AP). During an AP, all the contending users try to access the system, and the AS replies to their requests. At the end of an AP, the communication channel allocation occurs that marks the beginning of the *access free period* (FP) in the SP. Thus, an AP can also be considered the precommunication channel allocation segment of an SP, whereas an FP can be considered the postcommunication channel allocation segment. Similar time segmentation is used in [22] as well.

We present a graphical representation of our system model in Fig. 2. Consider an *n*th SP, and suppose there were  $I_{n-1}$  number of *idle users* during the (n-1)th SP. These are the users that were not busy in communications and thus were not occupying the communication channels, during the (n-1)th FP. Out of these  $I_{n-1}$  idle users, some or all users will contend for system access in the next slot, i.e., the *n*th SP. Let  $L_n$  be the number of



Fig. 2. System model.

contenders during the *n*th SP that decide to contend out of  $I_{n-1}$  idle users. Out of these  $L_n$  contenders, some users successfully get access to the system, whereas the rest of the contenders are blocked due to collision. The details of the access procedure considered for this paper are provided in the following. In Section III-C, we also describe how to incorporate any other access control protocol in the model.

Access Control Procedure: Assume that a call arrives for an idle user during an (n-1)th SP. The user then contends for the system access in the *n*th AP, by sending a call request to the AS over a randomly selected control channel. A call request over a control channel successfully gets access to the system to enter the communication channel allocation process if and only if only one contender selects that channel. Otherwise, if two or more contenders select the same control channel in the same AP, then all such call requests are blocked due to collision, and the collided users immediately go into a collision resolution procedure (CRP) mode, which will be explained in the following. We assume that a contending user does not change the selected control channel for the CRP mode. If an access success does not occur, even during the CRP, the call is blocked and is counted as a collision loss for the user. The user then immediately goes to the idle state.

Collision Resolution Procedure: We consider a simple CRP in which each contender either contends for the system access with probability  $\sigma \in (0, 1)$ , at the beginning of each access slot during an AP, or it waits during that access slot with probability  $1 - \sigma$ . We call  $\sigma$  as the *CRP contending rate per contender*, and it is same for all the users. Moreover, we assume that an AP can have at the most  $s \in \mathbb{N}^+$  access slots for CRP. Therefore, s is the maximum number of access retrials per user allowed during an AP. Thus, the CRP ends either when an access success occurs, or after s access slots in an AP. Note that an access success occurs for an access control channel when exactly one user selects that channel during a given access slot. This CRP is a special case of the standard *collision avoidance* technique, where the random backoff method is employed [23]. In our CRP, the backoff time, in terms of the number of access slots, is geometrically distributed with parameter  $1 - \sigma$ .

Let  $X_n$  be the number of users or calls successfully getting access to the system out of  $L_n$  contenders, and  $Q_n = L_n - X_n$ be the number of users or call requests blocked due to collision, during the *n*th AP. We also call  $X_n$  as the system access state. Out of  $X_n$  users successfully getting access to the system, some of the users are allocated communication channels at the beginning of the *n*th FP depending on the number of communication channels available, whereas the rest of the users are blocked due to congestion, i.e., unavailability of communication channels. Let  $Z_n$  be the number of busy communication channels during the *n*th FP. Note that the number of busy communications are the same. Then,  $Z_{n-1}$  will be the number of busy communication channels during the (n-1)th FP. Out of these  $Z_{n-1}$ communication channels, some number of channels, e.g.,  $Y_n$ , still remain busy during the *n*th AP and FP. Thus,  $Y_n$  is also the number of communication channels monitored busy during the *n*th AP, before the communication channel allocation for the *n*th SP. Let  $\omega$  be the probability that a communication channel that is busy in a given FP remains busy for the next FP as well. We call  $\omega$  as the *communication channel occupancy rate* per busy user. We assume that  $\omega$  is same for all the users and communication channels. Given  $Z_{n-1}$ ,  $Y_n$  is binomially distributed as follows:

$$\Pr(Y_n = y | Z_{n-1} = z_1) = \begin{cases} \binom{z_1}{y} \omega^y (1 - \omega)^{z_1 - y}, & \text{if } y \in \Omega_{\mathrm{Y}}(z_1) \\ 0, & \text{otherwise.} \end{cases}$$
(1)

 $\forall y \in \overline{\Omega}_{Y}$  and  $\forall z_{1} \in \Omega_{Z}$ . Here,  $\Omega_{Y}(z_{1}) = \{0, 1, \dots, z_{1}\}$ , and  $\overline{\Omega}_{Y} = \Omega_{Z} = \{0, 1, \dots, N_{c}\}$ . Note that, here, we assume a memoryless and stationary discrete-time call-service process. It is similar to the commonly used exponential service time process, which is also memoryless but continuous in time, as in [24].

Communication Channel Allocation: The number of idle communication channels during the *n*th AP is  $N_c - Y_n$  before the allocation of communication channels to the new calls. If  $X_n \leq N_c - Y_n$ , i.e., the number of calls successfully getting access to the system is less than or equal to the number of idle communication channels, then there will be no congestion. In this case, all new  $X_n$  calls are allocated communication channels without any call blocking, making  $Z_n = X_n + Y_n$ . Otherwise,  $X_n > N_c - Y_n$ , and the system randomly selects  $N_c - Y_n$  calls out of  $X_n$  new calls, making all the channels busy during the *n*th FP and thus  $Z_n = N_c$ . We call this a *random call (or user) selection*. The remaining  $X_n - (N_c - Y_n)$ calls are blocked due to congestion. Thus, we can write the number of busy communication channels  $Z_n$  in terms of the  $X_n$  and  $Y_n$  as follows:

$$Z_n = \begin{cases} X_n + Y_n, & \text{if } X_n + Y_n \le N_c \\ N_c, & \text{otherwise.} \end{cases}$$
(2)

Let  $G_n$  be the calls lost due to congestion during an *n*th SP. We can also write  $G_n$  in terms of the  $X_n$  and  $Y_n$  as follows:

$$G_n = \begin{cases} 0, & \text{if } X_n + Y_n \le N_c \\ X_n + Y_n - N_c, & \text{otherwise.} \end{cases}$$
(3)

Note that, in finite-source wireless systems, e.g., trunked radio systems, a talk group busy in communications does not generate a new call request. Thus, only an idle user, not busy in communications, can generate a new call request and contend for system access. The number of idle users during (n - 1)th SP is  $I_{n-1} = M - Z_{n-1}$ , where  $Z_{n-1}$  is the number of channels, or equivalently users, busy in communications during the (n-1)th FP. Out of these  $I_{n-1}$  idle users,  $L_n$  users will contend for system access in the next slot, i.e., the *n*th SP. Let  $\lambda$ be the probability that a call arrives for an idle user during an SP. We call  $\lambda$  as the *call arrival rate per idle user* and assume that it is same for all the users in the system. Now,  $L_n$  is binomially distributed and depends on the number of idle users  $M - Z_{n-1}$ in the (n-1)th SP, i.e.,

$$\Pr(L_n = l | Z_{n-1} = z_1)$$

$$= \begin{cases} \binom{M-z_1}{l} \lambda^l (1-\lambda)^{M-z_1-l}, & \text{if } l \in \Omega_L(z_1) \\ 0, & \text{otherwise.} \end{cases}$$
(4)

 $\forall l \in \overline{\Omega}_L \ \forall z_1 \in \Omega_Z$ . Here,  $\Omega_L(z_1) = \{0, 1, \dots, M - z_1\}$ , and  $\overline{\Omega}_L = \{0, 1, \dots, M\}$ . Note that, for a sufficiently large M and small  $\lambda$ , the binomial traffic model (4) approaches the Poisson model, which is commonly used for analysis of communication systems, e.g., in [24]–[26]. Moreover, note that  $\Pr(L_n = l | Z_{n-1} = z_1)$  is a function of  $L_n$ ,  $Z_{n-1}$ , M, and  $\lambda$ , and can be easily updated to incorporate any other call traffic model of interest.

As explained earlier, the call traffic model (4) models the arrival of calls such that the new calls are generated only by the idle users that are not busy in communications. This also agrees with the operation of practical finite source systems, with both unicast traffic and multicast traffic, because, in such systems, a user busy in communications does not generate a new call request. Due to the same phenomenon, the collision loss at the control layer is affected by the communication layer performance, as discussed in Section I. In this paper, we assume that the time horizon begins at n = 0, with the initial values of all the processes being 0, i.e.,  $L_0 = 0$ ,  $X_0 = 0$ ,  $Y_0 = 0$ ,  $Q_0 = 0$ , and  $G_0 = 0$ , with probability 1. Since  $X_0 = 0$  and  $Y_0 = 0$ , therefore,  $Z_0 = 0$  according to (2). Moreover, all the expressions in this paper are for  $n \in \mathbb{N}^+$ , where  $\mathbb{N}^+$  is a set of all strictly positive natural numbers, i.e., natural numbers without 0.

As described so far and shown in Fig. 2,  $L_n$ ,  $X_n$ ,  $Y_n$ ,  $Q_n$ , and  $G_n$  depend on  $Z_{n-1}$ . Moreover, given  $Z_{n-1}$ ,  $L_n$ ,  $X_n$ ,  $Y_n$ , and therefore  $Q_n$  and  $G_n$  are stationary processes. Furthermore, given  $Z_{n-1}$ ,  $\{L_n\}_{n=1}^{\infty}$ ,  $\{X_n\}_{n=1}^{\infty}$ ,  $\{Y_n\}_{n=1}^{\infty}$ , and therefore,  $\{Q_n\}_{n=1}^{\infty}$  and  $\{G_n\}_{n=1}^{\infty}$  are sequences of *conditionally independent and identically distributed* random variables. Moreover, (2) shows that  $Z_n$  depends on  $X_n$  and  $Y_n$  both of which, in turn, depend on  $Z_{n-1}$ . Thus,  $Z_n$  depends on  $Z_{n-1}$ ,  $\forall n \in \mathbb{N}^+$ . Moreover,  $Z_n$  depends on its previous history but only through  $Z_{n-1}$ , i.e.,  $Z_n$  does not depend on the rest of the past values given  $Z_{n-1}$ . Therefore, the process  $Z_n$  forms a firstorder DTMC in our model.

*Remarks on Visibility of Parameters:* In practice, multiple access systems only keep records of the calls that successfully get access to the system. However, no record is kept for the call requests that are lost due to collision. Therefore, we classify our model parameters, which are also shown in Fig. 2, into two types. The first type of parameters is of those that are recorded by or known to the practical systems, namely,  $\omega$ ,  $X_n$ ,  $Z_n$ ,  $G_n$ , M, and N. We call them the *visible* parameters. The

second type of parameters is of those that are not recorded by or unknown to the practical systems, namely,  $\lambda$ ,  $L_n$ , and  $Q_n$ . We call them the *invisible* parameters. The key invisible parameter is  $\lambda$ , and we also call it the *invisible actual traffic load*. In Section IV, we demonstrate how we can use our model to estimate  $\lambda$ , using the known values of the visible parameters, under the model proposed here. Note that this capability of estimating  $\lambda$  provided by our model brings the *traffic awareness* characteristic to our proposed channel allocation scheme, as described in Section IV. Moreover, in Section V-A, we remark how we can acquire the required knowledge of  $L_n$  and  $Q_n$ based on the estimated value of  $\lambda$ .

# **III. PROBLEM FORMULATION**

#### A. Performance Metric

The performance metric in this paper is the *total loss rate* of the system. We define it as the *fraction of calls blocked* during an SP in the long run, i.e., infinite time horizon or  $n \to \infty$ . It is represented as  $\beta$  and is given by

$$\beta = \lim_{k \to \infty} \frac{\sum_{n=1}^{k} Q_n + G_n}{\sum_{n=1}^{k} L_n} = \beta_Q + \beta_G.$$
 (5)

Here,  $\beta_Q$  is the measure of collision loss at the control layer, and  $\beta_G$  is the measure of congestion loss at the communication layer, as explained in the following.

Collision Loss Rate: We define the collision loss rate  $\beta_Q$ as the fraction of calls blocked due to collision, during an SP in the long run. Mathematically, irrespective of any assumed access control and communication channel allocation protocol, for  $\sum_{\forall z_1 \in \Omega_Z} E[L_n | Z_{n-1} = z_1] \cdot \pi_{z_1} \neq 0$ , we have

$$\beta_Q = \lim_{k \to \infty} \frac{\sum_{n=1}^k Q_n}{\sum_{n=1}^k L_n} \tag{6}$$

$$= \frac{\sum_{\forall z_1 \in \Omega_Z} E[Q_n | Z_{n-1} = z_1] \cdot \pi_{z_1}}{\sum_{\forall z_1 \in \Omega_Z} E[L_n | Z_{n-1} = z_1] \cdot \pi_{z_1}}.$$
 (7)

Here,  $\Omega_Z = \{0, 1, \dots, N_c\}$  is the sample space of  $Z_n, E[\cdot]$  is the expectation operator, and  $\pi_{z_1}$  is the steady-state probability of the Markov chain  $Z_n$  for state  $Z_n = z_1$ , i.e.,  $\lim_{n\to\infty} \Pr(Z_n = z_1)$ . Note that we get (7) from (6) by using the Law of Large Numbers, and the fact that given  $Z_{n-1}$ , both  $\{L_n\}_{n=1}^{\infty}$  and  $\{Q_n\}_{n=1}^{\infty}$  are sequences of *conditionally* independent and identically distributed random variables. The steady-state probability  $\pi_{z_1}$  is presented in Section III-B. The numerator in (7) is the average number of calls blocked due to collision during an SP, over the long run, whereas the denominator is the average number of contenders or arrived calls in an SP, over the long run. Since, given  $Z_{n-1}$ ,  $L_n$  is binomially distributed as shown in (4), therefore  $E[L_n|Z_{n-1}] =$  $z_1 = (M - z_1)\lambda$ . Moreover, we know that  $Q_n = L_n - L_n$  $X_n$ ; therefore,  $E[Q_n|Z_{n-1}=z_1]=E[L_n|Z_{n-1}=z_1]-E[X_n|$  $Z_{n-1} = z_1$ ]. If we represent the sample space of  $X_n$  as the set  $\Omega_x = \{0, 1, ..., N_x\}$ , then we have  $E[X_n | Z_{n-1} = z_1] =$  $\sum_{\forall x \in \Omega_r} x \Pr(X_n = x | Z_{n-1} = z_1)$ . We call  $\Pr(X_n = x | Z_{n-1} = z_1)$  $z_1$ ) as the conditional distribution of the system access state, and is presented in Section III-C.

Congestion Loss Rate: We define the congestion loss rate  $\beta_G$  as the fraction of calls blocked due to congestion, during an SP in the long run. Mathematically, similar to (7), for  $\sum_{\forall z_1 \in \Omega_Z} E[L_n | Z_{n-1} = z_1] \cdot \pi_{z_1} \neq 0$ 

$$\beta_G = \lim_{k \to \infty} \frac{\sum_{n=1}^k G_n}{\sum_{n=1}^k L_n} \tag{8}$$

$$= \frac{\sum_{\forall z_1 \in \Omega_Z} E[G_n | Z_{n-1} = z_1] \cdot \pi_{z_1}}{\sum_{\forall z_1 \in \Omega_Z} E[L_n | Z_{n-1} = z_1] \cdot \pi_{z_1}}.$$
 (9)

The numerator in (9) is the average number of calls blocked due to congestion during an SP, over the long run. If we represent the sample space of  $G_n$  as  $\Omega_G = \{0, 1, \ldots, N_x\}$ , then we have  $E[G_n|Z_{n-1} = z_1] = \sum_{\forall g \in \Omega_G} g \cdot \Pr(G_n = g|Z_{n-1} = z_1)$ . We call  $\Pr(G_n = g|Z_{n-1} = z_1)$  as the *conditional distribution of the congestion loss.* Its evaluation is similar to (10)–(12) and provided in detail in [5, Sec. 4.3.3].

# B. Steady-State Distribution of $Z_n$

As mentioned in Section II,  $Z_n$  forms a DTMC. Moreover, this chain is irreducible and ergodic. Therefore, a steadystate distribution exists for  $Z_n$ . Let  $\pi_z$  be the steady-state probability of the Markov chain  $Z_n$  for state  $Z_n = z$ , i.e.,  $\pi_z = \lim_{n\to\infty} \Pr(Z_n = z)$ . Let  $\underline{\Pi}$  be a column vector such that its *z*th element is  $\pi_z$ . Then, the steady-state distribution is the solution of a system of linear equations, i.e.,  $\underline{\Pi} = \mathbf{P}_z^t \underline{\Pi}$  and  $\sum_{\forall z \in \Omega_Z} \pi_z = 1$ . Here,  $\mathbf{P}_z^t$  is the transpose of matrix  $\mathbf{P}_z$ , which is the transition probability matrix for DTMC  $Z_n$ . Thus, the element of matrix  $\mathbf{P}_z$  at  $z_1$ th row and  $z_2$ th column is the state transition probability, i.e.,  $\mathbf{P}_z(z_1, z_2) = \Pr(Z_n = z_2 | Z_{n-1} = z_1)$ ,  $\forall z_1, z_2 \in \Omega_Z$ . This transition probability can be evaluated by marginalizing the conditional joint distribution  $\Pr(Z_n = z_2, X_n = x, Y_n = y | Z_{n-1} = z_1)$ , as follows:

$$\Pr(Z_n = z_2 | Z_{n-1} = z_1) = \sum_{\forall (x,y) \in \overline{\Omega}_{XY}} \Pr(Z_n = z_2, X_n = x, Y_n = y | Z_{n-1} = z_1)$$
(10)

 $\forall z_1, z_2 \in \Omega_Z$ . Moreover,  $\overline{\Omega}_{XY} = \Omega_X \times \overline{\Omega}_Y$ . The summand in (10) can be broken down using the *chain rule of probability* and the facts that given  $X_n$  and  $Y_n, Z_n$  is independent of  $Z_{n-1}$ , and given  $Z_{n-1}, Y_n$  is independent of  $X_n$ , i.e.,

$$\Pr(Z_n = z_2, X_n = x, Y_n = y | Z_{n-1} = z_1) = \Pr(Z_n = z_2 | Y_n = y, X_n = x) \times \Pr(Y_n = y | Z_{n-1} = z_1) \times \Pr(X_n = x | Z_{n-1} = z_1)$$
(11)

 $\forall x \in \Omega_X, \forall y \in \overline{\Omega}_Y, \text{ and } \forall z_1, z_2 \in \Omega_Z.$ 

Consider the first term in (11). It is given by the following, based on our knowledge of (2):

$$\Pr(Z_n = z_2 | Y_n = y, X_n = x) = \begin{cases} 1, & \text{if } (x + y \le N_c \text{ and } z_2 = x + y) \\ & \text{or } (x + y > N_c \text{ and } z_2 = N_c) \\ 0, & \text{otherwise} \end{cases}$$
(12)

 $\forall x \in \Omega_X \ \forall y \in \overline{\Omega}_Y$ , and  $\forall z_1, z_2 \in \Omega_Z$ . The second term in (11),  $\Pr(Y_n = y | Z_{n-1} = z_1)$ , is given by (1), whereas the last term  $\Pr(X_n = x | Z_{n-1} = z_1)$  is presented in Section III-C. Thus, the summand in (10) can be evaluated using (11), which can help us in evaluating the transition probabilities using (10). These transition probabilities are then used to form the transition probability matrix  $\mathbf{P}_z$ , which in turn is used to evaluate the steady-state distribution  $\pi_z \ \forall z \in \Omega_Z$ .

#### C. Conditional Distribution of Access State $X_n$

Now, in the context of the access control procedure described in Section II, we evaluate the conditional distribution of  $X_n$ , given  $Z_{n-1}$ . This distribution can be evaluated by marginalizing the conditional joint distribution  $Pr(X_n = x, L_n = l | Z_{n-1} = z_1)$  as follows:

$$\Pr(X_n = x | Z_{n-1} = z_1) = \sum_{\forall l \in \overline{\Omega}_L} \Pr(X_n = x, L_n = l | Z_{n-1} = z_1)$$
(13)

 $\forall x \in \Omega_X$  and  $\forall z_1 \in \Omega_Z$ . The summand in (13) can be broken down using the chain rule of probability and the fact that given  $L_n$ ,  $X_n$  is independent of  $Z_{n-1}$ , which is also shown in Fig. 2. Thus

$$\Pr(X_n = x, L_n = l | Z_{n-1} = z_1)$$
  
=  $\Pr(X_n = x | L_n = l) \times \Pr(L_n = l | Z_{n-1} = z_1)$  (14)

 $\forall x \in \Omega_X, \forall l \in \overline{\Omega}_L, \text{ and } \forall z_1 \in \Omega_Z.$ 

The second product term in (14),  $\Pr(L_n = l | Z_{n-1} = z_1)$ , is given by (4), and in [5, Sec 4.3.5], we describe how to evaluate the first term,  $\Pr(X_n = x | L_n = l)$ , according to the assumed access protocol. Moreover, it is only  $\Pr(X_n = x | L_n = l)$  tht needs to be updated if the protocol changes, the rest of the framework remains unchanged. This concludes the evaluation of  $\Pr(X_n = x | Z_n = z_1)$ . Recall that we need this conditional distribution of the system access state for the analysis discussed so far. Moreover, note that the evaluation of this distribution requires values of the model parameters, namely, M,  $N_x$ ,  $N_c$ , s,  $\sigma$ ,  $\lambda$ , and  $\omega$ .

#### D. Optimization Problem: Joint Channel Allocation

Our objective is to find the optimal number of access control and communication channels. The criterion for optimality is the minimization of the total loss rate of the system  $\beta$ . At the same time, we also have two constraints. The first constraint is that there should always be at least one control and one communication channel in the system. According to this constraint, we need to select the optimal  $N_x$  and  $N_c$  out of the set  $\{1, \ldots, N-1\}$ . As mentioned in Section II, the second constraint is that the total number of channels available in the system is N. According to this constraint, we have  $N_x + N_c =$ N. This means that, for a fixed number of control channels  $N_x$ , all of the remaining  $N - N_x$  available channels are used for communications, in order to minimize the loss rate. Moreover, we also need to select an optimal  $\sigma$  that minimizes the loss rate, for any given channel allocation. Thus, for a given N total number of channels, the optimal number of control channels  $N_{xo}$  is as follows:

$$N_{xo} = \underset{N_x \in \{1, \dots, N-1\}}{\arg\min} \hat{\beta}(N_x, N_c), \text{ s.t. : } N_x + N_c = N.$$
  
= 
$$\underset{N_x \in \{1, \dots, N-1\}}{\arg\min} \hat{\beta}(N_x, N - N_x).$$
(15)

Here,  $\forall N_x \in \{1, \dots, N-1\}$ , and  $N_c = N - N_x$ , we have

$$\hat{\beta}(N_x, N_c) = \min_{\sigma \in (0,1)} \beta(N_x, N_c, \sigma).$$
(16)

Note that, here, we have considered the variation in  $\beta$  with respect to  $N_x$ ,  $N_c$ , and  $\sigma$ , although  $\beta$  depends on other model parameters as well, which we assumed known for the stated optimization problem. According to the constraint, the optimal number of communication channels is  $N_{co} = N - N_{xo}$ . Moreover, note that this is a discrete optimization problem and does not have a closed-form solution. Therefore, we numerically solve the problem by exhaustive search and analyze the results in Section VI. However, in a practical system, instead of finding the optimal channel allocation by exhaustive search, an already generated channel allocation map is stored in the system's memory as a lookup table that is used for an instantaneous channel allocation, whenever there is a change in traffic load. As discussed in detail in Section VI-C, the channel allocation map provides the optimal channel allocation for all possible values of the traffic parameters, namely,  $\lambda$  and  $\omega$ . Note that while generating a channel allocation map, the optimal values of  $\sigma$  are also obtained, which can also be stored in the form of a lookup table, for all values of  $\lambda$  and  $\omega$ .

#### **IV. TRAFFIC-AWARE CHANNEL ALLOCATION**

For the given values of system parameters M, N, and s, we can solve (15) and find the optimal number of control and communication channels, provided we know the values of the traffic parameters  $\lambda$  and  $\omega$ . Recall from Section II that  $\omega$  is a traffic parameter that is visible to the system, whereas  $\lambda$ is a parameter that is invisible. However, we can develop an estimator based on our system model, which uses the values of visible parameters, namely,  $X_n$ ,  $Y_n$ , and  $Z_n$ , from the available system data, to estimate the *invisible actual traffic load*  $\lambda$ . One such estimator is the maximum likelihood (ML) estimator, as discussed in [5, Sec. 4.4.1], which is given by

$$\hat{\lambda}_k = \underset{a \in (0,1)}{\arg\max} \prod_{n=1}^{k} \Pr(X_n = x_n | Z_{n-1} = z_{n-1}, \lambda = a).$$
(17)

Once we get an estimate of  $\lambda$ , e.g.,  $\hat{\lambda}$ , we can then allocate the control and communication channels optimally, for the known values of  $\omega$  and the system parameters, as demonstrated in [5, Fig. 4.4].

#### A. Learning the Optimal Channel Allocation

Recall from Section III-C that, for a given value of  $\lambda = a$ , the evaluation of the distribution  $\Pr(X_n = x_n | Z_{n-1} = z_{n-1}, \lambda = a)$  requires values of the model parameters, namely, M,  $N_x$ ,  $N_c$ , s,  $\sigma$ , and  $\omega$ . Since this distribution is needed to estimate  $\lambda$ ,

as shown in (17); therefore, we need to know the values of  $N_x$ ,  $N_c$ , and  $\sigma$  to get the ML estimate of  $\lambda$ , i.e.,  $\hat{\lambda}$ , for the given values of M, N, s, and  $\omega$ . Hence, we can start with arbitrary values of  $N_x$ ,  $N_c$ , and  $\sigma$  to get an initial estimate of  $\lambda$ , using our proposed model and solving (17). Later, we can use this estimate of the invisible traffic load to update the optimal values of  $N_x$ ,  $N_c$ , and  $\sigma$  by using our proposed model and solving (15) and (16). Recall from Section III-D that, in practical systems, instead of solving (15) and (16) every time the load varies, we use the already generated lookup tables stored in the system's memory to get the optimal values of  $N_x$ ,  $N_c$ , and  $\sigma$  for the given values of  $\lambda$  and  $\omega$ . This basic idea can help develop an iterative algorithm for the traffic-aware joint allocation of control and communication channels in the system. In this paper, we focus on the development and analysis of the model required for such an algorithm, whereas a more comprehensive study on the development of the algorithm is left for future work.

## V. REPORTED LOSS RATE

In contrast to the actual total loss rate  $\beta$ , which is defined in Section III, we now define the *reported loss rate* of the system based on the visible parameters. We define it as the *fraction of visible calls that are blocked* during an SP in the long run. Here, by *visible calls* we mean those calls that successfully get access to the system and are thus recorded in the practical system data. Moreover, the blocked calls that are visible to the system are only those that are blocked due to congestion. Therefore, similar to (7) and (9), we can evaluate the reported loss rate as follows, for  $\sum_{\forall z_1 \in \Omega_Z} E[X_n | Z_{n-1} = z_1] \cdot \pi_{z_1} \neq 0$ :

$$\tilde{\beta} = \lim_{k \to \infty} \frac{\sum_{n=1}^{k} G_n}{\sum_{n=1}^{k} X_n}$$
(18)

$$= \frac{\sum_{\forall z_1 \in \Omega_Z} E[G_n | Z_{n-1} = z_1] . \pi_{z_1}}{\sum_{\forall z_1 \in \Omega_Z} E[X_n | Z_{n-1} = z_1] . \pi_{z_1}}.$$
 (19)

Recall from Section III-A that the numerator in (19) is the expected number of calls blocked due to congestion, whereas the denominator is the expected number of calls successfully getting access to the system, during an SP in the long run. Moreover, note that from (7), (9), and (19), and using the fact that  $Q_n = L_n - X_n$ , we can easily derive,  $\tilde{\beta} = \beta_G/(1 - \beta_Q)$ . We call  $\tilde{\beta}$  as the reported loss rate because it is usually reported by the practical system administrators as a system performance metric. The administrators evaluate this loss rate using the data of call records stored by the practical system. In Section VI-A, we demonstrate that the reported loss rate, which is used by the practical system administrators, is a misleading performance metric, as compared with the actual loss rate that can be estimated using our model as explained in Section V-A.

# A. Estimating the Actual Loss Rate

Recall that the actual loss rate can be evaluated with the help of our model, as explained in Section III. However, as discussed here, this requires the knowledge of the number of contenders  $L_n$  and the collision loss  $Q_n = L_n - X_n$ . Note that both  $L_n$ and  $Q_n$  are invisible system parameters and depend on the invisible parameter  $\lambda$ , as discussed in Section III. In Section IV,



Fig. 3. Reported versus actual loss rate ( $N_x = 2$ ,  $N_c = 3$ , M = 10,  $\omega = 0.5$ , s = 10, and  $\sigma = 0.4$ ).



Fig. 4. Existence of  $N_{xo}$  ( $N = 5, M = 10, \lambda = 0.1, \omega = 0.5$ , and s = 10).

we explain how we can use our model to estimate  $\lambda$ , which can then be used to evaluate the expected values of  $L_n$  and  $Q_n$ , and finally enable us to evaluate the actual loss rate for the system.

#### VI. RESULTS AND DISCUSSION

The problem under consideration is represented by (15). Recall that it is a discrete optimization problem and does not have a closed-form solution. Thus, here, we numerically solve this optimization problem and analyze the results. Moreover, we assume that  $\lambda$  is known. Note that we only consider  $\lambda$  and  $\omega$  lying in open intervals (0, 1). We ignore the values 0 and 1 since they are not practical cases. Moreover, note that to find the optimal channel allocation, we first minimize the total loss rate over  $\sigma$ , for each possible channel allocation, and later minimize the same over all possible allocations. Moreover, here, we consider M = 10 and N = M/2 = 5, which is a frequent practical scenario observed in public safety radio networks [6]. Note that in finite source systems, e.g., public safety trunked radio systems, M represents the number of talk groups, instead of the actual number of radio units since a single talk group can have many radio units as its members. Indeed, from the meta data collected on Louisiana Wireless Information Network, M = 10groups could correspond to as many as hundreds of radio units



Fig. 5. Numerical results with CRP (s = 10, N = 5, and M = 10). (a) Channel allocation map. (b) % age collision in total loss. (c) % age loss increment for  $N_x = 1$  w.r.t. optimal.

[6]. Due to the limitation of scope, we do not present the simulation results here that validate our numerical findings, but these are presented in [5, Sec 4.6.1].

# A. Reported Versus Actual Loss Rate: Misleadingness of Reported Loss

Both the actual and the reported loss rates of the system monotonically increase with increase in call arrival traffic  $\lambda$ , as shown in Fig. 3. Recall that the actual loss rate  $\beta$  incorporates both the collision and congestion losses, whereas the reported loss rate  $\tilde{\beta}$  only incorporates the congestion loss. Due to this reason, a significant difference between both the performance measures is evident in Fig. 3. This clearly shows that the reported loss rate underestimates the system loss, and therefore, it is quite misleading to have it as a performance metric.

# B. Numerical Results on Existence of $N_{xo}$ -Collision and Congestion Tradeoff

Increasing the number of control channels  $N_x$  in the system, while keeping the total number of channels constant, decreases the number of communication channels  $N_c$ , and thus has a twofold effect on the total system loss. On one hand, an increase in  $N_x$  decreases the collision loss by increasing the expected number of calls successfully getting access to the system. On the other hand, due to the same reason, the traffic load for the reduced number of communication channels increases, which increases the congestion loss. Thus, with an increase in  $N_x$ , there exists a tradeoff between collision and congestion losses. In Fig. 4, this tradeoff is clearly evident. Due to the analytical complexity of the problem as explained in Section III, the results shown in Fig. 4 are obtained using a numerical approach without analytical proofs. In particular, note that to find the optimal channel allocation, we first minimize the total loss rate over  $\sigma$ , for each possible channel allocation, and later minimize the same over all possible allocations, as shown in (16). Therefore, In Fig. 4, we have also plotted the corresponding optimal values of  $\sigma$ .

#### C. Channel Allocation Map

We define the *channel allocation map* as a color map that is used to present the optimal number of control channels  $N_{xo}$ , for all possible values of the traffic parameters, namely,  $\lambda$  and  $\omega$ , for the given values of system parameters, namely, M, N, and s. Moreover, note that the optimal number of communication channels are simply  $N_{co} = N - N_{xo}$ . One such map is shown in Fig. 5(a), for N = 5 and M = 10, for a system with CRP, and s = 10. In this map, the color indicates the value of  $N_{xo}$ for the system, for a certain  $(\lambda, \omega)$  pair. Further interesting details, along with a comparison with a system without CRP, are provided in [5, Sec. 4.6.4].

## D. Significance of Collision Loss

Fig. 5(b) shows the percentage of collision loss in total loss, for the optimal channel allocations shown in Fig. 5(a), for different values of  $\lambda$  and  $\omega$ . Fig. 5(b) shows that for N = 5 and M = 10, more than 40% of the total loss is due to collision. In fact, for most of the traffic region, collision loss is more than 70% of the total loss. This indicates that, even for the optimal channel allocation, collision loss is a significant part of the total loss and hence cannot be ignored. Now, we shall explain why the percentage of collision is so high. First, consider the case when collision is very high, e.g., for very high  $\lambda$ ; thus, the expected number of calls successfully getting access to the system is very low. This results in a very low congestion loss. Therefore, in this scenario, collision loss becomes a significant part of the total loss. On the other hand, for a relatively lower collision loss, one can reason that the collision loss per SP can go as high as the total number of users in the system, whereas the congestion loss per SP can only go as high as the maximum number of users that can successfully get system access, which is equal to the number of control channels. Since the number of users is usually much greater than the number of channels, therefore, most of the time, even if the absolute magnitudes of the collision and congestion loss rates are low, the collision loss can proportionately be a significant part of the total loss, as compared to congestion.

#### E. Comparison With Single Control Channel System

Fig. 5(c) shows the percentage increase in total loss rate when a single control channel is used instead of the optimal allocation, for N = 5 and M = 10. Let  $\Delta_p$  represent this percentage increase in loss rate, then we define it mathematically as  $\Delta_p = ((\beta(N_x = 1) - \beta(N_x = N_{xo}))/\beta(N_x = N_{xo})) \times 100.$  Fig. 5(c) shows that, for most of the traffic region,  $\Delta_p \ge 20\%$ , for N = 5 and M = 10. Hence, for these values of system parameters, most of the time, using a single control channel, instead of the optimal channel allocation, results in at least 20% more loss rate, as compared to the optimal allocation. This shows the significance of the optimal channel allocation as compared with the conventional single control channel system.

# VII. CONCLUSION

We have proposed a novel model for finite-source multiple access systems which jointly models the access control and communication layers. We then formulated the optimization problem to jointly find the optimal number of control and communication channels that minimizes the actual total loss rate, for a given number of channels available in total. We introduced the concept of a *channel allocation map* to represent the optimal channel allocation for all possible values of the traffic parameters. As a further contribution, we used our model to quantify the reported system loss rate and elaborated its misleadingness as compared with the actual loss rate. We also demonstrated a mechanism, based on our proposed model, to estimate the invisible actual traffic load required for deciding the trafficaware optimal channel allocation, and for estimating the actual loss rate for practical systems. Using numerical results, we demonstrated that the optimal channel allocation provides a significant improvement in performance as compared with the conventional strategy of using a single control channel. Our results also showed that, although we spend time resources at the control layer to alleviate collision, we may still need more than one frequency channel for access-control, depending on the traffic parameters.

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