

# Using Misbehavior to Analyze Strategic versus Aggregate Energy Minimization in Wireless Sensor Networks

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**Abstract**—We present a novel formulation of the problem of energy misbehavior and develop an analytical framework for quantifying its impact on other nodes. Specifically, we formulate two versions of the power control problem for wireless sensor networks with latency constraints arising from duty cycle allocations. In the first version, strategic power optimization, nodes are modeled as rational agents in a power game, who strategically adjust their powers to minimize their own energy. In the other version, joint power optimization, sensor nodes adjust their transmission powers to minimize the aggregate energy expenditure. Our analysis of these models yields insights into the different energy outcomes of strategic versus joint power optimization. We show that while joint power optimization fits the accepted paradigm of cooperation among sensor nodes (for example large number of sensor nodes cooperating for a task such as target tracking), it comes with both advantages and disadvantages when energy misbehavior is taken into account. One advantage is that it can (sometimes) be energy-dominant, i.e. the optimal energy cost for each node under joint energy minimization is lower than its strategically optimal energy cost. We then develop a model for characterizing energy misbehavior and show that joint optimization is disadvantageous because it is impossible to prevent misbehavior under any channel quality and load constraints, whereas strategic optimization is more resilient. We prove that it is impossible for a node to unilaterally and un-detectably follow a different energy optimization strategy than the other nodes and hence the only threat to the network is misbehavior through false advertisement. We then provide sufficient conditions under which misbehavior through false advertisement can be prevented under a strategic optimization regime. Our analytical results reveal optimal strategies for attacking nodes in an enemy network through energy depletion and help develop effective defense mechanisms for protecting our own wireless network against energy attacks by an intelligent adversary.

This work was supported in part by AFRL and NSF under ITR-0312632 and IIS-0329738. The views expressed in this articles are those of the authors and do not necessarily reflect the official policy or position of the United States Air Force, the U.S. Department of Defense or the U.S. Government.

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## I. INTRODUCTION

Energy-efficiency is a critical concern in many wireless networks, such as cellular networks, ad-hoc networks or wireless sensor networks (WSNs) that consist of large number of sensor nodes equipped with un-replenishable and limited power resources. Since wireless communication accounts for a significant portion of node energy consumption, network lifetime and utility are dependent on the design of energy-efficient communication schemes including low-power signaling and energy-efficient multiple access protocols.

Power-control multiple access (PCMA) schemes have become an essential feature of many energy-constrained interference-limited wireless networks. Several approaches for maximizing information transmission over a shared channel subject to average power constraints have been proposed [1], [2], [3], [4], [5], [6]. [7] addresses the issue of minimizing transmission power, subject to a given amount of information being successfully transmitted and derives PCMA algorithms for autonomous channel access. [4] describes an aggregate power control scheme for a group of interfering users subject to minimal signal-to-noise (SNR) constraints. They also show that this power vector solution is strictly Pareto-optimal since each individual nodes power is also minimized by this vector. In other words, the strategic or node-centric solution coincides with the aggregate or network-centric solution. [2], [3] then propose joint scheduling and power-control algorithms for wireless networks based on this system model.

A hidden feature of such PCMA schemes is the fact that they are based on implicit trust agreements between interfering nodes which makes them highly vulnerable to energy-depletion attacks. Compromised nodes can *mis-behave* by maliciously adjusting their transmission powers in order to increase energy consumption at ‘good’ nodes who are faithfully following a power-control regime. In this paper, we present a novel formulation of the problem of energy misbehavior and develop an analytical framework for quantifying its impact on other nodes. Our analytical results reveal optimal strategies for

attacking nodes in an enemy network through energy depletion.

We consider misbehavior in the context of the following problem: minimizing transmission energy for users transmitting information to their receivers over a shared wireless channel over overlapping intervals or duty cycles, as in the case of wireless sensor networks. We formulate two versions of the power control problem for such duty-cycle constrained networks. In the first version, strategic power optimization, nodes are modeled as rational agents in a power game who adjust their powers strategically in response to the power behaviors of the other nodes, in order to minimize their own energy consumption. We also develop a second version of the power control problem called joint power optimization, in which nodes adjust their transmission powers to minimize the aggregate energy expenditure. For strategic energy minimization, we develop a simple game-theoretic model of a 2-player power game and analytically derive conditions for the existence of Nash equilibria in this game. We then derive the power vectors for joint energy minimization and then investigate the relationship between the energy outcomes of the two approaches.

Note that the model of joint power optimization fits the accepted paradigm of cooperative sensor network operation in which large numbers of sensor nodes are cooperatively working towards a group objective such as target tracking. Thus while joint power optimization could result in higher energy depletion at *some* sensor nodes, it could be beneficial in improving overall sensor network lifetime, since the network contain a large number of redundant nodes that can afford to lose energies at differential rates. On the other hand, strategic power optimization could increase also network utility/performance since critical sensor nodes, for example, clusterheads or data aggregators might strategically consume less energy.

Our primary motivation for investigating these two optimization models is to gauge the energy outcome of selective node misbehavior. Misbehavior can occur in insecure networks if nodes are compromised by adversaries and then do not follow agreed upon transmission policies. In this paper, we develop a model for characterizing energy misbehavior by dishonest nodes and find that the common sensor network operational paradigm of joint energy optimization comes with both advantages and disadvantages when misbehavior is taken into account. We show that while joint optimization can sometimes be energy-dominant and there exist channel and load conditions under which *all* nodes consume less energy as opposed to strategic optimization (i.e the energy vector for joint energy minimization is strictly lesser than the energy vector for strategic energy minimization), it is however more vulnerable to misbehavior

by compromised nodes. We also show that a useful side effect of following a strategic energy optimization regime is the discouragement of node misbehavior, since it does not always lead to performance gains for the misbehaving node.

## II. MOTIVATION AND ASSUMPTIONS

We make several simplifying assumptions in order to gain fundamental theoretical insight into the problem of misbehavior by wireless nodes. First, similar to the approach followed by [8], [9] in which nodes periodically exchange duty-cycle information to enable the construction of interleaved duty-cycles, we assume that nodes exchange relevant information about duty-cycle lengths and traffic load with each other (though not necessarily in an honest manner). This information is then used by the nodes to calculate optimal solutions for both the strategic as well as joint energy minimization approach. We will show that this optimal solution (based on the advertised information) is sufficient to develop a strategy for preventing misbehavior. Second, rather than considering a general  $N$ -node scenario, we model wireless communication between two transmitter-receiver pairs over an interference channel: This model represents two strongly interfering nodes (close neighbors), who are also collaborating with each other by sharing information to solve the problem of mutual energy minimization (joint or strategic), i.e each wireless node selects a close neighbor with whom it engages in energy optimization with the remaining nodes treated as background interference. As a simplifying assumption for obtaining theoretical insights, this is similar in philosophy to the single user receiver assumption used in multi-user communication networks, where the summation of remaining interfering signals is treated as Gaussian noise based on the central limit theorem. Third, our analysis focuses on the case of a slowly fading channel where the delay constraints are on the order of channel coherence time, i.e these parameters remain fixed over the active periods. It is also assumed that these channels experience independent fading and channel state information at the receivers is known and advertised between the nodes.

## III. SYSTEM MODEL

Let  $N_{Tx_1}$  (node 1) and  $N_{Tx_2}$  (node 2) be two transmitter-receiver pairs transmitting to their respective receivers  $N_{Rx_1}$  and  $N_{Rx_2}$  over a Gaussian interference channel. Each transmitter has its own information to send to its receiver within its active duty-cycle deadline. The duty-cycles of the two nodes partially overlap. We assume that node 1 transmits first over the period  $T^I = T_1 + T_2$ , while node 2 starts its transmission later over the period  $T^{II} = T_2 + T_3$ ,  $T = T_1 + T_2 + T_3$ , and  $T^I \neq T^{II}$

in general. This can happen in wireless networks where different nodes initiate periodic transmissions at different times, for example in wireless sensor networks, nodes operate under duty cycles over an interval  $T$  which is divided into active (awake) and inactive (asleep) periods [8], [9]. We represent the average transmission load on each node by  $\bar{R}_1 = B_1/T$  and  $\bar{R}_2 = B_2/T$ , where  $B_1$  and  $B_2$ , respectively, represent the total amount of information to be transmitted by each node within its deadline. We use the notation  $\mu_1$  and  $\mu_2$  to represent the active ratios of the two nodes respectively, i.e.  $\mu_1 = (|T_1 + T_2|/|T|)$  and  $\mu_2 = (|T_2 + T_3|/|T|)$ . Figure III illustrates this transmission setup.

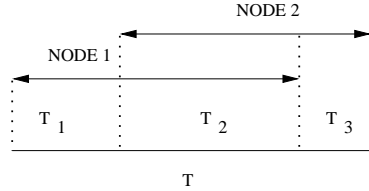


Fig. 1. Duty cycle transmission model for interfering nodes.

Let  $\alpha^{(i,j)}$ ,  $i, j \in \{1, 2\}$  be the channel attenuation factors between  $N_{Tx_i}$  and  $N_{Rx_j}$ , which captures the effects of path-loss, shadowing and frequency non-selective fading. Under our assumptions, this two-user interference channel system can be modeled as

$$\begin{aligned} r_{11}(t) &= \alpha^{(11)} s_{11}(t) + n_{11}(t), t \in T_1 \\ r_{12}(t) &= \alpha^{(11)} s_{12}(t) + \alpha^{(21)} s_{22}(t) + n_{12}(t), t \in T_2 \\ r_{22}(t) &= \alpha^{(12)} s_{12}(t) + \alpha^{(22)} s_{22}(t) + n_{22}(t), t \in T_2 \\ r_{23}(t) &= \alpha^{(22)} s_{23}(t) + n_{23}(t), t \in T_3 \end{aligned} \quad (1)$$

where  $r_{ij}(t)$  are the received baseband signals at node  $N_{Rx_i}$  in the  $j$ th interval,  $s_{ij}(t)$  are the transmit narrow-band signals from node  $N_{Tx_i}$  over  $j$ th interval with power  $E[|s_{ij}|^2(t)] = P_{ij}$ , and  $n_{ij}(t)$  are the additive complex white Gaussian noise with power  $\eta_i$ . It is assumed transmitters and receivers have full access to channel state information (CSI) such that channel coding over two independent blocks by each transmit node enables error free transmissions over two periods. Single user decoding is assumed at each receiver  $N_{Rx_i}$  to decode the information from its own transmit node  $N_{Tx_i}$  while treating other party's information as Gaussian interference. The normalized mutual information between  $N_{Tx_i}$  and  $N_{Rx_i}$  over the active periods are

$$\begin{aligned} \bar{R}_1 &= (1 - \mu_2)R_{11} + (\mu_1 + \mu_2 - 1)R_{12} \\ \bar{R}_2 &= (1 - \mu_1)R_{23} + (\mu_1 + \mu_2 - 1)R_{22} \end{aligned} \quad (2)$$

where  $R_{ij} = \log_2(1 + \rho_{ij})$  is the rate of node  $i$  in the  $j$ th interval with signal-to-interference-noise-ratio (SNR)

defined as follows:

$$\begin{aligned} \rho_{12} &= \frac{G^{(11)}P_{12}}{G^{(21)}P_{22} + \eta_1}, \quad \rho_{22} = \frac{G^{(22)}P_{22}}{G^{(12)}P_{12} + \eta_2} \\ \rho_{11} &= \frac{G^{(11)}P_{11}}{\eta_1}, \quad \rho_{23} = \frac{G^{(22)}P_{23}}{\eta_2} \end{aligned} \quad (3)$$

where  $G^{(ij)} = |\alpha^{(ij)}|^2$ . Further, let  $\beta_1 = \eta_1/G^{(11)}$ ,  $\alpha_1 = G^{(21)}/G^{(11)}$ ,  $\beta_2 = \eta_2/G^{(22)}$  and  $\alpha_2 = G^{(12)}/G^{(22)}$ . Defining  $\eta_{12} = \beta_1 + \alpha_1 P_{22}$  and  $\eta_{22} = \beta_2 + \alpha_2 P_{12}$ , we can more conveniently express  $\rho_{12} = P_{12}/\eta_{12}$  and  $\rho_{22} = P_{22}/\eta_{22}$ .

#### IV. PROBLEM SETUP

We model the problem of duty-cycle constrained strategic energy minimization as a simple two player power game with the following parameters: Node 1 selects its transmit power during periods  $T_1$  and  $T_2$  from the space  $\mathbb{P}$  of achievable transmit powers. Thus the strategy choice of node 1 is represented by  $l_1 = (P_{11}, P_{12}) \in \mathbb{P} \times \mathbb{P}$ . Likewise, the strategy choice of node 2 is given by  $l_2 = (P_{22}, P_{23}) \in \mathbb{P} \times \mathbb{P}$ . We consider only pure strategies here as opposed to the more general mixed strategy model where nodes choose their  $P_{ij}$ 's from a probability distribution. For notational simplicity, we define  $P_{13} = P_{23} = 0$  since the nodes are not active during these time intervals.

Let  $E_1$  and  $E_2$  denote the transmission energy functions

$$\begin{aligned} E_1 &= T [P_{11}(1 - \mu_2) + P_{12}(\mu_2 + \mu_1 - 1)] \\ E_2 &= T [P_{23}(1 - \mu_1) + P_{22}(\mu_2 + \mu_1 - 1)] \end{aligned} \quad (4)$$

Let  $R_{ij}$  represent the transmission rate of node  $i$  during period  $T_j$ , where

$$\begin{aligned} R_{11} &= \log_2 \left( 1 + \frac{P_{11}}{\beta_1} \right), \quad R_{23} = \log_2 \left( 1 + \frac{P_{23}}{\beta_2} \right), \\ R_{12} &= \log_2 \left( 1 + \frac{P_{12}}{\beta_1 + \alpha_1 P_{22}} \right), \\ R_{22} &= \log_2 \left( 1 + \frac{P_{22}}{\beta_2 + \alpha_2 P_{12}} \right) \end{aligned} \quad (5)$$

Also,  $\bar{R}_1 = B_1/T$  and  $\bar{R}_2 = B_2/T$  represent the average transmission rates for node 1 and node 2. Then nodes 1 and 2 operate under load constraints  $L_1$  and  $L_2$  defined as

$$\begin{aligned} L_1 &= R_{11}(1 - \mu_2) + R_{12}(\mu_2 + \mu_1 - 1) - \bar{R}_1 = 0 \\ L_2 &= R_{23}(1 - \mu_1) + R_{22}(\mu_2 + \mu_1 - 1) - \bar{R}_2 = 0 \end{aligned} \quad (6)$$

Let  $l = (l_i, l_{-i})$  represent a particular strategy profile of the power game. In this case,  $l_{-1} = l_2$ ,  $l_{-2} = l_1$  and  $l$  also represents a particular energy outcome of the game.

We define the payoff at node  $i$  under strategy profile  $l$  as:

$$\Pi_i(l) = -E_i$$

Strategy  $l_i$  is defined to be the best-response of player  $i$  to a given  $l_{-i}$  if

$$\Pi_i(l'_i, l_{-i}) \leq \Pi_i(l_i, l_{-i}) \text{ for all strategies } l'_i.$$

Let  $BR_i(l_{-i})$  denote the set of player  $i$ 's best response to  $l_{-i}$ . A strategy profile  $l = (l_1, l_2)$  is optimal if the nodes are playing a Nash Equilibrium[10] i.e.  $l_i \in BR_i(l_{-i})$  for each sensor node  $i$ .

Note that the best-response power strategy of node 1 minimizes its individual energy consumption and satisfies its load constraint for a given power strategy employed by node 2, without accounting for the load constraint of the other node. However at the Nash equilibrium point, node 2 is also playing its best response to node 1, i.e. both users are simultaneously satisfying their load constraints as well as minimizing their individual energies for each others power vector solutions. We will shortly identify system conditions (for example, load and channel quality) under which the two players arrive at Nash equilibrium in the power game.

We also consider the joint minimization approach in which nodes jointly adjust their powers during overlapping periods in order to minimize the aggregate energy, i.e. minimize  $\sum_i E_i$ , subject to the load constraints  $L_i$ . Joint minimization is important in itself since there are circumstances under it is preferable from the application point of view, for example data aggregation in sensor networks with large number of redundant nodes. More importantly, while strategic energy optimization naturally suggests energy benefits to some nodes, we investigate whether there are conditions under which joint energy minimization can strictly dominate the strategic approach, with respect to all individual node energies. This is indeed the case as shown below.

## V. ANALYTICAL RESULTS

We first obtain optimal strategic power vectors followed by power vectors for joint energy minimization. Optimal strategic power vectors correspond to the Nash equilibrium points of the two player power game defined above. Let power vectors  $P_1^s = (P_{11}^s, P_{12}^s)$  and  $P_2^s = (P_{23}^s, P_{22}^s)$  represent node 1 and node 2's best-responses to each other in the two player power game, with  $R_{12}^s = \log_2(1 + P_{12}^s/(\beta_1 + \alpha_1 P_{22}^s))$  and  $R_{22}^s = \log_2(1 + P_{22}^s/(\beta_2 + \alpha_2 P_{12}^s))$  the corresponding best-response rates. Also let  $C_1 = 2^{\bar{R}_1/(1-\mu_2)}$  and  $C_2 = 2^{\bar{R}_2/(1-\mu_1)}$  be load related terms. Finally, define  $n = \mu_1/(1 - \mu_2)$ ,  $m = \mu_2/(1 - \mu_1)$ ,  $0 < \mu_1, \mu_2 < 1$ ,  $x_s = 2^{R_{12}^s}$  and  $y_s = 2^{R_{22}^s}$ . Then we have,

**Proposition 1:** *The Nash equilibria of the two player power game are determined by the solutions to the system of bivariate functions  $\{\mathcal{F}(x_s, y_s) = 0, \mathcal{G}(x_s, y_s) = 0\}$  defined by*

$$\begin{aligned} \mathcal{F} : \quad & \beta_1 x_s^n y_s^m + \alpha_1 \beta_2 C_2 x_s^n y_s \\ & - \alpha_1 \beta_2 C_2 x_s^n - \beta_1 C_1 y_s^m = 0 \\ \mathcal{G} : \quad & \beta_2 x_s^n y_s^m - \beta_2 C_2 x_s^n \\ & + \alpha_2 \beta_1 C_1 x_s y_s^m - \alpha_2 \beta_1 C_1 y_s^m = 0 \end{aligned} \quad (7)$$

where  $x_s \geq 1, y_s \geq 1$ .

*Proof:* Please see Appendix. ■

Generally games can have several Nash equilibria or none at all [10], depending on specific conditions (in this case channel quality and load). If there is no solution to the above proposition, the two-player power game does not have explicit equilibria for the given parameters and the players cannot have meaningful overlapping periods. When Nash equilibrium does not exist, the nodes have several options. They can choose to change parameters such as  $\mu$  values and use the above necessary conditions to ensure equilibrium. Alternately they can choose TDM allocation in a 'fair' manner or agree on a different energy optimization function. Since these options involve changing the definition of the game (non-cooperative to cooperative) they are beyond the scope of this paper.

We now discuss under what conditions equilibria exist and if so, how many. Meaningful equilibria correspond to non-negative power allocations  $P_{12}^* \geq 0$  and  $P_{22}^* \geq 0$  are therefore those non negative real-valued solutions to the equilibrium functions which satisfy  $1 \leq x \leq 2^{\bar{R}_1/\mu_1}$  and  $1 \leq y \leq 2^{\bar{R}_2/\mu_2}$ . When one of the power solutions is zero, it corresponds to TDM–Time Division Multiplexing. We now provide explicit load and channel quality conditions for the existence of Nash equilibria for the power game.

**Proposition 2:** *The strategic power game does not have Nash equilibrium points only if either S.1 and T.2 are simultaneously true or T.1 and S.2 are simultaneously true. However if S.1 and T.1 are simultaneously true or simultaneously false, then there exist at most three Nash equilibria.*

$$\begin{aligned} S \quad & : \quad \frac{\beta_1}{\alpha_1} (2^{\bar{R}_1/\mu_2} - 1) < \beta_2 (2^{\bar{R}_2/\mu_2} - 1) \text{ S.1} \\ & \frac{(n-1)\beta_2 C_2 A}{(n-1)\beta_2 A + \alpha_2 \beta_1 C_1} > \left[ 1 + \frac{(n-1)\beta_1 (C_1 - A)}{\alpha_1 (\beta_2 C_2 A (n-1) + \alpha_2 \beta_1 C_1)} \right]^m \text{ S.2} \end{aligned}$$

$$\begin{aligned}
& T : \\
& \frac{\beta_2}{\alpha_2} (2^{\frac{\bar{R}_2}{1-\mu_1}} - 1) < \beta_1 (2^{\frac{\bar{R}_1}{\mu_1}} - 1) T.1 \\
& \frac{(m-1)\beta_1 C_1 B}{(m-1)\beta_1 B + \alpha_1 \beta_2 C_2} > \left[ 1 \right. \\
& \quad \left. + \frac{(m-1)\beta_2 (C_2 - B)}{\alpha_2 (\beta_1 C_1 B (m-1) + \alpha_1 \beta_2 C_2)} \right]^n T.2
\end{aligned}$$

where  $0 < \mu_1, \mu_2 < 1$ .

*Proof:* Please see Appendix. ■

**Corollary 1:** *For given channel quality and load conditions, there always exist duty cycle values under which the nodes can find meaningful equilibrium.*

As seen from conditions S.1 and T.1, for any channel quality and load, we can always find  $\mu_1$  and  $\mu_2$  such that the LSH of condition S.1 and T.1 exceed their RHS. Therefore, both S.1 and T.1 can be made false and thus equilibrium exists.

Next we identify the power vectors for the case when the two nodes carry out joint energy minimization. Let  $R_{12}^j$  and  $R_{22}^j$  be the rate solutions during the overlapping period and denote  $x_j = 2^{R_{12}^j}$  and  $y = 2^{R_{22}^j}$ . The corresponding power solutions are denoted by  $P^j$ .

**Proposition 3:** *The optimal power vectors for joint energy minimization are determined by the solutions  $(x_j, y)$  to*

$$\begin{aligned}
\mathcal{P} : & \beta_1 [1 + \alpha_1 (x_j - 1)] [C_1 (1 - \alpha_1 \alpha_2 (x_j - 1)(y - 1)) - \\
& x_j^n (1 + \alpha_2 (y - 1))] y^m \\
& = \alpha_1 \beta_2 C_2 (1 - \alpha_1 \alpha_2 (x_j - 1)(y - 1)) [1 + \alpha_2 (y - 1)] \\
& (y - 1) x_j^n \\
\mathcal{Q} : & \beta_2 [1 + \alpha_2 (y - 1)] [C_2 (1 - \alpha_1 \alpha_2 (x_j - 1)(y - 1)) - \\
& y^m (1 + \alpha_1 (x_j - 1))] x_j^n \\
& = \alpha_2 \beta_1 C_1 (1 - \alpha_1 \alpha_2 (x_j - 1)(y - 1)) [1 + \alpha_1 (x_j - 1)] \\
& (x_j - 1) y^m
\end{aligned}$$

*Proof:* Please see Appendix. ■

In the case of joint energy minimization, the non-existence of feasible power vectors, (i.e.  $P_{12}^j \geq 0$  and  $P_{22}^j \geq 0$ ) implies that one node is creating significant interference at the other nodes receiver. In this case, (since the nodes are cooperating, unlike in the strategic optimization case) one of the nodes can choose to zero its power output during the overlapping period.

**Proposition 4:** *There exist load, channel quality and duty-cycle conditions under which joint energy minimization is dominant over strategic energy minimization, i.e. the optimal energy cost for each node under joint energy minimization is strictly lower than its strategically optimal energy cost.*

*Proof:* Consider  $n = m = 2$ , identical loads ( $C_1 = C_2 = C$ ) and channel quality ( $\alpha_1 = \alpha_2 = \alpha$ ) and normalized  $\beta_1 = \beta_2 = T = 1$  at each node. It can be easily seen that the optimal joint power allocation of node 1 is equal to node 2, i.e.  $P_{12}^J = P_{22}^J = P^J = \frac{\sqrt{C}-1}{\alpha+1}$ . Likewise, it can be seen that at strategic equilibrium  $P_{12}^S = P_{22}^S = P^S$  where  $(\alpha+1)^2 (P^S)^2 - (2\alpha+2-\alpha C)P^S + 1 - C = 0$ . Further  $P_{11}^J = P_{23}^J$  and  $P_{11}^S = P_{23}^S$ . Therefore let  $E^J$  and  $E^S$  represent the optimal energy output of the nodes under the two energy minimization schemes, respectively. By definition,  $2E^J \leq 2E^S$ . After some simplification, it can be shown that  $E^J = P^J + (1+\alpha P^J)\sqrt{C}-1$  and  $E^S = P^S + \sqrt{(1+\alpha P^S)\sqrt{C}-1}$ . Looking at the expressions for  $P^J$  and  $P^S$ , there exist  $\alpha$  and  $C$  that  $E^J < E^S$  and thus joint optimization is dominant. By continuity arguments, there exists a range of channel quality and loads centered around the symmetric point for which joint optimization remains dominant. ■

As will be seen from the examples in the numerical results section, joint energy minimization is not always dominant over strategic optimization. In such cases, one of the nodes consumes less energy, while the other consumes more. Thus both joint as well as strategic optimization have their advantages. Either scheme can be preferable depending on the applications and specific parameters of data loads, channel qualities, and duty cycles.

## VI. NODE MISBEHAVIOR AND IMPACT ON ENERGY OPTIMIZATION

We now investigate the impact of misbehavior by sensor nodes in the network. We have assumed that nodes share duty cycle, load and channel quality parameters with each other so they can obtain the optimal power allocations as specified by either the strategic or joint optimization regime. Under these assumptions, is it possible for a sensor node to misbehave by selectively adjusting its power output. In general, we define node misbehavior as follows: A node will misbehave only if it can adjust its power output leading to lower transmission energy costs for itself, higher energy costs for others, **and** its misbehavior cannot be detected.

There are two ways in which nodes can misbehave. Since nodes must share information, a misbehaving node can easily affect the optimal power vector solutions by falsely advertising its duty-cycle or load parameters. We consider this approach first. Later we show that it is impossible for a node to misbehave without false advertisement and thus the results in the first part are strict (i.e. apply to all misbehavior). Essentially, we show that without false advertisement, the only possibility is for the misbehaving node to unilaterally adopt strategic minimization when the other node is expecting joint

energy minimization. We prove that such unilateral deviation from the agreed upon optimization strategy is always detectable and thus impossible.

#### A. Misbehavior through False Advertisement

We model misbehavior by assuming that a compromised node can falsely advertise its transmission load but make misbehavior difficult by assuming that duty cycle lengths ( $\mu_1, \mu_2$ ) along with node power outputs during the **overlapping portion** of the duty cycle can be monitored. However power outputs during the non-overlapping part of the duty cycle are assumed to be not monitored (since the other node is off during this period). Thus a misbehaving node **must** conform to the overlapping period power solutions obtained using the falsely advertised load, thereby preventing it from falsely advertising an overwhelmingly large load.

Using this model, we now analytically derive conditions for node misbehavior. First, we formally define misbehavior as follows: WLOG, assume that node 1 is the good node while node 2 can misbehave. Let  $E_1^t$  and  $E_2^t$  represent energy consumptions if node 2 does not misbehave, i.e the two nodes perform strategic/joint energy optimization with true load information  $B_2^t$ , with  $E_1^f$  and  $E_2^f$  the energy consumptions when node 2 falsely advertises a load of  $B_2^f$ .

*Definition 1: The misbehavior gain of (the misbehaving) node 2 is defined as  $EG_2 = E_2^t - E_2^f$ . Similarly, the misbehavior loss of node 1 is defined as  $EL_1 = E_1^f - E_1^t$ .*

We define the necessary condition for misbehavior as

$$\exists B_2^f \text{ s.t } EL_1 \geq 0, EG_2 \geq 0. \quad (8)$$

Thus node 2 will misbehave only if there exists  $B_2^f$  such that it has a misbehavior gain **and** node 1 has a misbehavior loss.

As shown previously, strategic transmission is energy optimal for one node in most cases (the exception being when joint energy optimization turns out to be strongly pareto-optimal). Thus given freedom of choice, this node will choose to optimize transmission energy strategically thereby forcing the other node to minimize its energy by also performing strategic optimization. We show below that there can be an additional rationale for strategic optimization, namely preventing misbehavior. In particular, the following propositions show that joint energy optimization is conducive to misbehavior, while strategic energy optimization is not.

We summarize the main results in this section below. In the derivations, we assume normalized duty cycle interval  $T = 1$  and background interference parameters  $\beta_1 = 1$  and  $\beta_2 = 1$ .

**Theorem 1:** *It is impossible to prevent misbehavior under joint energy optimization for all channel quality, duty-cycle and load values  $\alpha_1, \alpha_2, \mu_1, \mu_2, B_1, B_2, 0 < \mu_1, \mu_2 < 1$ .*

**Theorem 2:** *Under strategic energy optimization, both nodes can guarantee good behavior from each other by choosing loads and duty-cycle lengths  $B_1, B_2, \mu_1, \mu_2, 0 < \mu_1, \mu_2 < 1$ , such that*

$$\rho_{12} > \frac{1 - \mu_2}{\mu_1 + \mu_2 - 1} \quad (9)$$

$$\rho_{22} > \frac{1 - \mu_1}{\mu_1 + \mu_2 - 1} \quad (10)$$

1) *Misbehavior under Strategic Energy Optimization:* We first consider misbehavior under strategic optimization. First we show that it is necessary and sufficient for the bad node to advertise a larger false load, in order to penalize the good node (i.e make  $EL_1 > 0$ ). This is not true under joint optimization, as we shall prove later. Later we provide a necessary condition for profitable misbehavior at the bad node ( $EG_1 > 0$ ).

**Proposition 5:** *If both nodes are following a strategic optimization regime, then  $EL_1 > 0$  if and only if  $B_2^f > B_2^t$ .*

*Proof:* Let  $C_1 = 2^{\bar{R}_1/(1-\mu_2)}$  and  $C_2 = 2^{\bar{R}_2/(1-\mu_1)}$ . If node 2 falsely advertises a load  $B_2^f$ , then the equilibrium power solutions in Prop 1 are changed, because  $C_2$  has changed to  $C_2^f = 2^{\bar{R}_2^f/(1-\mu_1)}$ . For any given load pair  $(B_1, B_2^f)$ , let  $E_1^f$  denote the energy consumed by the good node at (false load) strategic equilibrium.

$$E_1^f = (1 - \mu_2) \left\{ (n - 1)P_{12}^f + P_{11}^f \right\} \quad (11)$$

where  $n = \mu_1/(1 - \mu_2)$ .  $P_{12}^f$  (along with  $P_{22}^f$ ) is the equilibrium strategic power solution obtained using  $C_1$  and  $C_2^f$  (for notational simplicity, we have dropped the  $s$  superscript).  $P_{11}^f = (C_1/x_f^{n-1}) - 1$  (from Eqs. 41 and 43), where  $x_f - 1 = \rho_{12}^f$  is the equilibrium SNR at node 1.

We consider the rate of change of  $E_1^f$  with respect to  $C_2^f$  i.e the partial  $E_1^f = \partial E_1^f / \partial C_2^f$ . Differentiating Eq. 11, we get

$$E_1^f = (\mu_1 + \mu_2 - 1) \left( P_{12}^f - \frac{C_1}{x_f^n} x_f' \right) \quad (12)$$

where  $P_{12}^f$  and  $x_f'$  are the partial derivatives with respect to  $C_2^f$ . From Eq 47, we get  $C_1/x_f^n = \eta_{12}^f$ , where  $\eta_{12}^f = 1 + \alpha_1 P_{22}^f$ . Next from Eq. 49, we have  $(x_f - 1)\eta_{12}^f = P_{12}^f$ . Taking the partial derivative and simplifying, we get

$$x_f' \eta_{12}^f = P_{12}^f - \alpha_1 \rho_{12}^f P_{22}^f \quad (13)$$

Thus we have

$$E'_1 = \alpha_1 \rho_{12}^f P'_{22} \quad (14)$$

To obtain  $P'_{22}$  and  $P'_{12}$  rewrite Eqs. 47 and 48 as

$$P_{12}^f = (\eta^f)_{12}^{1-1/n} \left( C_1^{1/n} - (\eta^f)_{12}^{1/n} \right) \quad (15)$$

$$P_{22}^f = (\eta^f)_{22}^{1-1/m} \left( (C_2^f)^{1/m} - (\eta^f)_{22}^{1/m} \right) \quad (16)$$

Taking partial derivatives w.r.t  $C_2^f$  and simplifying, we get

$$n P'_{12} = \alpha_1 P'_{22} (\rho_{12}^f (n-1) - 1) \quad (17)$$

$$m P'_{22} = \alpha_2 P'_{12} (\rho_{22}^f (m-1) - 1) + \frac{1}{y^{m-1}} \quad (18)$$

Solving for  $P'_{22}$  above we get

$$P'_{22} = \frac{1}{m y_f^m \left( 1 - \alpha_1 \alpha_2 \left[ \frac{\rho_{12}^f (n-1) - 1}{n} \right] \left[ \frac{\rho_{22}^f (m-1) - 1}{m} \right] \right)} \quad (19)$$

Using Eqs 49 and 50,  $P'_{12}$  and  $P'_{22}$  can be expressed in terms of the equilibrium SNRs as

$$P'_{12} = \frac{\rho_{12}^f (1 + \alpha_1 \rho_{22}^f)}{1 - \alpha_1 \alpha_2 \rho_{12}^f \rho_{22}^f}$$

$$P'_{22} = \frac{\rho_{22}^f (1 + \alpha_2 \rho_{12}^f)}{1 - \alpha_1 \alpha_2 \rho_{12}^f \rho_{22}^f}$$

Thus  $1 - \alpha_1 \alpha_2 \rho_{12}^f \rho_{22}^f > 0$  for all equilibrium  $\rho_{12}^f$  and  $\rho_{22}^f$ . Furthermore, since  $\rho_{12}^f > 0$ ,  $\rho_{22}^f > 0$  at equilibrium, we also have  $(\rho_{12}^f (n-1) - 1) < n \rho_{12}^f$  and  $(\rho_{22}^f (m-1) - 1) < m \rho_{22}^f$ . Utilizing this in Eq 19, we get  $P'_{22} > 0$  and therefore  $\forall C_2^f : E'_1 = \partial E_1 / \partial C_2^f > 0$ . Hence under strategic energy optimization,  $E_1^f - E_1^t > 0$  if and only if  $C_2^f > C_2^t$  (and therefore  $B_2^f > B_2^t$ ). ■

Thus the good node is penalized if and only if the bad node falsely advertises a higher load. Next, we derive a necessary condition for the the bad node to also profit through misbehavior.

**Proposition 6:** *Under strategic energy optimization, the bad node can profit from misbehavior ( $EG_1 > 0$ ), only if the equilibrium SNR of the good node during the overlapping period satisfies*

$$\rho_{12}^f < \frac{1 - \mu_2}{\mu_1 + \mu_2 - 1} \quad (20)$$

*Proof:* The energy consumed by the bad node can be expressed as

$$E_2^f = (1 - \mu_1) \left\{ (m-1) P_{22}^f + P_{23}^f \right\} \quad (21)$$

where  $m = \mu_2 / (1 - \mu_1)$  and  $P_{22}^f$  (along with  $P_{12}^f$ ) is derived from proposition 1 using  $C_1$  and  $C_2^f$ . Note that  $P_{23}^f = (C_2^t / y_f^{m-1}) - 1$  (from Eqs. 44 and 46) and therefore

considering the partial derivative w.r.t  $C_2^f$ , we get  $P'_{23} = -(m-1) C_2^t y_f^m / y_f^m$ . Thus we have

$$E'_2 = (\mu_1 + \mu_2 - 1) \left( P'_{22} - \frac{C_2^t}{y_f^m} y_f^m \right) \quad (22)$$

Next, taking the partial derivative of Eq. 50 and simplifying, we get

$$\begin{aligned} y_f \eta_{22}^f &= P'_{22} - \alpha_2 \rho_{22}^f P'_{12} \\ &= P'_{22} \left( 1 - \alpha_1 \alpha_2 \left[ \frac{\rho_{12}^f (n-1) - 1}{n} \right] \rho_{22}^f \right) \end{aligned} \quad (23)$$

Note that  $C_2^f = \eta_{22}^f y_f^m$  (from Eq. 48). Substituting this in Eq. 22 along with Eq 23, we get

$$E'_2 = (\mu_1 + \mu_2 - 1) P'_{22} \left( 1 - \frac{C_2^t}{C_2^f} \left\{ 1 - \alpha_1 \alpha_2 \left[ \frac{\rho_{12}^f (n-1) - 1}{n} \right] \rho_{22}^f \right\} \right)$$

For the bad node to profit from misbehavior, it is necessary that  $E'_2 < 0$ , which reduces to

$$\alpha_1 \alpha_2 \left[ \frac{\rho_{12}^f (n-1) - 1}{n} \right] \rho_{22}^f < 1 - \frac{C_2^f}{C_2^t}$$

Since  $C_2^f \geq C_2^t$  (from proposition 5), a necessary condition for misbehavior by node 2 is  $\rho_{12}^f (n-1) - 1 < 0$  or

$$\rho_{12}^f > \frac{1}{n-1} \quad (24)$$

Substituting  $n = \mu_1 / (1 - \mu_2)$  leads to the proposition as stated. ■

Thus if both nodes are following strategic energy optimization, there exist values of true-load at the good node for which the bad node cannot misbehave. Prop 6 shows that node 1 can defend against misbehavior by node 2 (make it less likely) by choosing larger loads and larger values of  $n$ . However, as we show below, there is no such defense against misbehavior under joint energy optimization.

2) *Misbehavior under Joint Energy Optimization:* We begin by proving the following useful result.

**Lemma 1:** *If both nodes are following a joint energy optimization regime, then*

$$E'_1|_{C_2^f=C_2^t} + E'_2|_{C_2^f=C_2^t} = 0 \quad (25)$$

and hence  $EG_1 = -EL_2$  for small values of  $\Delta C_2 = C_2^f - C_2^t$ .

*Proof:*  $E_1^f$  and  $\partial E_1/\partial C_2^f$  are defined as in Eqs. 12 and 11 (with all the parameters there and below referring to joint rather than strategic optimization). From Eq. 60, we get  $C_1/x_f^n = \frac{(1+\alpha_2\rho_{22}^f)\eta_{12}^f}{1-\alpha_1\alpha_2\rho_{12}^f\rho_{22}^f}$ . Substituting the above in Eq. 12 along with  $x_f^f$  (from Eq.13) and simplifying, we get,

$$E_1' = \left( \frac{\mu_1+\mu_2-1}{1-\alpha_1\alpha_2\rho_{12}^f\rho_{22}^f} \right) \left( \alpha_1\rho_{12}^f(1+\alpha_2\rho_{22}^f)P_{22}' - \alpha_2\rho_{22}^f(1+\alpha_1\rho_{12}^f)P_{12}' \right) \quad (26)$$

$E_1'$  at  $C_2^f = C_2^t$  is obtained by replacing the  $\rho^f$  terms with  $\rho^t$  in the expression above and evaluating  $P_{22}'$  and  $P_{12}'$  at  $C_2^f = C_2^t$ .

Likewise  $E_2' = (\mu_1+\mu_2-1)(P_{22}' - C_2^t y_f^m / y_f^m)$  (from Eq. 22). When evaluated at  $C_2^f = C_2^t$ ,  $C_2^t / y_f^m = \frac{(1+\alpha_1\rho_{12}^t)\eta_{22}^t}{1-\alpha_1\alpha_2\rho_{12}^t\rho_{22}^t}$  (from Eq. 61) Substituting the above in 22 along with  $y_f^f$  evaluated at  $C_2^f = C_2^t$  (from Eq. 23) and simplifying, we get

$$E_2'|_{C_2^f=C_2^t} = \left( \frac{\mu_1+\mu_2-1}{1-\alpha_1\alpha_2\rho_{12}^t\rho_{22}^t} \right) \left( \alpha_2\rho_{22}^t(1+\alpha_1\rho_{12}^t)P_{12}' - \alpha_1\rho_{12}^t(1+\alpha_2\rho_{22}^t)P_{22}' \right) \quad (27)$$

Therefore we have  $E_2'|_{C_2^f=C_2^t} = -E_1'|_{C_2^f=C_2^t}$  as desired. ■

**Proposition 7:** *If both nodes are following a joint energy optimization regime, then for any channel quality, duty-cycle and true-load values,  $\alpha_1, \alpha_2, \mu_1, \mu_2, B_1, B_2^f$ , there always exists a false load value using which the bad node can obtain an energy gain while the good node suffers an energy loss.*

*Proof:* From lemma 1, it suffices to show that  $E_2' \neq 0$  at  $C_2^t$  (i.e  $C_2^t$  is not a local minimum for  $E_2^f$  viewed as a function of  $C_2^f$ ), since this will imply that  $E_2^f < E_2^t$  (and therefore  $E_1^f > E_1^t$ ) always, in either the positive or negative neighborhood of  $C_2^t$ . After simplifying Eqs. 26 and 27, showing  $E_2' \neq 0$  is equivalent to showing

$$\alpha_2\rho_{22}^t(1+\alpha_1\rho_{12}^t)P_{12}' \neq \alpha_1\rho_{12}^t(1+\alpha_2\rho_{22}^t)P_{22}' \quad (28)$$

where  $P_{22}'$  and  $P_{12}'$  are evaluated at  $C_2^f = C_2^t$ . For notational simplicity, we drop the  $t$  and  $f$  subscripts in the rest of this derivation.

Simplifying Eqs. 60 and 61 (using normalized  $\beta_1 = \beta_2 = 1$ ), we get

$$C_1 = \frac{1+\alpha_2(P_{12}+P_{22})}{1+\alpha_2P_{12}+\alpha_1P_{22}}\eta_{12}^2x^n \quad (29)$$

$$C_2 = \frac{1+\alpha_1(P_{12}+P_{22})}{1+\alpha_2P_{12}+\alpha_1P_{22}}\eta_{22}^2y^n \quad (30)$$

Differentiating both equations above with respect to  $C_2$ , and after some algebraic manipulation, we have

$$P_{12}' = \frac{A}{BC+AD} \quad P_{22}' = \frac{B}{BC+AD} \quad (31)$$

where

$$A = \frac{\eta_2(\alpha_2-\alpha_1)}{(\eta_1+\alpha_2P_{12})(\eta_2+\alpha_2P_{22})} + \frac{2\alpha_1}{\eta_1} - n\alpha_1P_{12}\eta_1(\eta_1+P_{12}) \quad (32)$$

$$B = \frac{\alpha_2(\alpha_2-\alpha_1)P_{22}}{(\eta_1+\alpha_2P_{12})(\eta_2+\alpha_2P_{22})} - \frac{n}{\eta_1+P_{12}} \quad (33)$$

The  $C$  and  $D$  terms are obtained by interchanging  $\alpha_1$  with  $\alpha_2$ ,  $n$  with  $m$  and the 12 subscripts with 22 in  $B$  and  $A$ , respectively. Consider two cases:

- **BC+AD < 0:** Then we have  $A > \alpha_1\rho_{12}B$  after some manipulation of Eqs. 32 and 33 and therefore  $P_{12}' < \alpha_1\rho_{12}P_{22}'$ . Also,  $\alpha_2\rho_{22}(1+\alpha_1\rho_{12}) < 1 + \alpha_2\rho_{22}$  since  $\alpha_1\alpha_2\rho_{12}\rho_{22} < 1$ . Hence Eq. 28 is satisfied and  $E_2' < 0$  in this case.
- **BC+AD > 0:** Then we have  $\alpha_2\rho_{22}A > B$  from Eqs. 32 and 33 and therefore  $\alpha_2\rho_{22}P_{12}' > P_{22}'$ . Likewise,  $1 + \alpha_1\rho_{12} > \alpha_1\rho_{12}(1 + \alpha_2\rho_{22})$ . Hence Eq. 28 is satisfied and  $E_2' > 0$  in this case.

Hence  $E_2' \neq 0$  at  $C_2^f = C_2^t$ , and there always exists  $C_2^f$  in the positive or negative neighborhood of  $C_2^t$  such that  $EL_1 > 0, EG_2 > 0$ . Therefore misbehavior by the bad node cannot be prevented. ■

### B. Misbehavior through Unilateral Deviation

Suppose a node cannot falsely advertise its load values as assumed in the previous section. In this case the only way a node can misbehave is if it unilaterally follows an energy optimization strategy that is different from the one being followed by the other nodes. Clearly, if the good node is performing strategic optimization, then the bad node cannot decrease the good nodes energy by performing joint optimization (By definition of strategic optimization. Note that both nodes are using true load values). The only possibility is if the good node assumes that both nodes will follow the joint optimization regime, however the bad node unilaterally deviates and follows strategic optimization. We now show that it is impossible for the bad node to remain undetected and hence the only threat of energy misbehavior in the network is through false advertisement.

**Proposition 8:** *If all nodes in the network are following a joint optimization regime, a bad node cannot un-detectably obtain a misbehavior gain by unilaterally following strategic energy optimization.*



*Proof:* WLOG, assume node 2 is the bad node performing unilateral strategic energy minimization while node 1 adheres to the power vectors produced under joint energy minimization. Clearly, due to the nature of strategic optimization, the bad node always has an energy gain from this type of misbehavior. Now assume that the good node cannot detect the others misbehavior provided the good node's packet can still be successfully transmitted. Based on our interference channel model, this is only possible when the good node's SNR during the overlapping period  $T_2$  is not decreased.

Let  $R_{22}^{s|j}$  and  $P_{22}^{s|j}$  denote the rate and power of node 2 over  $T_2$ , respectively, given that node 1 uses power  $P_{12}^j$  and  $P_{11}^j$  determined under the joint energy minimization protocol. The necessary condition for undetectable misbehavior by node 2 is  $P_{22}^{s|j} \leq P_{22}^j$  i.e  $R_{22}^{s|j} \leq R_{22}^j$ .

Given  $P_{12}^j > 0$ , to minimize node 2's total energy,  $R_{22}^{s|j}$  and  $R_{23}^{s|j}$  satisfy the following two equations:

$$\begin{aligned} R_{23}^{s|j}(1 - \mu_1) + (\mu_2 + \mu_1 - 1)R_{22}^{s|j} &= \frac{B_2}{T} \\ R_{22}^{s|j} + \log_2 \eta_{22}^j &= R_{23}^{s|j} + \log_2 \beta_1 \end{aligned} \quad (34)$$

where the first equation is the load constraint, the second one comes from solving the Lagrange multiplier equation for strategic minimization and  $\eta_{22}^j = P_{12}^j \alpha_2 + \beta_2$ . The power vectors for joint energy minimization must also satisfy  $P_{22}^j = \eta_{22}^j(y - 1)$  along with

$$P_{22}^j = (y - 1) \frac{\beta_2 + \alpha_2 \beta_1 (x - 1)}{1 - \alpha_1 \alpha_2 (x - 1)(y - 1)}$$

Combining these results, we solve Eq. (34) and obtain

$$2^{R_{22}^{s|j}} = 2^{\frac{B_2}{T\mu_2}} \left[ \frac{\beta_2 (1 - \alpha_1 \alpha_2 (x - 1)(y - 1))}{\beta_2 + \alpha_2 \beta_1 (x - 1)} \right]^{\frac{1 - \mu_1}{\mu_2}} \quad (35)$$

Define  $m = \frac{\mu_2}{1 - \mu_1}$  and  $n = \frac{\mu_1}{1 - \mu_2}$ . To satisfy the necessary condition of misbehavior  $R_{22}^{s|j} \leq R_{22}^j$ , and using  $y = 2^{R_{22}^j}$ , we have

$$y^m \geq 2^{\frac{mB_2}{T\mu_2}} \cdot \frac{\beta_2 (1 - \alpha_1 \alpha_2 (x - 1)(y - 1))}{\beta_2 + \alpha_2 \beta_1 (x - 1)} \quad (36)$$

Next, from the Lagrangean for joint energy minimization, we have

$$\begin{aligned} \lambda_2 &= \beta_2 2^{\frac{mB_2}{T\mu_2}} y^{1-m} \\ (1 + \alpha_1(x-1))(\beta_2 + \alpha_2 \beta_1(x-1)) &= \\ \frac{\lambda_2}{y} (1 - \alpha_1 \alpha_2 (x-1)(y-1))^2 & \end{aligned} \quad (37)$$

which yields

$$\begin{aligned} y^m (1 + \alpha_1(x-1))(\beta_2 + \alpha_2 \beta_1(x-1)) &= \\ \beta_2 2^{\frac{mB_2}{T\mu_2}} (1 - \alpha_1 \alpha_2 (x-1)(y-1))^2 & \end{aligned} \quad (38)$$

Combining Eq. (38) and (36), we obtain

$$1 - \alpha_1 \alpha_2 (x-1)(y-1) \geq 1 + \alpha_1(x-1), \quad (39)$$

which is only true for  $x = 1$  implying  $P_{12}^j = 0$ , i.e. it requires the good node to shut off its transmitter during  $T_2$ , which contradicts the assumption that  $P_{12}^j > 0$ . ■

Proposition 8 demonstrates that it is impossible for the bad node to misbehave using unilateral strategic approach without compromising the good nodes performance. Thus a bad node can misbehave only by falsely advertising its load as analyzed in the previous section.

## VII. NUMERICAL RESULTS

This section contains numerical results for optimal power allocation and misbehavior given the duty cycle  $\mu = \mu_1 = \mu_2$  for both the strategic and total energy minimization approach. It is assumed normalized  $\beta_1 = \beta_2 = 1$  and  $T = 1$ .

Figure 2-5 compare individual energies  $E_1$  and  $E_2$ , as well as the total energy  $E_1 + E_2$ , under both joint and strategic energy minimization schemes, respectively. It has been shown in [4] that joint energy minimization is strongly Pareto-optimal when duty-cycle overlap is complete, i.e.  $\mu = 1$ . Figure 2 and 3 demonstrate the case when joint energy minimization is still strongly Pareto-optimal even for partial overlap, i.e.  $\mu < 1$ . These observations agree with Proposition 4. It can also be seen that the dominance of the joint minimization scheme over the strategic one becomes greater as overlap  $\mu$  increases.

For intermediate  $\mu$  values, Figure 4 illustrates the benefit of the strategic approach in terms of energy gains by the user having smaller load and higher interference. Since the goal of the strategic scheme is to minimize individual energies, node 1 saves its energy at the price of higher energy consumption by node 2 compared to the joint energy minimization scheme.

Figures 4 and 5 reflect the converging tendency of these two schemes in the sense that the difference between individual energies is decreasing. We could expect as  $\mu \rightarrow 1$  (complete overlap of duty cycles), joint and strategic energy minimization will yield the same energy expenditures. If the node with higher load has better channel quality in terms of smaller  $\alpha_j$ , there exists a crossing point of  $\mu$  beyond which the joint minimization scheme becomes dominant, as shown in Figure 5.

Figure 6 demonstrates misbehavior by node 2 when both nodes perform joint energy minimization. Node 2's misbehavior results in energy savings for itself while simultaneously leading to higher energy costs for the good node. However, as shown in 7 if both nodes perform strategic energy minimization, misbehavior through false

advertisement not only increases the energy of the good node, but also that of the bad node.

The figures show that both joint as well as strategic optimization have their advantages. Either scheme can be preferable depending on the applications, specific parameters of data loads, channel qualities and duty cycles and tolerance for misbehavior.

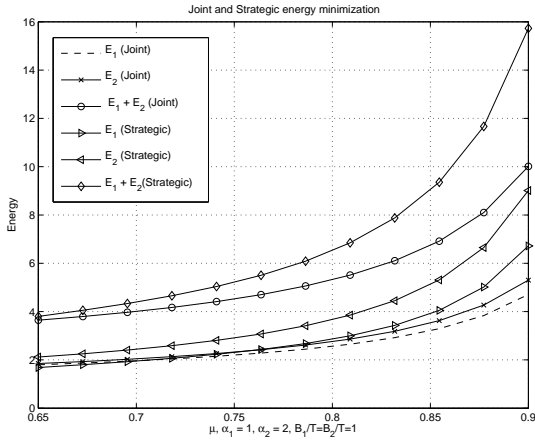


Fig. 2. Energy versus active cycle length for joint versus strategic energy minimization. Figure shows joint energy dominance.  $B_1/T = B_2/T = 1$ ,  $\alpha_1 = 1$ ,  $\alpha_2 = 2$ .

VIII. CONCLUSIONS

Power-control multiple access (PCMA) schemes have become an essential feature of many energy-constrained interference-limited wireless networks. A hidden feature of such PCMA schemes is the fact that they are based on implicit trust agreements between interfering nodes which makes them highly vulnerable to energy-depletion attacks. Compromised nodes can maliciously

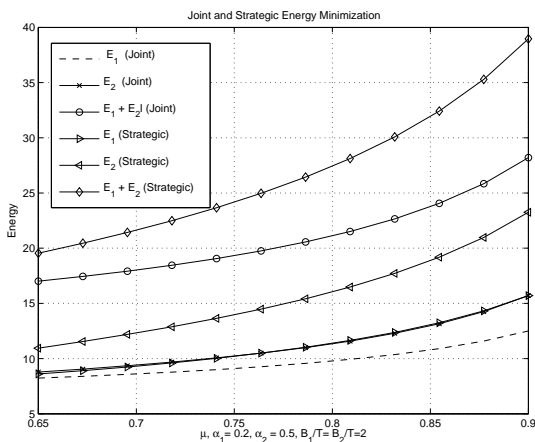


Fig. 3. Joint energy dominance for  $B_1/T = B_2/T = 2$ ,  $\alpha_1 = 0.2$ ,  $\alpha_2 = 0.5$ .

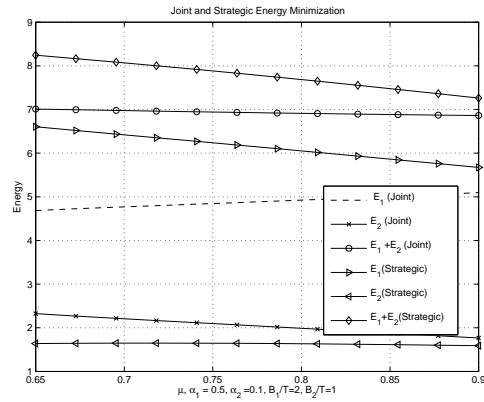


Fig. 4. Joint versus strategic energy minimization. Node 1 benefits from strategic minimization.  $B_1/T = 2$ ,  $B_2/T = 1$ ,  $\alpha_1 = 0.1$ ,  $\alpha_2 = 0.5$ .  $B_1^T/T = 2$ ,  $B_2^T/T = 1$ ,  $\alpha_1 = 0.5$ ,  $\alpha_2 = 0.1$ .

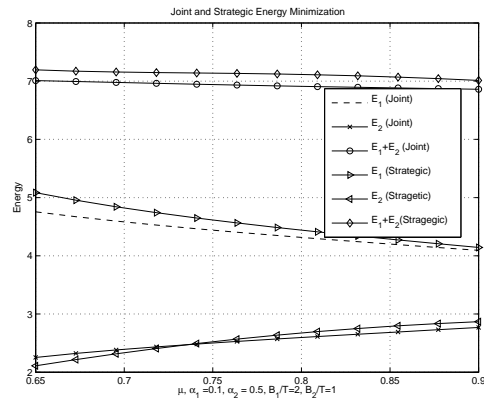


Fig. 5. Joint versus strategic energy minimization. Both nodes benefit from strategic minimization at different  $\mu$  values.  $B_1/T = 2$ ,  $B_2/T = 1$ ,  $\alpha_1 = 0.1$ ,  $\alpha_2 = 0.5$ .

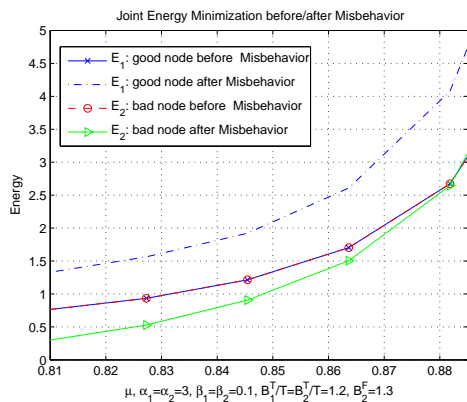


Fig. 6. Impact of misbehavior by node 2 on  $E_1$  and  $E_2$  for joint energy minimization by declaring false load  $B_2^F/T$ . Node 2 profits from misbehavior.  $\alpha_1 = \alpha_2 = 3$ ,  $\beta_1 = \beta_2 = 0.1$ ,  $B_1^T/T = B_2^T/T = 1.2$ ,  $B_2^F/T = 1.3$ .

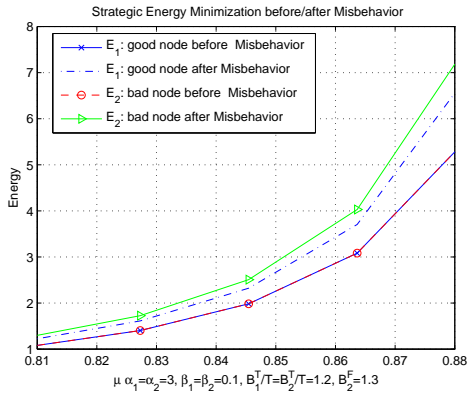


Fig. 7. Impact of misbehavior by node 2 on  $E_1$  and  $E_2$  for strategic energy minimization by declaring false load. Node 2 does not profit from misbehavior.  $\alpha_1 = \alpha_2 = 3$ ,  $\beta_1 = \beta_2 = 0.1$ ,  $B_1^T/T = B_2^T/T = 1.2$ ,  $B_2^F/T = 1.3$ .

adjust their transmission powers resulting in increased energy consumption at 'good' nodes who are faithfully following a power-control regime. In this paper, we present a novel formulation of the problem of energy misbehavior and develop an analytical framework for quantifying its impact on other nodes. Our analytical results reveal optimal strategies for attacking nodes in an enemy network through energy depletion. We also develop effective defense mechanisms for protecting our own wireless network against energy attacks by an intelligent adversary. Specifically, we formulate two versions of the power control problem for wireless networks with latency constraints arising from duty cycle allocations. In the first version, strategic power optimization, wireless nodes are modeled as rational agents in a power game, who strategically adjust their powers to minimize their own energy. In the other version, joint power optimization, wireless nodes jointly minimize the aggregate energy expenditure. We show that a node cannot unilaterally misbehave by transmitting strategically without being detected. We then show quantitatively how an enemy network can be attacked by falsely advertising traffic load information in order to minimize our energy consumption while maximally depleting the enemies'. While joint energy optimization is sometimes energy dominant, it is more vulnerable to energy misbehavior than strategic optimization. We provide sufficient conditions under which strategic optimization inoculates our network against an enemies' misbehavior. Extensions of our misbehavior model to the case of multiple nodes is described in [11].

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## APPENDIX

Proof of Proposition 1. *Proof:* For given  $P_{22}$ ,  $T$ ,  $\mu_1$  and  $\mu_2$ , the best-response ( $P_{11}^*$ ,  $P_{12}^*$ ) of node 1 is the solution to the constrained minimization problem  $\min E_1$  s.t  $L_1 = 0$  or equivalently,

$$\begin{aligned} \text{Min} \quad & P_{11} + (n-1)P_{12} \\ \text{s.t} \quad & \log_2 \left( 1 + \frac{P_{11}}{\beta_1} \right) + (n-1) \log_2 \left( 1 + \frac{P_{12}}{\beta_1 + \alpha_1 P_{22}} \right) - \bar{R}_1 = 0 \end{aligned}$$

Since this is a minimization of a convex objective function with convex constraints ( $E_1$  and  $L_1$  are convex functions of  $P_{11}$  and  $P_{12}$ ), the global minimum can be obtained by considering the function  $H_1(\lambda_1, P_{11}, P_{12}) = E_1 - \lambda_1 L_1$ , where  $\lambda_1$  is the Lagrange multiplier [12]. The necessary and sufficient condition for the global minimum of  $E_1$  is

$$\nabla_{\lambda_1, P_{11}^*, P_{12}^*} H_1 = \bar{0} \quad (40)$$

Solving Eq. 40 leads to

$$\lambda_1 / \ln 2 = P_{11}^* + \beta_1 \quad (41)$$

$$\lambda_1 / \ln 2 = P_{12}^* + \beta_1 + \alpha_1 P_{22} \quad (42)$$

$$\lambda_1 / \ln 2 = \frac{\beta_1 C_1}{(x^*)^{n-1}} \quad (43)$$

where  $x^* - 1 = P_{12}^* / (\beta_1 + \alpha_1 P_{22})$ . Further, it can be seen that  $\nabla^2 H_1(P_{11}^*, P_{12}^*)$  is a non-negative definite matrix. Thus the necessary condition is also a sufficient condition and the local minimum is a Global minimum [12].

Similarly, the best-responses ( $P_{23}^*$ ,  $P_{22}^*$ ) of node 2 for a given  $P_{12}$  can be obtained from  $H_2(\lambda_2, P_{23}, P_{22}) = E_2 - \lambda_2 L_2$  in an identical manner as:

$$\lambda_2 / \ln 2 = P_{23}^* + \beta_2 \quad (44)$$

$$\lambda_2 / \ln 2 = P_{22}^* + \beta_2 + \alpha_2 P_{12} \quad (45)$$

$$\lambda_2 / \ln 2 = \frac{\beta_2 C_2}{(y^*)^{m-1}} \quad (46)$$

where  $y^* - 1 = P_{22}^s / (\beta_2 + \alpha_2 P_{12})$ .

The above equations describe the best-responses of each node to an arbitrary power value of the other node. At the Nash equilibrium point, these power values are not arbitrary and must in fact be best-responses to each other. Let  $(P_{11}^s, P_{12}^s)$  and  $(P_{23}^s, P_{22}^s)$  represent the Nash power vectors. They can be obtained by solving Eqs. 41-46, where all the power variables are replaced by the  $P_{ij}^s$ 's.

Combining Eqs. 42 and 43 and Eqs. 45 and 46, we get

$$\beta_1 + \alpha_1 P_{22}^s = \frac{\beta_1 C_1}{x_s^n} \quad (47)$$

$$\beta_2 + \alpha_2 P_{12}^s = \frac{\beta_2 C_2}{y_s^m} \quad (48)$$

We also have, by definition,

$$x_s - 1 = \frac{P_{12}^s}{\beta_1 + \alpha_1 P_{22}^s} \quad (49)$$

$$y_s - 1 = \frac{P_{22}^s}{\beta_2 + \alpha_2 P_{12}^s} \quad (50)$$

Combining Eqs. 47-50 and simplifying, we get equilibrium functions  $\mathcal{F}$  and  $\mathcal{G}$  as stated. ■

Next we prove Proposition 2. *Proof:* We provide a simple algebraic and graphical proof of the proposition. The Nash equilibrium functions  $\mathcal{F}$  and  $\mathcal{G}$  from Prop. 1 can be rewritten as

$$\mathcal{F} : \frac{y_s^m}{y_s - 1} = \frac{\alpha_1 \beta_2 C_2}{\beta_1} \frac{x_s^n}{C_1 - x_s^n} \quad (51)$$

$$\mathcal{G} : \frac{x_s^n}{x_s - 1} = \frac{\alpha_2 \beta_1 C_1}{\beta_2} \frac{y_s^m}{C_2 - y_s^m} \quad (52)$$

Meaningful equilibria correspond to  $1 \leq x_s \leq C_1^{1/n}$  and  $1 \leq y_s \leq C_2^{1/m}$ . The LHS of Eq. 51 is  $U$ -shaped with the minimum value occurring at  $y_s = m/(m-1)$  while the RHS is increasing in  $x_s$  with the minimum value at  $x_s = 1$ . For simplicity, we assume  $n$  and  $m$  are integers. Condition S.1 indicates whether the minimum value of the RHS of Eq. 51 i.e.  $(\alpha_1 \beta_2 C_2) / (\beta_1 (C_1 - 1))$  is greater than the RHS for  $y_s = C_2^{1/m}$  i.e.  $C_2 / (C_2^{1/m} - 1)$ . Thus it can be seen that within these  $x_s$  and  $y_s$  boundaries,  $\mathcal{F}$  forms a curve with bottom endpoint at  $(x_s \rightarrow C_1^{1/n}, y_s = 1)$  and top endpoint intersecting *only* the vertical line  $x_s = 1$  if condition S.1 is true and the horizontal line  $y_s = C_2^{1/m}$  *only if* condition S.1 is false. A similar observation can be made about  $\mathcal{G}$  using Eq. 52, i.e. a curve with top endpoint at  $(x_s = 1, y_s \rightarrow C_2^{1/m})$  and bottom endpoint intersecting either only  $y_s = 1$  or  $x_s = C_1^{1/n}$  depending on condition T.1.

Combining the two observations, note that if S.1 and T.1 are simultaneously true or false,  $\mathcal{F}$  and  $\mathcal{G}$  must intersect within the prescribed  $x_s, y_s$  boundary, thereby creating Nash equilibria. Further they must intersect at most at three points. For there to be no Nash equilibrium, exactly one of the conditions S.1 or T.1 must be true. Assume T.1 is true. Then, for there to be no Nash equilibrium, the leftmost point of  $\mathcal{F}$  within the boundary should lie above  $\mathcal{G}$ , i.e. have a bigger  $y_s$  value. Algebraically, this translates to condition S.2 being true. ■

Next we prove Proposition 3. *Proof:* The joint objective function to be minimized by both nodes is

$$\text{Min } (E_1 + E_2) \text{ s.t } \{L_1 = 0, L_2 = 0\}$$

Consider the function  $H(\lambda_1, \lambda_2, P_{11}, P_{12}, P_{23}, P_{22}) = \sum_i E_i - \lambda_i L_i$ , where  $\lambda_i$  is the Lagrange multiplier,  $i = 1, 2$ . Unlike the strategic optimization case, the constraints are non-convex. Thus the necessary condition for the local minima of  $\sum_i E_i$  is

$$\nabla_{\lambda_1, \lambda_2, P_{11}^j, P_{12}^j, P_{23}^j, P_{22}^j} H = \bar{0} \quad (53)$$

First we have, by definition,

$$x_j - 1 = \frac{P_{12}^j}{\beta_1 + \alpha_1 P_{22}^j} \quad (54)$$

$$y_j - 1 = \frac{P_{22}^j}{\beta_2 + \alpha_2 P_{12}^j} \quad (55)$$

Differentiating  $H$  with respect to the  $P_{ij}^j$ 's and simplifying leads to

$$P_{11}^j + \beta_1 = \frac{x_j (\beta_1 + \alpha_1 P_{22}^j) (1 + \alpha_2 (y_j - 1))}{1 - \alpha_1 \alpha_2 (x_j - 1)(y_j - 1)} \quad (56)$$

$$P_{23}^j + \beta_2 = \frac{y_j (\beta_2 + \alpha_2 P_{12}^j) (1 + \alpha_1 (x_j - 1))}{1 - \alpha_1 \alpha_2 (x_j - 1)(y_j - 1)} \quad (57)$$

Differentiating  $H$  with respect to  $\lambda_1$  and  $\lambda_2$  leads to

$$P_{11}^j + \beta_1 = \frac{\beta_1 C_1}{x_j^{n-1}} \quad (58)$$

$$P_{23}^j + \beta_2 = \frac{\beta_2 C_2}{y_j^{m-1}} \quad (59)$$

Combining the above equations together leads to

$$\beta_1 C_1 = \frac{x_j^n (\beta_1 + \alpha_1 P_{22}^j) (1 + \alpha_2 (y_j - 1))}{1 - \alpha_1 \alpha_2 (x_j - 1)(y_j - 1)} \quad (60)$$

$$\beta_2 C_2 = \frac{y_j^n (\beta_2 + \alpha_2 P_{12}^j) (1 + \alpha_1 (x_j - 1))}{1 - \alpha_1 \alpha_2 (x_j - 1)(y_j - 1)} \quad (61)$$

Substituting for  $P_{12}^j$  and  $P_{22}^j$  above using Eqs. 54 and 55 and simplifying, we get  $\mathcal{P}$  and  $\mathcal{Q}$  as stated. Note that since the constraints are non-convex the solutions to  $\mathcal{P}$  and  $\mathcal{Q}$  specify all minima and must be evaluated exhaustively for global minima. ■

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