Approximation Algorithms for Minimum Energy Transmission in Rate and Duty-Cycle Constrained Wireless Networks*

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Abstract

We consider a constrained energy optimization problem for wireless networks, where the constraints arise because of interference between wireless nodes that limits their transmission rates along with load and duty-cycle (on-off) restrictions. Since traditional optimization methods using Lagrange multipliers do not work well and are computationally expensive given the non-convex constraints, we develop fully polynomial approximation schemes (FPAS) for finding the optimal (minimum energy) transmission schedule by discretizing power levels over the interference channel. For any $\epsilon > 0$, we develop an algorithm for computing the optimal number of discrete power levels per time slot ($O(1/\epsilon)$), and use this to design a $(1, 1+\epsilon)$-FPAS that consumes no more energy than the optimal while violating each rate constraint by at most a $1+\epsilon$ factor. For wireless networks with low-cost transmitters, where nodes are restricted to transmitting at a fixed power over active time slots, we develop a 2-factor approximation for finding the optimal fixed transmission power value P_{opt} that results in the minimum energy schedule.

1 Introduction

Energy-efficiency is a critical concern in many wireless networks, such as cellular networks, ad-hoc networks or wireless sensor networks (WSNs) that consist of large number of sensor nodes equipped with unreplenishable and limited power resources. Since wireless communication accounts for a significant portion of node energy consumption, network lifetime and utility are dependent on the design of energy-efficient communication schemes including low-power signaling and energy-efficient multiple access protocols.

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Delay is also an important constraint in many wireless network applications, for example battlefield surveillance or target tracking in which data with finite lifetime-information must be delivered before a deadline. Delay constraints in wireless networks can also be examined in terms of node operation under periodic duty cycles, in which time is divided into active (awake) and inactive (asleep) periods. [1], [2, 3] establish the idea of duty cycles in WSNs as a practical means of conserving node energy. Minimizing transmission energy subject to latency constraints has been studied [4, 5] while [6] studies energy-latency tradeoffs for data gathering. Several approaches for maximizing information transmission over a shared channel subject to average power constraints have been proposed [7, 8, 9, 10, 11]. [12] addresses the issue of minimizing transmission power, subject to a given amount of information being successfully transmitted and derives power control multiple access (PCMA) algorithms for autonomous channel access.

In this paper, we consider a constrained energy optimization problem for wireless networks, where the constraints arise because of interference between wireless nodes that limits their transmission rates along with load and duty-cycle (on-off) restrictions. We consider N wireless nodes transmitting to their destinations over a typical Additive White Gaussian Noise (AWGN) interference channel over a time period T. These nodes could represent reasonably close neighbors communicating as part of some MAC protocol. Their receivers could be distinct or identical, representing the case when all nodes are transmitting to the same basestation or clusterhead. We assume that time T is divided into M slots of equal duration. Let P_{it} be the transmit power used by node i during time slot t, $1 \le t \le M$. Let R_{it} represent the achievable transmission rate for node i during time slot t over this N-node interference channel. Single user decoding is assumed at each receiver to decode the information from its own transmitter while treating the remaining information as Gaussian interference.

Thus we have,

$$R_{it} = \frac{1}{2}\log_2\left(1+\frac{\alpha_{ii}^t P_{it}}{\mathcal{N}_i^t+\sum_{j\neq i}\alpha_{ji}^t P_{jt}}\right), \quad 1\le i\le N,\ \ 1\le t\le M \quad (1)$$

where α_{ji}^t represent the channel attenuation at i's receiver due to transmitter j, which captures the effects of path-loss, shadowing and frequency nonselective fading, and $\mathcal{N}_i^t$ represents the background interference (usually $\mathcal{N}_i^t = \mathcal{N}_0$), during time slot t. We assume these parameters remain fixed over a (short) time slot of duration T/M but can vary from slot to slot.

We are interested in the following scheduling and energy minimization problem (labeled MESP: minimum energy scheduling problem)

$$\begin{aligned} &\min f : \sum_{i=1}^{N}\sum_{t=1}^{M} P_{it} \\ \text{s.t}\quad & g : \sum_{t=1}^{M} A_{it}R_{it} \ge \tilde{R}_i \quad i=1,2,\ldots,N \\ & A_{it} = \begin{cases} 0 \text{ if node } i \text{ is idle} \\ 1 \text{ otherwise}\end{cases} \\ & \sum_{t=1}^{M} A_{it} \le \mu_i \quad i=1,2,\ldots,N \end{aligned} \quad (2)$$

The objective function in MESP is to determine the schedule which minimizes the total energy. Since all slots are assumed to be of fixed duration, this is equivalent to minimizing the total transmitted power. Each node must maintain an average rate constraint $\tilde{R}_i$ over the M slots. Further, we assume that nodes operate under duty-cycles where time T is divided into active and idle time slots, wireless sensor networks for example, operate under such constraints [2, 1]. The duty-cycle constraint of node i is given by μ_i: the maximum number of time slots it can remain active, $1 \le \mu_i \le M$, $i = 1, 2, \ldots N$. $A_{it} \in \{0, 1\}$ depending on whether the node is idle or active during slot t, $1 \le t \le M$.

Note that the system model described above can be easily translated from the time domain to the frequency domain for orthogonal frequency division multiplexed (OFDM) systems. Each slot in the time domain now becomes a frequency sub-band over which a subset of the N users transmit to a basestation. The duty cycle constraint for each user in the time domain now changes to the number of frequency bands each user can use at maximum. Each users information transmitted through its assigned sub-bands is decoded by treating other users' information as pure interference, i.e single-user decoding is deployed [13]. Therefore, without loss of generality, we focus on the time domain model in the rest of this paper. We assume at the beginning of each duty cycle, all channel coefficients can be obtained through training sequences [14, 15]. The measured channel coefficients are then fedback by the basestation/receivers to the transmitters and used to develop the optimal energy schedule in 2. As typically assumed (for example, [16, 17]), measurements and feedback of channel fading variables are assumed perfect.

It can be seen that the rate constraints above are non-convex in the power variables P_{it}, even for the restricted version of MESP with two users ($N = 2$). Unfortunately this implies that traditional analytical optimization methods such as Lagrange multipliers [18] will not work well, since convexity of the constraints is a necessary condition for obtaining the global minimum using the Lagrangean $H = f + \lambda_k g_k$ (where g_k are the constraints), and computing $\bigtriangledown_{P_{it},\lambda_k} = 0$. Moreover finding the global minimum through exhaustive search of all possible solutions of $\partial h/\partial P_{it} = 0$ is likely to be computationally expensive. Alternately computing the optimal dual $\max_\lambda \min_x h()$ introduces a duality gap which vanishes only under certain conditions on the number of constraints and parameters N and M [18, 19]. As an example of this technique, the authors in [16, 17], consider the problem of maximizing the sum transmission rate of a group of users with maximum power constraints using OFDM. The constraint functions could be non-concave. They consider the dual problem (whose solution has a duality gap with respect to the optimal primal) and provide an iterative search based algorithm that searches over the entire range of Lagrange multiplier values. They do not analyze the complexity of their algorithm (which appears to be exponential) but indicate correctness by showing that there are conditions under which the duality gap vanishes for large number of frequencies.

In this paper, rather than solve the objective function exactly by analytical techniques (with hard to evaluate complexity), we develop an algorithmic methodology based on power discretization and rounding. We provide a fully-polynomial approximation scheme that will solve the rate and duty-cycle constrained energy objective while violating some of the constraints, both within given arbitrarily small factors and find the optimal number of power levels required for the approximate optimal schedule.

From the algorithmic perspective, the MESP problem is NP-hard and related to the generalized assignment problem [20]. We develop fully polynomial approximation schemes (FPAS) for MESP using ideas related to bin-packing and the knapsack problem [20]. We first show a simple dynamic programming solution (of exponential complexity in M) that optimally solves the restricted problem. We then develop an algorithm for computing the optimal number of discrete power levels per time slot ($O(1/\epsilon)$), and use this to design a $(1, 1+\epsilon)$-FPAS that consumes no more energy than the optimal while violating each rate constraint by at most a $1+\epsilon$ factor. For two fixed transmit power levels, we then develop a 2-factor approximation for finding the optimal fixed transmit power level per time slot, P_{opt}, that generates the optimal (minimum) energy schedule.

2 Basic Dynamic Programming Solution

First, we consider a simplified version of the minimum energy scheduling problem using two discrete transmit power levels. In the restricted version, a node is allowed to be either idle or transmit with a given (fixed) power P during its active slot. We illustrate our schemes using two nodes ($N = 2$) over M time slots. Even this restricted two node case is not amenable to traditional optimization methods like the Lagrangean and is also NP-hard. Later in section 6, we extend the approximations to the N-node, M-time slot case.

The restricted optimization problem is described by:

$$\min \sum_{i=1}^{2} \sum_{t=1}^{M} P_{it}$$

$$\text{s.t} \ \sum_{t=1}^{M} R_{it} \geq \tilde{R}_i, \quad i = 1, 2$$

$$P_{it} \in \{0, P\}, \quad i = 1, 2; \quad t = 1, \ldots, M \qquad (3)$$

$$A_{it} = \begin{cases} 0 \text{ if } P_{it} = 0 \\ 1 \text{ otherwise} \end{cases}$$

$$\sum_{t=1}^{M} A_{it} \leq \mu_i, \quad i = 1, 2 \qquad (4)$$

We assume that $\mu_1 + \mu_2 \geq M$, i.e the two nodes have to interleave during some of the slots. A more restricted version of 4 with $\alpha_{ji}^t = \alpha_{ji}$ independent of t is analyzed in [21].

Let $\bar{R}_{i,j}^{kP,a,b} = \{(R_1, R_2)\}$ represent the set of rate vectors (list of rate pairs) corresponding to cumulative transmission rates for user 1 and user 2 from time slots i through j, $1 \leq i \leq j \leq M$, while using a total power (node 1 + node 2) of kP and having a total of a and b active slots, respectively, where $0 \leq a, b \leq j - i + 1$. Since a node uses fixed power P during an active slot, $a + b = k$, in this case. For notational simplicity, if $i = j$, we drop one of the redundant subscripts in the rate vector. In the above definition, $R_l = \sum_{t=i}^{j} R_{lt}$, where R_{lt}, $l = 1, 2$, is the achievable rate for node l during time slot t, depending on the actions of the other node i.e active/asleep. Thus for a given time slot t, we have four different rate vectors specified by,

$$\begin{aligned} \bar{R}_t^{0,0,0} &= (0, 0) \\ \bar{R}_t^{P,0,1} &= (0, \frac{1}{2}\log_2\left(1 + \frac{\alpha_{22}^t P}{\mathcal{N}_2^t}\right)) \\ \bar{R}_t^{P,1,0} &= (\frac{1}{2}\log_2\left(1 + \frac{\alpha_{11}^t P}{\mathcal{N}_1^t}\right), 0) \\ \bar{R}_t^{2P,1,1} &= (\frac{1}{2}\log_2\left(1 + \frac{\alpha_{11}^t P}{\mathcal{N}_1^t + \alpha_{21}^t P}\right), \\ &\quad \frac{1}{2}\log_2\left(1 + \frac{\alpha_{22}^t P}{\mathcal{N}_2^t + \alpha_{12}^t P}\right)) \end{aligned} \qquad (5)$$

The restricted version of the problem consists of finding a transmission schedule of minimum total energy in which active nodes transmit at a fixed power during each active time slot while also satisfying the given duty-cycle and rate constraints. For fixed power level P, the optimal schedule is easily specified by the following dynamic program which maintains the current best-solution of rate vectors for each total power level and duty-cycle value. The boundary conditions are given by the rate vectors in Eq. 5. The recursive formula for each power level kP and duty-cycles a, b, $1 \leq k \leq (\mu_1 + \mu_2)$, $0 \leq a \leq \mu_1$, $0 \leq b \leq \mu_2$ is

$$\begin{aligned} \bar{R}_{i,j}^{kP,a,b} = \text{vectormax}\Big\{ & \bar{R}_{i,j-1}^{kP,a,b} \\ & \bigcup \left(\bar{R}_{i,j-1}^{(k-1)P,a-1,b} + \bar{R}_j^{P,1,0}\right) \\ & \bigcup \left(\bar{R}_{i,j-1}^{(k-1)P,a,b-1} + \bar{R}_j^{P,0,1}\right) \\ & \bigcup \left(\bar{R}_{i,j-1}^{(k-2)P,a-1,b-1} + \bar{R}_j^{2P,1,1}\right) \Big\} \end{aligned} \qquad (6)$$

where the rate vectors in each union operation above are computed using pairwise addition of the individual vectors. The vectormax operation eliminates all dominated vectors from the set, i.e. $\forall\{(R_1,R_2),(R_3,R_4)\} \in \bar{R}^{kP,a,b}_{i,j}$ either $R_1 > R_3$ and $R_2 \le R_4$ or vice versa. Using the recursive function, the table of values is evaluated in increasing order of time slots from $i = 1,\ j = 1,2,\ldots M$. There are $O(M\mu_1\mu_2)$ table entries corresponding to all possible total power consumption (kP)and duty-cycle solutions, $1 \le k \le 2M$, $0 \le a \le \mu_1$, $0 \le b \le \mu_2$ The number of rate vectors corresponding to each table entry can be exponential as described below. On termination of the algorithm, the set of feasible schedules correspond to those rate vectors $\ge (\tilde{R}_1,\tilde{R}_2)$ under the usual meaning of vector comparison. The optimal schedule for a given transmit power level P is the one whose rate vector satisfies

$$\bar{R}^P_{opt} = \operatorname*{argmin}_{k=1,2\ldots,2M} \Big\{(R_1,R_2) \in \bar{R}^{kP,\mu_1,\mu_2}_{1,M} | \quad (R_1,R_2) \ge (\tilde{R}_1,\tilde{R}_2)\Big\} \quad (7)$$

In practice, it is likely that many of the vectors in $\bar{R}^{kP,a,b}_{i,j}$ would be dominated and hence eliminated by the vectormax operation. However in the worst-case, even after the vectormax operation, the size of $\bar{R}^{kP,a,b}_{i,j}$ can quadruple with each additional slot. Thus the above dynamic program is clearly exponential in terms of the slot parameter M, even though each slot contains only four rate vectors. This motivates us to consider a $(1, 1+\epsilon)$ FPAS for the problem, as described next.

3 Minimum Energy Schedule with Multiple Power Levels

We now consider the scheduling problem with multiple discretized power levels, where each node can choose from a set of power levels per time slot. As shown below, if the power levels are chosen appropriately, the cost of the resulting minimum energy schedule approximates the cost of the optimal schedule to within an ϵ-factor.

For the optimization problem with multiple power levels, let P and L_t denote the maximum allowable transmit power and the number of discrete power levels available per time slot, respectively, with values as defined below. For this problem, the constraint 3 of Eq. 4 is replaced with

$$P_{it} \in \{P_l\}, \quad l = 0,1,\ldots L_t; \quad 0 = P_0 \le P_l \le P_{L_t} = P; \quad i = 1,2; \quad t = 1,\ldots,M. \quad (8)$$

Note that the corresponding optimal version of the minimum energy scheduling problem contains the constraint

$$0 \le P_{it} \le P, \qquad i = 1,2; \quad t = 1,\ldots,M \quad (9)$$

Let $\mathcal{A}^{P^*}$ denote the optimal algorithm for the above restricted version of MESP with per slot maximum power constraints (Eq. 9), i.e nodes select an optimal power value $0 \le P^*_{it} \le P$ in each slot, to satisfy their rate and duty-cycle constraints. Let R^*_{it} denote the corresponding optimal rate achieved per time slot, $i = 1,2,\ t = 1,2,\ldots M$. Finally, let $P^* = \sum\sum P^*_{it}$ and $R^*_i = \sum_t R^*_{it}$ denote the overall optimal power and rate allocations. In general, an (α,β) approximation of the optimal minimum energy scheduling problem is one which provides a feasible schedule with total power $\hat{P} \le \alpha P^*$ and each rate constraint violated by at most a β-factor i.e $\beta\hat{R}_i \ge R^*_i$, for each node i. Note that $R^*_i \ge \tilde{R}_i$ and hence $\beta\hat{R}_i \ge \tilde{R}_i$. Given some $\epsilon > 0$, we first show the construction of a more computationally expensive $(1+\epsilon, 1+\epsilon)$-approximation in order to illustrate our approach and then describe a more efficient $(1, 1+\epsilon)$-approximation to the optimal.

Let $P' = P'_1 + P'_2$, where P'_i is the solution to the problem

$$\begin{aligned}
\min P'_i &= \sum_{t=1}^{M} P_{it}, \quad i = 1,2 \\
\text{s.t} \sum_{t=1}^{M} \frac{1}{2}\log_2\left(1 + \frac{\alpha^t_{ii}P_{it}}{\mathcal{N}^t_i}\right) &\ge \tilde{R}_i, \quad i = 1,2 \\
P_{it} &\ge 0 \quad i = 1,2; t = 1,..,M \\
\sum_{t=1}^{M} A_{it} &\le \mu_i, \quad i = 1,2 \\
A_{it} &= \begin{cases} 0 \text{ if } P_{it} = 0 \\ 1 \text{ otherwise} \end{cases}
\end{aligned} \quad (10)$$

P'_j is the solution to the problem of zero-interference scheduling of node j with variable (non-discrete) power levels and can be found using standard Lagrange multiplier techniques [18]. Thus P' is a lower bound for the minimum energy scheduling problem using discrete power levels. Now define $q = \min_{i,t} \left\{\frac{P'}{M}, \frac{\alpha^t_{ii}}{\alpha^t_{ji}}\left(2^{\epsilon\tilde{R}_i/M} - 1\right)\right\}$, $i,j = 1,2$, $1 \le t \le M$. Let k be the largest solution to the equation $kq = 2\ln kP$ such that

$$e/P < k \leq \frac{2(2^{\epsilon \tilde{R}_i/M} - 1)}{q\left(1 + \epsilon - 2^{\epsilon \tilde{R}_i/M}\right)} \tag{11}$$

else set $k = 0$. For the given $\epsilon > 0$, choose $\delta_1 = \frac{\epsilon q}{2+kq}$. If $k = 0$, let $r_0 = \lfloor \frac{2P}{q\epsilon} \rfloor$, otherwise $r_0 = \lceil \frac{2+kq)}{\epsilon kq} \rceil$. Let $s_0 = \lfloor \ln_{1+k\delta_1} P/r_0\delta_1 \rfloor$.

Allocate power to nodes in each time slot by dividing the total available power P into the following $L_t = r_0 + s_0 + 2$ discrete power levels.

$$P_r = \begin{cases} r\delta_1, & 0 \leq r \leq r_0 \\ (1 + k\delta_1)^{r-r_0} P_{r_0}, & r_0 + 1 \leq r \leq r_0 + s_0 \\ P, & r = r_0 + s_0 + 1 \end{cases} \tag{12}$$

Lemma 1 *For given max power level P and constraints $\tilde{R}_i$, the number of discrete power levels per slot L_t is $O(\frac{1}{q\epsilon})$.*

Proof: Note that we are allocating power levels by dividing the range of available power into two types of intervals: the first r_0 intervals of fixed size δ_1 and remaining intervals of geometrically increasing size. Since geometric intervals are small in the beginning, the total number of power levels would be much larger if we only used geometrically increasing intervals. Therefore we use intervals of fixed size initially and choose integer r_0 such that the size of the first geometric interval, $k\delta_1^2 r_0$ is the same as the size of the previous fixed interval δ_1. The overall objective is to find optimal values of k and δ_1 that minimize the total number of power levels, yet allow us to closely approximate the overall energy consumption and rate constraints. From the energy approximation requirements (as shown below), we will get the constraint $\delta_1 = q\epsilon/(2 + kq)$. Hence $k\delta_1 < \epsilon$ and thus for small ϵ, the total number of levels $L_t = r_0 + s_0 = 1/(k\delta_1) + \ln_{1+k\delta_1} kP$ can be approximated by $\frac{1+\ln kP}{k\delta_1} = (1/\epsilon)(1+\ln kP)(1+2/(kq))$. Thus the objective is to find k that minimizes L_t. The solution to this minimization is $\ln kP = kq/2$ subject to $\ln kP > 1$. If k does not satisfy these conditions then $\delta_1 = q\epsilon/2$ and the number of power levels is $\lceil \frac{2P}{q\epsilon} \rceil$. ■

The remaining constraints on k as specified in Eq. 11, are obtained from the rate approximation requirements shown below.

Theorem 1 *For small $\epsilon > 0$, let $\mathcal{A}^{\hat{P}}$ denote the modified version of the (exponential) dynamic programming algorithm $\mathcal{A}^P$ in which each node can select from discrete power levels per time slot as specified by Eq. 12, subject to overall duty-cycle and rate constraints $\tilde{R}_i(1-\epsilon)$. Then $\mathcal{A}^{\hat{P}}$ is a $(1+\epsilon, 1+\epsilon)$-approximation of $\mathcal{A}^{P^*}$.*

Proof: Divide the set of time slots $T = \{1, 2, \ldots, M\}$ into disjoint sets T_{11} and T_{12} (resp. T_{21} and T_{22}) such that

$$\begin{array}{lll} t \in T_{11}(\text{resp. } T_{21}) & \text{if} & P^*_{1t}(\text{resp. } P^*_{2t}) \in [0, r_0\delta_1] \\ t \in T_{12}(\text{resp. } T_{22}) & \text{if} & P^*_{1t}(\text{resp. } P^*_{2t}) \in (r_0\delta_1, P] \end{array} \tag{13}$$

Let $\hat{P}_{it}$ and $\hat{R}_{it}$ denote the (discrete) power levels and rate allocations per node per time slot under $\mathcal{A}^{\hat{P}}$. Since $\mathcal{A}^{\hat{P}}$ considers combinations of power levels over M slots, the errors in power levels and rate allocations per slot (either absolute or relative) must be bounded from above. Consider the solution in $\mathcal{A}^{\hat{P}}$ that simply rounds up the optimal power level in each slot to the nearest (larger) discrete power level. For this solution, the absolute error is bounded by $\hat{P}_{it} - P^*_{it} < \delta_1, \quad t \in T_{i1}$, and the relative error by $\hat{P}_{it} < (1 + k\delta_1)P^*_{it}, \quad t \in T_{i2}$, $i = 1, 2$. Therefore we have

$$\begin{aligned} \hat{P} &= \sum_i \sum_{t \in T_{i1}} \hat{P}_{it} + \sum_i \sum_{t \in T_{i2}} \hat{P}_{it} \\ &\leq P^* + \frac{q\epsilon\left(|T_{11}| + |T_{21}|\right)}{2 + kq} + \frac{kq\epsilon}{2 + kq} \sum_i \sum_{t \in T_{i2}} P^*_{it} \\ &\leq P^* + \frac{2Mq\epsilon}{2 + kq} + \frac{\epsilon kq}{2 + kq} P^* \end{aligned} \tag{14}$$

The overall relative error in energy P_{err}, of this solution $\hat{P}$ is defined as

$$P_{err} = \frac{\hat{P} - P^*}{P^*}$$

Therefore we can bound the relative error as

$$P_{err} = \frac{2\epsilon}{2 + kq} \cdot \frac{Mq}{P^*} + \frac{\epsilon kq}{kq + 2} \leq \epsilon \tag{15}$$

since $q \leq P'/M \leq P^*/M$ as P' is a lower bound for the optimal energy value P^*. Hence this particular solution of algorithm $\mathcal{A}^{\hat{P}}$ approximates the optimal energy value of the minimum energy schedule to within an ϵ factor.

To complete the proof, we just need to show that the above power allocation is also a feasible solution in terms of the rate constraints i.e the overall rates achieved by $\mathcal{A}^{\hat{P}}$ also approximate each rate constraint to within an ϵ factor. First consider the achieved rate $\hat{R}_{1t}$, for the case $t \in T_{21}$.

$$\begin{aligned}\hat{R}_{1t} &\geq \frac{1}{2}\log_2\left(1+\frac{\alpha_{11}^t P_{1t}^*}{\mathcal{N}_1^t+\alpha_{21}^t(P_{2t}^*+\delta_1)}\right)\\ &\geq \frac{1}{2}\log_2\left(1+\frac{\alpha_{11}^t P_{1t}^*}{\mathcal{N}_1^t+\alpha_{21}^t P_{2t}^*}\cdot\frac{1}{1+\frac{\alpha_{21}^t\delta_1}{\mathcal{N}_1^t+\alpha_{21}^t P_{2t}^*}}\right)\\ &\geq R_{1t}^*-\frac{1}{2}\log_2\left(1+\frac{\delta_1}{P_{2t}^*+\frac{\mathcal{N}_1^t}{\alpha_{11}^t}\cdot\frac{\alpha_{11}^t}{\alpha_{21}^t}}\alpha_{21}^t)\right) \quad (16)\end{aligned}$$

Using the fact that $P_{2t}^* \geq 0$, and the background noise $\mathcal{N}_1^t/\alpha_{11}^t \geq 1$ for each time slot $t \in T_{11}$, we can bound the absolute R_1 rate error $= R_1^* - \hat{R}_1$ over all such time slots by

$$\frac{M}{2}\log_2\left(1+\max_t\left(\frac{\alpha_{21}^t}{\alpha_{11}^t}\right)\delta_1\right) \leq \frac{\epsilon\tilde{R}_1}{2}$$

by using the fact that $\delta_1 \leq \epsilon q \leq \min_t\left(\frac{\alpha_{11}^t}{\alpha_{21}^t}\right)\epsilon\left(2^{2\epsilon\tilde{R}_1/M}-1\right)$.

Next, for $t \in T_{22}$ (when $k > 0$), we get

$$\begin{aligned}\hat{R}_{1t} &= \frac{1}{2}\log_2\left(1+\frac{\alpha_{11}^t\hat{P}_{1t}}{\mathcal{N}_1^t+\alpha_{21}^t\hat{P}_{2t}}\right)\\ &\geq \frac{1}{2}\log_2\left(1+\frac{\alpha_{11}^t P_{1t}^*}{\mathcal{N}_1^t+\alpha_{21}^t P_{2t}^*(1+k\delta_1)}\right)\\ &\geq \frac{1}{2}\log_2\left(1+\frac{1}{1+k\delta_1}\cdot\frac{\alpha_{11}^t P_{1t}^*}{\frac{\mathcal{N}_1^t}{1+k\delta_1}+\alpha_{21}^t P_{2t}^*}\right)\end{aligned}$$

Since $k\delta_1 \geq 0$, this implies

$$\begin{aligned}\hat{R}_{1t} &\geq \frac{1}{2}\log_2\left(1+\frac{\alpha_{11}^t P_{1t}^*}{\mathcal{N}_1^t+\alpha_{21}^t P_{2t}^*}\right)-\frac{1}{2}\log_2(1+k\delta_1)\\ &= R_{1t}^*-\frac{1}{2}\log_2(1+k\delta_1) \quad (17)\end{aligned}$$

Hence the total error in R_1 over all the time slots when $t \in T_{22}$ is at most $(M/2)\log_2(1+k\delta_1) \leq \epsilon\tilde{R}_1/2$ using the upper bound on k as specified in Eq. 11. Combining the two cases, the total absolute error in $R_1 = \tilde{R}_1 - \hat{R}_1 \leq \epsilon\tilde{R}_1$ and thus the relative error in R_1 is bounded by ϵ i.e $\hat{R}_1 \geq \tilde{R}_1(1-\epsilon)$. The analysis is identical for rate R_2. Since algorithm $\mathcal{A}^{\hat{P}}$ uses $\tilde{R}_i(1-\epsilon)$ as the rate constraint for user i, therefore the choice of power levels described above is a feasible choice and hence the algorithm is a $(1+\epsilon, 1+\epsilon)$ approximation. ■

For the algorithm above, note that the number of discrete power levels per slot L_t, is a function of the channel quality parameters $\alpha_{ji}^t/\alpha_{ii}^t$. While the α's are exponentially distributed random variables with typically small means [22], the ratios can still be quite large, thereby increasing the number of levels. Therefore we consider a more optimal scheme where the rate and energy approximations are obtained independent of channel quality parameters.

Let $\tilde{R}_m = \min(\tilde{R}_1, \tilde{R}_2)$ and $k_1 = (M\log_2(1+P) - 2\tilde{R}_m)/\log_2\left(\frac{1+P}{1+1/k}\right)$. Define $\delta_1 > 0$ and $k > 0$ as the solutions to

$$\begin{aligned}&\min \frac{1}{k\delta_1} + \ln_{1+k\delta_1} kP\\ &\text{s.t } k_1\delta_1 + M\log_2(1+k\delta_1) = 2\epsilon\tilde{R}_m\\ &k > \frac{1}{2^{2\tilde{R}_m/M}-1} \quad (18)\end{aligned}$$

δ_1 and k can be obtained using standard constrained minimization techniques such as Lagrange multipliers [18]. However if no solution exists above, then δ_1 and k are the solutions obtained by replacing the constraints in Eq. 18 above by the constraint

$$\delta_1 + \log_2(1+k\delta_1) = \frac{2\epsilon\tilde{R}_m}{M} \quad (19)$$

If no solution still exists, then $\delta_1 = \epsilon\tilde{R}_m/M$ and $k = (2^{\epsilon\tilde{R}_m/M}-1)/\delta_1$. Now divide the available power per time slot into discrete power levels as specified by Eq. 12 using the δ_1 and k values above.

Theorem 2 *For $\epsilon > 0$, let $\mathcal{A}^{\overline{P}}$ denote the (exponential) dynamic programming algorithm for finding a minimal energy schedule using the discrete power levels defined above, subject to overall duty-cycle and rate constraints $\tilde{R}_i(1-\epsilon)$. Then $\mathcal{A}^{\overline{P}}$ is a $(1, 1+\epsilon)$-approximation of $\mathcal{A}^{P^*}$.*

Proof: For each slot t, round down the optimal power level choice P_{it}^* to the nearest discrete power level, represented by $\overline{P}_{it}$ and let $\overline{R}_{it}$ denote the corresponding achieved rate per slot. As before, divide the M time slots into sets $T_{ij},\ i,j = 1,2$, based on the value of P_{it}^*. We show below that $\overline{P}_{it}$ represents a feasible allocation of power levels under the rate constraints $\tilde{R}_i/(1-\epsilon)$. Hence $\mathcal{A}^{\overline{P}}$ is a $(1, 1+\epsilon)$-approximation since the total energy consumption of $\mathcal{A}^{\overline{P}}$ is at most $\sum\sum\overline{P}_{it} \leq \sum\sum P_{it}^*$.

First, for $t \in T_{12}$, using $\overline{P}_{1t} \geq P_{1t}^*/(1+k\delta_1)$ and $\overline{P}_{2t} \leq P_{2t}^*$, we get

$$\begin{aligned}\overline{R}_{1t} &\geq \frac{1}{2}\log_2\left(1+\frac{\alpha_{11}^t P_{1t}^*}{(1+k\delta_1)(\mathcal{N}_1^t+\alpha_{21}^t\overline{P}_{2t})}\right)\\ &\geq R_{1t}^*-\frac{1}{2}\log_2(1+k\delta_1) \quad (20)\end{aligned}$$

Thus the absolute error in R_{1t} per time slot for this case is $\leq \frac{1}{2}\log_2(1+k\delta_1)$.

Next, for $t \in T_{11}$, define the total interference, $\overline{I}_{1t} = (\mathcal{N}_1^t + \alpha_{21}^t \overline{P}_{2t})/\alpha_{11}^t$, and likewise I_{1t}^*, where $I_{1t}^* \geq \overline{I}_{1t} \geq 1$ (minimum total interference ≥ 1). Therefore we have,

$$R_{1t}^* - \overline{R}_{1t} \quad \leq \quad \frac{1}{2}\log_2\left(1+\frac{P_{1t}^*}{\overline{I}_{1t}}\right) - \frac{1}{2}\log_2\left(1+\frac{\overline{P}_{1t}}{\overline{I}_{1t}}\right)$$

Using the fact that $\ln x - \ln y < x - y$ for $x > y > 1$, we get $R_{1t}^* - \overline{R}_{1t} < (P_{1t}^* - \overline{P}_{1t})/2 \leq \delta_1/2$. Thus the absolute error in R_{1t} per time slot for this case is $\leq \delta_1/2$.

Combining the two cases, we can bound the overall rate error over M time slots as

$$T_{err} = \frac{|T_{11}|\delta_1}{2} + \frac{|T_{12}|\log_2(1+k\delta_1)}{2} \tag{21}$$

For $\mathcal{A}^{\overline{P}}$ to be a $(1, 1+\epsilon)$ algorithm, we must have $T_{err} \leq \epsilon\tilde{R}_1$. To finish the proof, note that the maximum R_1 rate we can obtain under this algorithm in any $t \in T_{12}$ is $\frac{1}{2}\log_2(1+P)$ and $\frac{1}{2}\log_2(1+r_0\delta_1) = \frac{1}{2}\log_2(1+1/k)$ in any $t \in T_{11}$. The maximum value of $|T_{12}|$ is M. (Clearly $\log_2(1+P)$ should be $\geq 2\tilde{R}_1(1-\epsilon)/M$, otherwise $\mathcal{A}^{\overline{P}}$ does not have a solution). However the maximum value of $|T_{11}|$ is $|T_{11}| \leq (M\log_2(1+P) - 2\tilde{R}_1)/\log_2\left(\frac{1+P}{1+1/k}\right)$ if $\log_2(1+1/k) < 2\tilde{R}_1/M$ else $|T_{11}| \leq M$. When $|T_{11}|$ takes the first value, the total number of power levels per slot is minimized by choosing δ_1 and k as in Eq. 18, whereas in the second case it is minimized by Eq. 19. If both cases do not yield a solution then we set the two error components $\delta_1 = \log_2(1+k\delta_1) = \epsilon\tilde{R}_m/M$ which makes the relative error over M slots $\leq \epsilon$ as desired. ■

Finally, we note that the worst-case values of k and $k\delta_1$ are $O(\epsilon\tilde{R}_m/M)$ and therefore

Theorem 3 *Given rate constraints $\tilde{R}_i$ and max power P, the number of discrete power levels per slot is $O(\frac{1}{\epsilon})$.*

Note that the time complexity of $\mathcal{A}^{\overline{P}}$ is still exponential. However, using the fact that the number of power levels per slot required to closely approximate rate and energy constraints is $O(\frac{1}{\epsilon})$, we will develop an FPAS in Section 4 .

4 An FPAS for Rate Constraints

We now describe a simple Fully Polynomial Approximation Scheme that solves the minimum energy scheduling problem by using a β-relaxation on the rate constraints for some arbitrary constant $\beta > 0$. For clarity, we describe the FPAS using two power levels 0 and P per time slot. The algorithm for the multiple power level case is a simple extension as described later.

The FPAS solves the same restricted problem of Eq. 4 with only each rate constraint replaced by

$$\sum_{t=1}^{M} R_{it} \geq (1-\beta)\tilde{R}_i \tag{22}$$

For any $\delta > 0$, define the following

Definition 1 *A rate vector (R_1, R_2) δ-dominates another vector (R_3, R_4) iff either $R_3(1-\delta) \leq R_1 \leq R_3$ and $R_2 \geq R_4$ or $R_3 \leq R_1(1-\delta)$ and $R_4(1-\delta) \leq R_2$. For $R_1 \geq \tilde{R}_1$, the δ-dominant vector is the one with* $\max R_2$ *among all such vectors.*

Note that dominance (under standard vector comparison) implies δ-dominance but not vice-versa.

Definition 2 *Let $\bar{R}$ be a set of rate vectors. Define the operation vectormaxdelta($\bar{R}$) as one that eliminates all δ-dominated and dominated vectors from $\bar{R}$.*

Operation vectormaxdelta is equivalent to dividing the two-dimensional vector space into horizontal and vertical strips, each of whose left endpoint is $(1-\delta)$ times its right endpoint and choosing at most one vector per strip. A simple algorithm for implementing vectormaxdelta($\bar{R}$) is as follows. Assume $\bar{R}$ has been sorted by R_1 values. First obtain the δ-dominant vector for $R_1 \geq \tilde{R}_1$ if such R_1's exist. Then find the δ-dominant vectors successively in the strips defined by R_1 intervals $(\tilde{R}_1(1-\delta), \tilde{R}_1]$, $(\tilde{R}_1(1-\delta)^2, \tilde{R}_1(1-\delta)]$ $(\tilde{R}_1(1-\delta)^3, \tilde{R}_1(1-\delta)^2]$ and so on. Dominated vectors are eliminated simultaneously. Since $\bar{R}$ has been sorted by R_1, this can be done in one pass through $\bar{R}$, in decreasing order of R_1 values.

Choose $\delta = \frac{\beta}{2M}$. Let $\mathcal{A}_\beta^P$ denote the following dynamic programming algorithm for the fixed power minimum energy scheduling problem. The boundary conditions (i.e rate vectors for each slot t) are the same as before in Eq. 5. The main recursive step in the algorithm is derived by replacing the vectormax operation with vectormaxdelta. Let $\hat{R}_{i,j}^{kP,a,b}$ represent the set of δ-dominating rate pairs corresponding to cumulative transmission rates for user 1 and user 2 from time slots i through j, $1 \leq i \leq j \leq M$, while using a total power of kP, $1 \leq k \leq 2M$.

$$\hat{R}_{i,j}^{kP,a,b} = \text{vectormaxdelta}\Big\{ \hat{R}_{i,j-1}^{kP,a,b} \bigcup \left(\hat{R}_{i,j-1}^{(k-1)P,a-1,b} + \hat{R}_{j}^{P,1,0}\right) \bigcup \left(\hat{R}_{i,j-1}^{(k-1)P,a,b-1} + \hat{R}_{j}^{P,0,1}\right) \bigcup \left(\hat{R}_{i,j-1}^{(k-2)P,a-1,b-1} + \hat{R}_{j}^{2P,1,1}\right) \Big\} \tag{23}$$

The terminating condition for the algorithm occurs when the rate vectors are $\geq \tilde{R}_i(1-\beta)$, $i = 1, 2$. The optimal schedule corresponds to the minimum total power rate vector that satisfies the terminating condition.

Theorem 4 *$\mathcal{A}_{\beta}^{P}$ is a FPAS for the minimum energy scheduling problem with two fixed transmit power choices 0 or P per slot.*

Proof: First we show that the running time of $\mathcal{A}_{\beta}^{P}$ is polynomial in M and $1/\beta$. The number of δ-dominant vectors in $\hat{R}_{i,j-1}^{kP,a,b}$ is bounded by

$$1 + \ln_{1+\delta} \tilde{R}_1 = 1 + \frac{\ln \tilde{R}_1}{\ln(1+\delta)} = O\left(\frac{M}{\beta} \cdot \ln \tilde{R}_1\right)$$

since we keep only one vector for each $1-\delta$-factor interval. and using $1/(1-\delta) = 1+\delta$. The running time for the creation of each $\hat{R}_{i,j}^{kP,a,b}$ is also polynomial since it includes sorting followed by the vectormaxdelta operation. There are $O(MP\mu_1\mu_2)$ such rate vector sets, each of size polynomial in $1/\beta$ and hence the overall running time is also polynomial in $1/\beta$.

Next we need to show that algorithm $\mathcal{A}_{\beta}^{P}$ provides a β-approximation of the rate constraints. Let $(R_1, R_2) \in \bar{R}_{1,j}^{kP,a,b}$ be an arbitrary non-dominated vector from the exponential time algorithm $\mathcal{A}^P$ up to time slot j. We can show by induction that $\exists (R_3, R_4) \in \hat{R}_{1,j}^{kP,a,b}$ such that $R_3 \geq R_1(1-\delta)^j$ and $R_4 \geq R_2(1-\delta)^j$. The 'parent' of (R_1, R_2) (the vector that produced (R_1, R_2) in stage $j-1$) is approximated within $(1-\delta)^{j-1}$ by the induction hypothesis. After combining with the vectors of stage j and implementing vectormaxdelta, at most a further $(1-\delta)$-factor error in R_1 and R_2 is introduced. Thus the total error in each dimension is bounded by $(1-\delta)^j$ after j slots. Therefore every rate vector in $\bar{R}_{1,M}^{kP,\mu_1,\mu_2}$ is approximated to within $(1-\delta)^M$ by a rate vector from algorithm $\mathcal{A}_{\beta}^{P}$. Using $\delta = \beta/2M$, we can see that there exist 'approximate' rate vectors $(R_3, R_4) \in \hat{R}_{1,M}^{kP,\mu_1,\mu_2}$ such that $R_3 \geq R_1(1-\beta)$ and $R_4 \geq R_2(1-\beta)$ for all 'actual' rate vectors $(R_1, R_2) \in \bar{R}_{1,M}^{kP,\mu_1,\mu_2}$. Hence $\mathcal{A}_{\beta}^{P}$ is a β-approximation. ∎

Algorithm $\mathcal{A}_{\beta}^{P}$ above can be easily modified to incorporate multiple power levels per slot. For any small $\alpha > 0$, choose $\epsilon = \beta = \alpha/2$ and then set δ_1 and k as per Eq. 18 with L_t power levels per user per slot. Eq. 5 is modified to reflect $(L_t)^2 = O(1/\alpha^2)$ (from Theorem 3) total rate vectors per time slot t, corresponding to all combinations of power levels. Define a new algorithm $\mathcal{A}_{\beta}^{P_{L_t}}$ in which the vectormaxdelta operation applies to combinations of these $(L_t)^2$ rate vectors. The total number of table entries (for rate vectors) in the modified dynamic program is now increased to $(L_t)^2 M\mu_1\mu_2$. However by applying the vectormaxdelta operation, the size of each rate vector set remains the same size, $O(1/\beta)$, as before.

Theorem 5 *For any $\alpha > 0$ and $\epsilon = \beta = \alpha/2$, $\mathcal{A}_{\beta}^{P_{L_t}}$ is a $(1, 1+\alpha)$-Fully Polynomial Approximation Scheme for the minimum energy scheduling problem with L_t power levels per slot.*

Proof: By choosing multiple power levels as defined above, each rate vector is no more than a $1-\epsilon = (1-\alpha/2)$-factor away from the ideal rate vector for that stage. For each such vector, the vectormax operation selects another which is at most another $1-\alpha/2$-factor away. Thus at the end of algorithm $\mathcal{A}_{\beta}^{P_{L_t}}$, the rate constraints are violated by at most a factor of $(1-\alpha/2)^2 < (1-\alpha)$. For given M, P, μ_1 and μ_2, the total number of table entries and related operations is $O(1/\alpha^2)$ and hence $\mathcal{A}_{\beta}^{P_{L_t}}$ is a $(1, 1+\alpha)$ FPAS. ∎

5 Multiple Node Case

We now consider the MESP problem with multiple $(N > 2)$ users transmitting over M time slots. Note that MESP is NP-hard even for the restricted case of users transmitting using only two power levels (0 and P). In this case, the basic dynamic programming algorithm of Section 2 is exponential both in the number of slots M and users N with 2^N feasible rate vectors per time slot and the size of each table entry (the rate vector set corresponding to feasible total power and duty cycle solutions) also growing exponentially with M. If the numbers of users is a fixed constant, N, then we can develop an FPAS for the general case where users can select from multiple power levels by extending the results of the previous section.

Proposition 1 *For a fixed number of users N transmitting over an arbitrary number of slots M using multiple power levels, there is a $(1+\alpha, 1+\alpha)$-FPAS for finding the optimal minimum energy schedule.*

We first note that the optimal number of power levels required to approximate each nodes rate and overall energy within a $(1+\epsilon)$-factor can be obtained by extending Theorem 2 to the general N-node case, since the bounding arguments apply even with interference from multiple nodes. Hence each node can select from $L_t = O(1/\epsilon)$ power levels per slot, where the levels are defined by Eq. 12 and Eq. 18 with $\tilde{R}_m$ in Eq. 18 changed to $\tilde{R}_m = \min\{\tilde{R}_i\}$, $i = 1, 2, \ldots N$. The number of feasible rate vectors per slot t is now $O((1/\epsilon)^N)$, selected from the N-dimensional hyperplane bounded by $\tilde{R}_1 \times \ldots \times \tilde{R}_N$. At each slot, we construct a polynomial number of table entries corresponding to total power and duty-cycle combinations of nodes. Each updated table entry consists of a set of feasible rate vectors up to the current slot that satisfy the total power and duty-cycle requirements. To keep the size of each of table entry polynomial in M and $1/\epsilon$, we extend the definition of δ-dominance to rate vectors over the N-dimensional hyperplane and use it to eliminate dominated rate vectors. The number of δ-dominant rate vectors per table entry is $O\left(\left(\frac{M \ln \tilde{R}_m}{\alpha}\right)^{N-1}\right)$, where $\tilde{R}_m = \min_i\{\tilde{R}_i\}$, since the number of δ-dominant vectors in the smallest dimension is $O((M \ln \tilde{R}_m)/\alpha)$ and we are considering dominant vectors over the N-dimensional hyperplane. The arguments of algorithm $\mathcal{A}_\beta^{P_{L_t}}$ can now be applied to show that the rate vectors output by the algorithm are within a $(1, 1+\alpha)$-factor of the optimal power and rate constraints. The algorithm is an FPAS since it is polynomial in M and $1/\alpha$.

6 2-Approximate Minimum Energy Schedule for Fixed Power Transmitters

Consider an interference channel based wireless network with N (low-cost) transmitters, where nodes are restricted to transmitting at a fixed power over their active time slots within the M slot duty-cycle. At the start of the duty-cycle, nodes must decide the optimal fixed transmission power value P_{opt} that results in a minimum energy schedule. Since we do not have a closed form analytical solution for this schedule as a function of P, we need an algorithmic solution for P_{opt}. The basic dynamic programming solution of Section 2 addresses only the restricted version of this problem, where the fixed transmit power value P is given as a prior.

For a given value of P, let $\mathcal{A}^P$ denote the FPAS (based on the previous section using only two power levels) for finding the minimum energy schedule. It is possible that a feasible schedule does not exist under $\mathcal{A}^P$, i.e. $\forall k, (\tilde{R}_1, \tilde{R}_2, \ldots, \tilde{R}_N) \not\leq \bar{R}_{1,M}^{kP,\mu_1,\mu_2,\ldots,\mu_N}$ and thus $\bar{R}_{opt}^P = \phi$. Thus the problem is to find the optimal fixed transmission power P_{opt} for which both a feasible schedule exists and the total energy cost $E_\mathcal{A}^{P_{opt}} = \sum_{i=1}^N \sum_{t=1}^M P_{it}$ is minimized, subject to $P_{it} \in \{0, P_{opt}\}$ in addition to the duty-cycle and rate constraints.

Unfortunately, P_{opt} cannot be found via simple binary search since the total energy of a schedule is not a convex function of P. $E_\mathcal{A}^P$ can have multiple local minima; increasing transmit power may increase or decrease the total energy depending on the specific channel interference coefficients [1]. Thus to find P_{opt} and the global minimum energy schedule, we first restrict the space of feasible transmit powers by finding upper and lower bounds P_{min} and P_{max}, such that 1) $P_{opt} \in [P_{min}, P_{max}]$; 2) $\mathcal{A}^P$ is infeasible for $P < \lfloor P_{min} \rfloor$; and 3) $\forall P > P_{max}, E_\mathcal{A}^{P_{opt}} \leq E_\mathcal{A}^P$. In this section, we describe a 2-approximation for finding $E_\mathcal{A}^{P_{opt}}$.

Consider two instances of the scheduling problem: One where nodes transmit at power P_1 during active slots and the other, where they transmit at P_2, with $P_1 < P_2$. It is straightforward to note that all nodes can achieve a higher total rate over the M slots under P_2, since during each slot, for the same combination of active nodes, the individual rates achieved by the nodes is higher under P_2 than P_1.

To find P_{opt}, we first find P_{min}, the minimum (fixed) transmit power level per active slot for which a feasible schedule exists. P_{min} can be found via binary search as follows: Initialize $P = \min\{P'_1/M, P'_2/M, \ldots, P'_N/M\}$, where P'_i is obtained by extending Eq 10 to N nodes. We will assume $P_{min} \geq 1$ for notational convenience below. While $\bar{R}_{opt}^P = \phi$, set $P = 2P$ and run algorithm $\mathcal{A}^P$. The values of all rate vectors increase with P and hence the process will terminate with $\bar{R}_{opt}^P \neq \phi$. Let P_m be the terminating value of P which is found in $\lceil \log_2 P_{min} \rceil$ calls. $\lceil P_{min} \rceil$ can then be obtained through binary search in the interval $[P_m/2, P_m]$ with $O(\log_2(P_m/2))$ further calls to $\mathcal{A}^P$. Thus we have,

Proposition 2 *$\lceil P_{min} \rceil$ can be found in $O(\lceil \log_2 P_{min} \rceil)$ calls to the FPAS $\mathcal{A}^P$.*

The following proposition defines an upper bound for P_{max}:

[1] Note that for the multiple power levels per slot case as in section 3 (with $l_t > 2$ levels per slot), a schedule with maximum power P encompasses smaller values as well and thus the optimum value of P can be found through a simple binary search. However this is not true when only two power levels 0 and P are available.

Proposition 3 *$P_{max} = \left(\frac{\sum_1^N \mu_i}{N}\right) P_{min}$ and P_{opt} can be found by searching in an interval of size $O((M-1)P_{min})$.*

Proof: Let P_{max} be as defined above. For any $P > P_{max}$ we have, $E_{\mathcal{A}}^P > \left(\frac{\sum_1^N \mu_i}{N}\right) P_{min} t_P$, where t_P is the total number of active slots under algorithm $\mathcal{A}^P$. Since each node is active for at least one slot in a valid schedule, we have $E_{\mathcal{A}}^P > P_{min} \sum_1^N \mu_i, \forall P > P_{max}$. Also, by definition we have $E_{\mathcal{A}}^{P_{opt}} \le E_{\mathcal{A}}^{P_{min}} \le P_{min} \sum_1^N \mu_i$. Combining the two, we get $E_{\mathcal{A}}^{P_{opt}} < E_{\mathcal{A}}^P$ for all $P > P_{max}$ as desired. Finally, since $\mu_i \le M$ and $P_{opt} \in [P_{min}, P_{max}]$, it can be found by searching in an interval of size $O((M-1)P_{min})$. ■

Note that the above bound on P_{max} is independent of the number of users N. For the special case of $N = 2$, we can obtain a smaller bound on P_{max} (and hence the search space for P_{opt}) by using the following observation based on the definition of the rate function:

Observation 1 *Let S denote any set of slots in which a node is transmitting solo with power P achieving a total rate of $R_S^P = \sum_{t \in S} (1/2) \log_2(1 + \alpha_{ii}^t P/\mathcal{N}_i^t)$. Increasing the transmit power over these slots to $2^n P$, $n = 1, 2, \ldots$, increases the achieved rate by less than $n|S|/2$, i.e $R_S^{2^n P} < R_S^P + n|S|/2$.*

Let S_1^P, S_2^P and T^P be the set of time slots occupied by node 1 only, node 2 only and both nodes, under the schedule created by $\mathcal{A}^P$. Let $R_{i,S}^P$ denote the total rate obtained by node i, $i = 1, 2$, over any set of slots S in this schedule. Let $S_{i,s}^P \subset S_i^P$ represent the set of $\lfloor |S_i^P|/2 \rfloor$ time slots with the smallest achievable *solo* rates (i.e $(1/2)\log_2(1 + \alpha_{ii}^t P/\mathcal{N}_i^t)$ among the slots in S_i^P and let $S_{i,l}^P$ denote the remaining $\lceil |S_i^P|/2 \rceil$ slots. Let $K_1 \subset T^P$ (K_2, resp.) be the smallest subset of slots such that the total rate obtained by node 1 (node 2, resp.) over these slots when transmitting solo at power $2P$ is $\ge R_{1,T^P}^P$ ($\ge R_{2,T^P}^P$, resp.). These slots can be determined by selecting the best slots for node 1/node 2 in T^P after sorting by decreasing solo rates using power $2P$. Also let K_i^s represent the worst set of $\lfloor |K_i|/2 \rfloor$ slots for node i in K_i, with $R_{i,K_i^s}^{2P}$ the corresponding total solo rates over these slots.

The following lemma provides a sufficient condition for finding P_{max} under a moderate interference regime, when the average solo achievable rate over the worst slots is $\ge 1/4$.

Proposition 4 *$P_{max} \in [P, 2P]$ if $\forall i$*

$$\begin{array}{llll} \textit{1.} & R_{i,S_{i,s}^P}^P & \ge & (\lceil |S_i^P|/2 \rceil)/2 \\ \textit{2.} & |K_1| + |K_2| & \ge & |T^P| \\ \textit{3.} & R_{i,K_i^s}^{2P} & \ge & (\lceil |K_i|/2 \rceil)/2 \end{array}$$

Proof: First we look at the rate impact of increasing the power over the best solo slots. We have, $R_{i,S_i^P}^P = R_{i,S_{i,l}^P}^P + R_{i,S_{i,s}^P}^P$, $i = 1, 2$. Using the first condition of the proposition, we get,

$$R_{i,S_{i,l}^P}^P \le R_{i,S_i^P}^P - (\lceil |S_i^P|/2 \rceil)/2 \tag{24}$$

Now consider the best set of $\lceil |S_i^P|/2^n \rceil$ solo slots for node i, $n = 1, 2 \ldots$. Suppose we transmit over these $\lceil |S_i^P|/2^n \rceil$ slots with power $2^n P$ and zero power over the rest of the slots from S_i^P. The new energy cost over S_i^P is at least as much as the energy cost using power P.

The new rate achieved over S_i^P is:

$$R_{i,S_i^P}^{2^n P} \quad < \quad R_{i,S_{i,l}^P}^P + \tfrac{n}{2} \lceil |S_i^P|/2^n \rceil \quad \le R_{i,S_i^P}^P$$

by using Observation 1 and then Eq. 24.

Thus increasing the power over S_i^P will reduce the rate while keeping the energy cost at least the same as before. Hence $P_{max} < 2P$ over the set of solo slots.

Next we consider the set of jointly active slots T^P. Since K_i represents the best slots for node i in T^P, condition 2, i.e. $K_1 + K_2 \ge T^P$, implies that the minimum total energy required to simultaneously obtain a rate of R_{i,T^P}^P, $i = 1, 2$, over any subset of T^P while using power $2P$ is $\ge 2PT^P$ (which is the energy consumption when both nodes are using power P). Now applying Observation 1 to condition 3 in a similar manner as for the solo slots, we find that increasing power after $2P$ will not lead to a more efficient energy solution, and thus $P_{max} < 2P$. ■

Finally, we use the above bounds on P_{max} to obtain a 2-approximation for $E_{\mathcal{A}}^{P_{opt}}$: the energy of the optimal (minimum energy) schedule for N nodes transmitting at fixed power P_{opt} over M slots as follows.

Theorem 6 *Let*

$$P^* = \underset{P = 2^t P_{min}, \quad t = 0, 1 \ldots, \lceil \log_2 \frac{P_{max}}{P_{min}} \rceil}{\operatorname{argmin}} E_{\mathcal{A}}^P.$$

Then $E_{\mathcal{A}}^{P^*}$ is a 2-approximation to $E_{\mathcal{A}}^{P_{opt}}$, the minimum energy schedule generated by the optimal transmit power P_{opt}. The algorithm for finding $E_{\mathcal{A}}^{P^*}$ uses $\lceil \log_2 \frac{P_{max}}{P_{min}} \rceil = O(\log_2 M)$ calls to $\mathcal{A}^P$.

Proof: We run the $\mathcal{A}^P$ algorithm starting with $P = P_{min}$ and doubling P with each iteration until we reach a P_{max} as defined by propositions 3 or 4. We claim that the energy of any solution using power $P_a : P \leq P_a \leq 2P$, satisfies $E_{\mathcal{A}}^{P_a} \geq (1/2)\min(E_{\mathcal{A}}^P, E_{\mathcal{A}}^{2P})$. Let t_P denote the total number of active slots for N users under power P. If $E_{\mathcal{A}}^P \leq E_{\mathcal{A}}^{2P}$, then we must have $t_P \geq t_{P_a} \geq t_{2P} \geq t_P/2$, using the fact that the number of active slots in a solution cannot increase as we increase the power. Thus $E_{\mathcal{A}}^{P_a} = P_a t_{P_a} \geq P t_P/2 = (1/2)E_{\mathcal{A}}^P$. Conversely if $E_{\mathcal{A}}^{2P} \leq E_{\mathcal{A}}^P$, then $P_a \geq P$ and $t_{P_a} \geq t_{2P}$ together imply that $E_{\mathcal{A}}^{P_a} \geq P t_{2P} = (1/2)E_{\mathcal{A}}^{2P}$.

When the algorithm above is implemented, the total energy can oscillate between $E_{\mathcal{A}}^{P_{min}}$ and $E_{\mathcal{A}}^{P_{max}}$ as we sequentially double the power. Let P^* be the power yielding the minimum energy among the iterations and choose $E_{\mathcal{A}}^{P^*}$ as the output of our algorithm. By the previous arguments, $E_{\mathcal{A}}^{P^*} \leq 2E_{\mathcal{A}}^{P_{opt}}$ and therefore this algorithm is a 2-approximation. Since $P_{max} = O(MP_{min})$, the number of iterations is $O(\log_2 M)$. ■

7 Conclusions

We have considered the problem of finding a minimum energy transmission schedule for duty-cycle and rate constrained wireless networks. Since traditional optimization methods using Lagrange multipliers are computationally expensive given the non-convex constraints, we develop fully polynomial time approximation schemes by considering restricted versions of the problem using discrete power levels. We derive a $(1, 1+\epsilon)$-FPAS for MESP that approximates the optimal energy consumption and rate constraints to within an $1+\epsilon$-factor.

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