Jamming and Counter-Measure Strategies in Parallel Gaussian Fading Channels with Channel State Information

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***Abstract*— Consider a parallel Gaussian channel with M subchannels each of which experiences independent flat fading. There exists a smart jammer which is capable of intercepting the feedback channel message such that channel state information (CSI) measured and sent back by a receiver can be perfectly known to the jammer, as well. We formulate a novel two-layer two-person zero-sum game. Under this zero-sum game framework, the following three fundamental questions are addressed. The first one is about whether to hop or to spread power over M frequency bands given the full CSI at both transmitter and jammer sides. We prove that spreading versus spreading is the Nash equilibrium point. The second question is about the impact of sending back CSI on the overall throughput, to feedback or not to feedback, considering the presence of a smart jammer capable to exploit CSI for its malicious purpose. We show that possessing the full CSI enables a better means to defend against the jammer's attacks. The last question is about whether the amount of feedback information can be reduced given the mutual restrictions between transmitter and jammer. We prove that the receiver should also get involved by adaptively sending CSI without compromising the sum rate at the equilibrium point.**

I. Introduction

Security issues in wireless systems and networks have attracted significant interests lately because of the tendency of nearly ubiquitous deployment of wireless devices and networks in our society, as well as the susceptibility of wireless systems to malicious attacks and eavesdropping as a result of the broadcasting nature of wireless transmissions. One of the prominent security problems is denial of service which includes one of the oldest strategies in electronic warfare systems, namely, jamming of legitimate transmissions in physical layer [1].

Spread spectrum using frequency hopping has been historically adopted as a viable solution to defend against jamming attacks [2]. The rationale underlying frequency hopping is to let a legitimate transmitter randomly pick a frequency band to communicate with its receiver such that the probability of getting jammed by a power-limited partial-band jammer will be reduced. Usually the transmitter will pick one band out of M bands with a uniform distribution determined by a pseudo-random sequence which is known at receiver and not known to the jammer.

The recent advances in communication and information theory have revealed the importance of exploiting channel state information in improving throughput, reliability and efficiency of wireless communications systems [3]. Channel state information (CSI) about fading realizations are measured and sent back by the legitimate receiver to its transmitter. This immediately poses a potential problem at the presence of a smart jammer which is able to eavesdrop the feedback channel and obtain the exact CSI, the same information as what transmitter possesses. Consequently, the jammer can further exploit its knowledge about CSI to produce more damages to the legitimate communication link. A natural question to ask immediately is what will transmitter respond given its knowledge about CSI when it has several frequency bands to consider? Will the transmitter still hop as the conventional wisdom implies? In [2], it was presumed the best response for the transmitter is to hop over M bands with non-uniform distribution which is a function of channel states, while the jammer spreads its power non-uniformly over all frequency bands based on CSI. We are going to show this is not an optimal strategy.

This paper is motivated by the questions raised above and intended to address these questions by formulating problems within the framework of a novel two-layer two-person zero-sum game. More specifically, we address the following three questions using the proposed two-person zero-sum game framework: (1) To hop or to spread powers for transmitter and jammer, given the complete CSI? (2) Will the feedback of CSI really help considering the malicious use of CSI by the jammer? (3) If feedback helps, should we always send back CSI given the mutual

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restraints imposed by each player on its opponent? Our contributions can be summarized as follows:

- A novel two-layer two-person zero-sum game is formulated to provide us a means to analyze whether to hop or to spread for jammer and transmitter, respectively.
- We prove that both spreading is the Nash equilibrium point as compared with the other combinations.
- We prove that the possessing of complete CSI for transmitter makes it more powerful in fighting against the smart jammer.
- We provide an adaptive feedback protocol for the receiver. We show that it is not always necessary for the receiver to send back the exact CSI. The inter-restraints between jammer and transmitter makes them reach the same Nash equilibrium point even under the partial knowledge about CSI.

The paper is organized as follows. In Section II, we present the system model and formulate a two-person two-layer zero-sum game. In Section III, we prove both spreading is the Nash equilibrium point for the game formulated in Section II. In Section IV, we prove that feedback always helps transmitter to mitigate the impact of jamming on the throughput. In Section V, we prove that without compromising the total sum rate, the receiver does not need to send back the exact CSI all the time under an adaptive CSI feedback protocol. Finally, conclusions are reached in Section VI

II. System Model and Problem Formulations

Consider a system of three nodes: source (S), destination (D) and jammer (J), where S sends data to D under the jammer's attacks. A parallel flat fading channel model is assumed in this paper, where each fading block has M parallel subchannels defined as follows:

$$Y_i[n] = H_i[n]X_i[n] + W_i[n] + Z_i[n], \tag{1}$$

for $i = 1, \cdots, M$, and $n = 1, 2, \cdots, LN$, where channel fading coefficients $H_i[n]$ remain constant over $(l-1)N+1 \leq n \leq lN$, $1 \leq l \leq L$, and varies independently over different blocks (of N channel uses) and sub-channels, satisfying the distribution: $H_i[n] \sim \mathcal{CN}(0, \sigma_i^2)$, where $\mathcal{CN}(\mu, \sigma^2)$ denotes the complex circular Gaussian distribution with mean μ and variance σ^2. The M subchannels model the M parallel frequency bands. Each codeword spans over $M \times N \times L$ channel uses. Channel additive white Gaussian noise sequences $W_i[n]$ are assumed i.i.d across MNL channel uses with distribution $\mathcal{CN}(0, N_0)$. Jamming signals $Z_i[n]$ are independent across all channel uses. In this paper, we investigate the jamming and counter-jamming problems under short-term and long-term average power constraints [4].

Short-term average power constraint:

$$\frac{1}{N}\sum_{i=1}^{M}\sum_{n=(l-1)N+1}^{lN+1}|X_i[n]|^2 \leq P_S,$$
$$\frac{1}{N}\sum_{i=1}^{M}\sum_{n=(l-1)N+1}^{lN+1}|Z_i[n]|^2 \leq P_J, \quad 1 \leq l \leq L, \tag{2}$$

Long-term average power constraint:

$$\frac{1}{LN}\sum_{i=1}^{M}\sum_{n=(l-1)N+1}^{lN+1}\sum_{l=1}^{L}|X_i[n]|^2 \leq P_S,$$
$$\frac{1}{LN}\sum_{i=1}^{M}\sum_{n=(l-1)N+1}^{lN+1}\sum_{l=1}^{L}|Z_i[n]|^2 \leq P_J \tag{3}$$

In the sequel, without delay constraint, we let the number of independent blocks L grow to infinity so that we only need to be concerned with the average sum rate of transmission from S to D under the jammer's attacks. We are interested in the following two problems. Firstly, assume channel state information (CSI) about $H_i[n]$ can be measured and sent back perfectly via an error free feedback channel from D to S, and moreover, this CSI is assumed to be eavesdropped by the jammer to be exploited for smart jamming attacks, what jamming and counter-measure strategies should be taken for J and S, respectively? Secondly, given the presence of a smart jammer which exploits CSI for its "evil" purpose, is it still always effective and efficient to send back full CSI?

In this paper, we adopt the average transmission rate as a metric and employ game theory to answer the two questions raised above. In particular, we model the jamming and counter-measures as a novel two-person zero-sum game. The novelty of this game is reflected by the two-layers of strategy spaces involved. The first layer of strategy spaces is defined as $C_i = \{\text{Hp}, \text{Sp}\}$, where $i \in \{S, J\}$, and Hp and Sp represent hopping over M bands and spreading over an entire frequency band, respectively. After each player decides on hopping or spreading, it needs to further decide what is the associated hopping pattern or power allocation function given CSI, which thus forms the second layer of game for a given selected option in $\{(S_{hp}, J_{hp}), (S_{hp}, J_{sp}), (S_{sp}, J_{sp}), (S_{sp}, J_{hp})\}$. Each player (jammer or source) chooses hopping or spreading, as well as the associated hopping patterns or power functions, accordingly. The two player over the first layer can be characterized by the matrix in Table I, where $u_{i,j}$ denotes the utility function over the first layer, which

	J_{hp}	J_{sp}
S_{hp}	$u_{1,1}$	$u_{1,2}$
S_{sp}	$u_{2,1}$	$u_{2,2}$

TABLE I

TWO-PERSON ZERO SUM GAME OVER HOPPING OR SPREADING

is the rate determined below, for each joint strategy $(c_{S,i}, c_{J,j})$ with $c_{S,i} \in C_S$ and $C_{J,j} \in C_J$, respectively. We focus herein on the Gaussian jamming [5], where the jamming sequence $\{Z_i[n]\}$ is assumed to be i.i.d Gaussian random variables with mean 0 and variance J_i, i.e $Z_i[n] \sim \mathcal{CN}(0, J_i)$, for $i = 1, \cdots, M$. The transmitter encodes using Gaussian codebook such that the transmitted sequence $\{X_i[n]\}$ is i.i.d and distributed as $\mathcal{CN}(0, P_i)$ and independent of $\{Z_i[n]\}$ and $\{X_j[n]\}$, for $j \neq i$.

Note: P_i and J_i are in general two functions of channel states $\mathbf{h} \triangleq \{h_1, \cdots, h_M\}$. The power constraints in (2) and (3) can be translated to

Short-term Power constraint:

$$P_M(\mathbf{h}) \triangleq \sum_{i=1}^{M} P_i(\mathbf{h}) \leq P_S, \quad J_M(\mathbf{h}) \triangleq \sum_{i=1}^{M} J_i(\mathbf{h}) \leq P_J, \tag{4}$$

Long-term Power constraint:

$$\sum_{i=1}^{M} E\left[P_i(\mathbf{h})\right] \leq P_S, \quad \sum_{i=1}^{M} E\left[J_i(\mathbf{h})\right] \leq P_J, \tag{5}$$

Next, we provide the pay-off function of the second layer zero-sum game where $u_{i,j}$ in Table I is the maxmin solution of the underlying zero-sum game, i.e. the robust solution from transmitter's perspective. For some cases, we will show in next section that $u_{i,j}$ is also the minimax solution, and consequently a Nash Equilibrium point [6].

Tx hopping (S_{hp}) versus Jx hopping (J_{hp}):

When both players decide to hop over M frequency bands, each of them picks up one frequency slot to send/jam randomly, the resulting average sum rate $\overline{R}_{h,h}$ over M frequency slots, which is the pay-off function of the two-person zero-sum game for this particular case, can be computed as

$$\overline{R}_{h,h} = \sum_{i=1}^{M} E\left[\alpha_i(1-\gamma_i)R_{i,0} + \alpha_i\gamma_i R_{i,J}\right] \tag{6}$$

where $\alpha_i \geq 0$ and $\gamma_i \geq 0$ for $i = 1, \cdots, M$ are the probability that node S and node J are active over the i-th slot, respectively, with constraints $\sum_{i=1}^{M} \alpha_i = 1$ and $\sum_{i=1}^{M} \gamma_i = 1$. Both of these distributions are functions of $\mathbf{h}$. The expectation is taken over the joint distribution of M channel statistics. As only one slot is picked by each player, the mutual information rate over that particular slot i is

$$R_{i,J} = \log\left(1 + \frac{P_i h_i}{J_i + N_0}\right), \quad \text{or} \quad R_{i,0} = \log\left(1 + \frac{P_i h_i}{N_0},\right) \tag{7}$$

where $R_{i,J}$ is the rate when jammer is present, while $R_{i,0}$ is the rate when jammer is absent.

Tx hopping (S_{hp}) versus Jx spreading (J_{sp}):

Given full channel state information (CSI), the jammer can unilaterally change to spread its power over M slots without hopping while the transmitter sticks with its hopping strategy. This is the strategy suggested in [2] in the same scenario. The resulting pay-off function in terms of average sum rate is therefore $\overline{R}_{h,s} = \sum_{i=1}^{M} E\left[\alpha_i R_{i,J}\right]$.

Tx spreading (S_{sp}) versus Jx hopping (J_{hp}):

Similarly, if transmitter unilaterally alters to spreading while jammer keeps on hopping. The average sum rate is $\overline{R}_{s,h} = \sum_{i=1}^{M} E\left[(1-\gamma_i)R_{i,0} + \gamma_i R_{i,J}\right]$.

Tx spreading (S_{sp}) versus Jx spreading (J_{sp}):

When the transmitter and jammer both spread power over M channels without hopping, the resulting average sum rate is

$$\overline{R}_{s,s} = \sum_{i=1}^{M} E\left[R_{i,J}\right]. \tag{8}$$

The power constraints specified in (4) and (5) are actually for the case where both players adopt spreading strategy. For other cases where at least one of them adopts hopping strategy, the power constraints will be modified as follows. Under the short-term power constraint, we have $P_i(\mathbf{h}) \leq P_S$ and $P_j(\mathbf{h}) = 0$ for transmitter and $J_i(\mathbf{h}) \leq P_J$ and $J_j(\mathbf{h}) = 0$ for jammer, $\forall j \neq i$, if the i-th frequency slot is picked up by either the transmitter or jammer. While under the long-term power constraint, the only non-zero power is the one allocated over the picked i-th frequency slot by transmitter (jammer) $P_i(\mathbf{h})$ ($J_i(\mathbf{h})$) with probability $\alpha_i(\mathbf{h})$ ($\gamma_i(\mathbf{h})$), which satisfies $\sum_{i=1}^{M} E\left[\alpha_i(\mathbf{h})P_i(\mathbf{h})\right] \leq P_S$, and $\sum_{i=1}^{M} E\left[\gamma_i(\mathbf{h})J_i(\mathbf{h})\right] \leq P_J$, for transmitter and jammer, respectively.

The short term power constraints imply that whoever selects hopping, the power used by it for that particular active frequency slot can not be greater than P_S or P_J for any given channel realization $\mathbf{h}$. As a contrast, under the long term power constraint, the power averaged over all channel realizations and over all frequency slots cannot exceed P_S or P_J.

III. Spreading or Hopping

We show next that when transmitter takes a robust perspective in the sense to seek the maxmin solution, i.e. to maximize the minimum average sum rate when jammer knows completely the strategy space of the transmitter, none of these two players will select hopping strategy, but instead spreads power over M frequency slots optimally by taking the S_{sp} versus J_{sp} approach.

Given the transmitter's robustness perspective, the pay-off functions in the matrix game of Table I are

$$u_{1,1} = \max_{\{\alpha_i(\mathbf{h}),P_i(\mathbf{h})\}} \min_{\{\gamma_i(\mathbf{h}),J_i(\mathbf{h})\}} \overline{R}_{h,h},$$
$$u_{1,2} = \max_{\{\alpha_i(\mathbf{h}),P_i(\mathbf{h})\}} \min_{\{J_i(\mathbf{h})\}} \overline{R}_{h,s},$$
$$u_{2,1} = \max_{\{P_i(\mathbf{h})\}} \min_{\{\gamma_i(\mathbf{h}),J_i(\mathbf{h})\}} \overline{R}_{s,h},\ u_{2,2} = \max_{\{P_i(\mathbf{h})\}} \min_{\{J_i(\mathbf{h})\}} \overline{R}_{s,s} \tag{9}$$

Theorem 1: The Nash equilibrium of the game in Table I is (S_{sp}, J_{sp}) when transmitter takes a robust perspective by taking the pay-off function $u_{i,j}$ of the matrix game as the maximin solution of the game in the second layer for each of these four cases.

The proof of Theorem 1 relies on the convexity of the function $f(x) = \log(1 + 1/x), x \geq 0$, as well as the following Lemma:

Lemma 1: When both players select spreading powers over M frequency slots, the power allocation profiles $\{P_i^*(\mathbf{h}), i = 1, \cdots, M\}$ and $\{J_i^*(\mathbf{h}), i = 1, \cdots, M\}$ resulted from solving $u_{2,2}$ is a Nash Equilibrium point, i.e.

$$u_{2,2} = \max_{\{P_i(\mathbf{h})\}} \min_{\{J_i(\mathbf{h})\}} \overline{R}_{s,s} = \min_{\{J_i(\mathbf{h})\}} \max_{\{P_i(\mathbf{h})\}} \overline{R}_{s,s} \tag{10}$$

The optimal power control functions are given by

$$P_i^*(\mathbf{h}) = \begin{cases} \left(\frac{1}{\lambda} - \frac{N_0}{h_i}\right)^+, & h_i \leq \frac{N_0\lambda}{1-N_0\nu} \\ \frac{h_i}{\lambda(h_i+\lambda/\nu)}, & h_i > \frac{N_0\lambda}{1-N_0\nu} \end{cases}, i = 1\cdots, M, \tag{11}$$

and

$$J_i^*(\mathbf{h}) = \begin{cases} 0, & h_i \leq \frac{N_0\lambda}{1-N_0\nu} \\ \frac{h_i}{\nu(h_i+\lambda/\nu)} - N_0, & h_i > \frac{N_0\lambda}{1-N_0\nu} \end{cases}, i = 1\cdots, M, \tag{12}$$

where λ and ν are two constants determined by the power constraints. Under the short-term one (4), $\lambda(\mathbf{h})$ and $\nu(\mathbf{h})$ are functions of the given channel realization vector $\mathbf{h}$. Under the long-term one (5), λ and ν are functions of channel statistics.

Proof:

The existence of Nash equilibrium point is due to the convexity of $\log\left(1 + \frac{h_iP_i}{J_i+N_0}\right)$ with respect to J_i and concavity of it with respect to P_i. The proof of the optimal power control functions is quite similar as the one for $M = 1$ [7] and is omitted here. ■

Proof of Theorem 1

Proof: The proof of Theorem 1 consists of two steps. We first demonstrate that given $\{P_i^*(\mathbf{h})\}$ and $\{J_i^*(\mathbf{h})\}$, when either jammer or transmitter deviates from spreading to hopping, it always ends up with the beneficial effect for its opponent, and thus prove that both spreading is **a** Nash equilibrium point for the game in Table I. Note that we cannot argue using the fact that $\{P_i^*(\mathbf{h})\}$ and $\{J_i^*(\mathbf{h})\}$ is the Nash equilibrium point for (S_{sp}, J_{sp}) in the second layer to show that spreading and spreading is a Nash equilibrium point in the first layer of the entire game.

Consider the strategy profile (S_{sp}, J_{hp}) where transmitter selects spreading and jammer selects hopping. We have the following inequalities hold:

$$\begin{aligned} u_{2,1} &= \max_{\{P_i(\mathbf{h})\}} \min_{\{\gamma_i(\mathbf{h}),J_i(\mathbf{h})\}} \sum_{i=1}^{M} E\left[(1-\gamma_i)\log\left(1+\frac{h_iP_i}{N_0}\right)\right. \\ &\quad \left. +\gamma_i \log\left(1+\frac{h_iP_i}{J_i+N_0}\right)\right] \\ &\geq \min_{\{\gamma_i(\mathbf{h}),J_i(\mathbf{h})\}} \sum_{i=1}^{M} E\left[(1-\gamma_i)\log\left(1+\frac{h_iP_i^*}{N_0}\right)\right. \\ &\quad \left. +\gamma_i \log\left(1+\frac{h_iP_i^*}{J_i+N_0}\right)\right] \\ &\geq \min_{\{\gamma_i(\mathbf{h}),J_i(\mathbf{h})\}} \sum_{i=1}^{M} E\left[\log\left(1+\frac{h_iP_i^*}{\gamma_iJ_i+N_0}\right)\right] \\ &= \sum_{i=1}^{M} E\left[\log\left(1+\frac{h_iP_i^*}{J_i^*+N_0}\right)\right] = u_{2,2}, \end{aligned} \tag{13}$$

where the first inequality is from a particularization of $P_i(\mathbf{h})$ to $P_i^*(\mathbf{h})$, the second inequality is due to the convexity of $\log(1+1/x)$ and the last equality is a result of treating γ_iJ_i as an equivalent J_i' under the constraint of $\sum_{i=1}^{M} E[J_i'(\mathbf{h})] \leq P_J$ (long-term) or $\sum_{i=1}^{M} J_i'(\mathbf{h}) \leq P_J$ (short-term), as well as the condition that $\{P_i^*(\mathbf{h})\}$ and $\{J_i^*(\mathbf{h})\}$ is a Nash equilibrium point of (S_{sp}, J_{sp}).

The inequality $u_{2,1} \geq u_{2,2}$ shows that jammer's unilateral deviation from spreading power to hopping only benefits the transmitter. Jammer therefore will not alter its spreading strategy unilaterally when transmitter spreads its power. Using the similar techniques, we can prove that $u_{1,2} \leq u_{2,2}$ [8]. Inequalities $u_{1,2} \leq u_{2,2}$ and $u_{2,1} \geq u_{2,2}$ together imply that (S_{sp}, J_{sp}) is a Nash equilibrium point of the matrix game over the first layer of the zero-sum game as characterized in Table I. The only remaining question is whether (S_{hp}, J_{hp}), i.e. two players both select hopping over M slots, is another equilibrium point.

Using the similar method, we can prove $u_{1,1} \geq u_{1,2}$ [8], which implies that jammer's unilaterally deviation from hopping to spreading when the transmitter stays with hopping further reduces the average sum rate, which indicates that (S_{hp}, J_{hp}) cannot be an equilibrium point of the matrix game in Table I.

Putting all these inequalities together, we can come to the conclusion that the strategy of both spreading (S_{sp}, J_{sp}) is the unique Nash equilibrium point of the matrix game in Table I, which completes the proof of Theorem 1.

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IV. Feedback or No Feedback

Without jammer's presence, it is a well known result that power control based on channel state information received perfectly at transmitter side helps in increasing the ergodic capacity of fading channels [9]. However, when we consider the counter-measure issues against a smart jammer who intercepts the feedback channel to obtain the perfect channel state information also about $\{\mathbf{h}\}$ as we have assumed in the system model part, it is not quite obvious at the first glimpse whether it remains a viable option for the legitimate transceiver to have $\{\mathbf{h}\}$ sent back considering the presence of such a smart jammer. Notice that the jammer could also exploit the CSI using the jamming power control function obtained in Lemma 1 for its malicious purpose. The question we address in this section is to justify that even when the smart jammer has the capability to explore CSI, it is still better to feedback CSI than not from the transmitter's perspective.

To answer the question raised above, we need to first examine how we attain the two power control functions in (11) and (12), respectively. Define a concave function $g(x) = \log(1+x)$ for $x \geq 0$. Under the strategy profile (S_{sp}, J_{sp}), the pay-off function $\overline{R}_{s,s}$ in (8) can be re-written as $\overline{R}_{s,s} = \sum_{i=1}^{M} E\left[g(x_i)\right]$ where $x_i = \frac{h_i P_i}{J_i + N_0}$. Since $g(x_i)$ is a concave function of P_i and a convex function of J_i, the KKT [10] condition yields

$$\frac{dg}{dx_i}\frac{h_i}{J_i+N_0} = \hat{\lambda} > 0,\ \frac{dg}{dx_i}\frac{h_i P_i}{(J_i+N_0)^2} = \hat{\nu} > 0, \quad (14)$$

for $i = 1, \cdots, M$, where $\hat{\lambda}$ and $\hat{\nu}$ are constants similar as those in (11) and (12) to meet the power constraints for transmitter and jammer, respectively.

From the conditions in (14), we can easily infer that the following condition holds as long as both jammer and transmitter are active in the i-th frequency slot:

$$\frac{P_i}{J_i+N_0} = \frac{\hat{\nu}}{\hat{\lambda}},\ P_i > 0,\ J_i > 0. \quad (15)$$

Without the concern of the sign of P_i and J_i, (15) immediately implies

$$\sum_{i=1}^{M} P_i(\mathbf{h}) = \frac{\hat{\nu}}{\hat{\lambda}}\left(\sum_{i=1}^{M} J_i(\mathbf{h}) + MN_0\right) \quad (16)$$

Under the power constraints in either (4) or (5), we always have

$$\frac{\hat{\nu}}{\hat{\lambda}} = \frac{P_S}{P_J + MN_0}, \quad (17)$$

which is exactly the ratio of $P_i/(J_i+N_0)$ when the only option for both transmitter and jammer is to divide their powers P_S and P_J uniformly over M frequency slots, i.e. $P_i = P_S/M$ and $J_i = P_J/M$ for $i = 1, \cdots, M$, if neither the CSI about exact channel coefficients $\{\mathbf{h}\}$ nor CSI about channel statistics is sent to back to the transmitter. [1].

However, since additional constraints of $P_i \geq 0$ and $J_i \geq 0\ \forall 1 \leq i \leq M$ have to be invoked, the optimal power control functions are in (11) and (12), which reflects such restrictions. We can therefore see that the constant ratio of $\hat{\nu}/\hat{\lambda}$ as determined in (17) is under an equivalent assumption that J_i could be negative, which cannot hold in real systems. Consequently, imposing non-negativeness for J_i is equivalently to confining jammer's jamming capability, which in turn favors the transmitter and hence the associated $R_{s,s}$ at the equilibrium point must be greater than $E\left[\sum_{i=1}^{M}\log\left(1+\frac{h_i P_S}{P_J+MN_0}\right)\right]$, the rate when no information is available to transmitter and jammer. In addition, it can be shown the true ratio of ν/λ in Lemma 1 is greater than $P_S/(P_J+MN_0)$ [8].

We can thus conclude that even when jammer is able to intercept the feedback CSI to conduct smart jamming, we still need to inform transmitter of the current $\{\mathbf{h}\}$ for the sake of improving the legitimate user's counter-measure capability.

V. When to Feedback and When not to Feedback

We have noticed from the last section that although the transmission power of transmitter and jammer at the equilibrium point are both functions of channel coefficients $\{\mathbf{h}\}$ as shown in (11) and (12), the ratio $P_i^*/(J_i^* + N_0)$ is a constant and has nothing to do with the exact h_i as long as $h_i \geq \Gamma$ holds, where $\Gamma = N_0\lambda/(1 - N_0\nu)$. The question we intend to address in this section is: Is it possible that the amount of

[1]When only channel statistics is known to both jammer and transmitter, we can reach a similar conclusion that none of them shall adopt hopping as a strategy, but instead both of them spread powers based on channel statistics

information sent back by receiver regarding $\{\mathbf{h}\}$ can be cut to some extent while the performance in terms of the average sum rate remains the same, given the presence of such a smart jammer? The importance of the answer to this question could be quite significant as we have been ignoring the cost associated with sending back perfect CSI $\{\mathbf{h}\}$. If we can show that receiver can deliver channel state information back to transmitter in a smarter way, instead of always sending back full CSI, without hurting the performance, it will imply the possibility of reducing the system complexity and cost as far as feedback is concerned.

It turns out not only is transmitter involved in adaptive power control in fighting against the jammer, the receiver should also perform some kind of adaptive feedback mechanism. The exact adaptive feedback protocol depends on whether we have short-term or long-term power constraints. Due to the space limitation, we only present the case under the long term power constraint. The protocol under the short-term power constraint can be treated in a similar way [8].

Define ν_L and λ_L as the multipliers under the long-term power constraint. We then accordingly define $\Gamma_L = \frac{N_0\lambda_L}{1-N_0\nu_L}$, as the threshold for power control functions under the long-term power constraint. Consequently, define $I_L(\mathbf{h}) \triangleq \{j : h_j > \Gamma_L, 1 \le j \le M\}$ as the the set of indexes of channel coefficients over which both jammer and transmitter allocate positive powers, and $I_L^c(\mathbf{h}) \triangleq \{j : h_j \le \Gamma_L, 1 \le j \le M\}$. Let $M_L(\mathbf{h}) = |I_L|$ denote the number of elements of the set I_L under the channel realization vector $\{\mathbf{h}\}$, and $\overline{M}_L$ denote the average number of frequency slots where jammer is active, i.e. $\overline{M}_L = \sum_{i=1}^{M} \Pr\left[h_i \ge \Gamma_L\right] = E\left[M_L(\mathbf{h})\right]$.

Let $\tilde{P}_{S,L}$ denote the amount of average power available to spend over frequency slots where $h_i \ge \Gamma_L$, which can be determined as

$$\tilde{P}_{S,L} = P_S - E\left[\sum_{i\in I_L^c(\mathbf{h})}\left(\frac{1}{\lambda_L} - \frac{N_0}{h_i}\right)^+\right]. \tag{18}$$

As seen from (12) and (11), for any i in $I_L(\mathbf{h})$, we have

$$\frac{P_{i,L}^*(\mathbf{h})}{J_{i,L}^*(\mathbf{h}) + N_0} = \frac{\nu_L}{\lambda_L}, \tag{19}$$

from which we can deduce

$$\frac{\nu_L}{\lambda_L} = \frac{\tilde{P}_{S,L}}{P_J + \overline{M}_L N_0}., \tag{20}$$

Using the Nash equilibrium points argument, the adaptive feedback protocols under the long-term power constraint is stated in the following Theorem.

Theorem 2: To counter-measure the jammer's attacks, the legitimate receiver measures $\{h_1, \cdots, h_M\}$ and decide whether or not to send back its measurement based on the following conditions about h_i: if $i \in I_L(\mathbf{h})$, no information is sent back to transmitter; if $i \in I_L^c(\mathbf{h})$, full CSI about h_i is feedback. The power control functions of jammer and transmitter are altered as follows: when $i \in I_L^c(\mathbf{h})$, the same power functions as J_i^* and P_i^* specified in (11) and (12) are deployed, i.e.

$$\tilde{P}_{i,L}^* = \left(\frac{1}{\lambda_L} - \frac{N_0}{h_i}\right)^+, \ \tilde{J}_{i,L}^* = 0. \tag{21}$$

When $i \in I_L(\mathbf{h})$, the power control functions at Equilibrium points are given by

$$\tilde{P}_{i,L}^* = \tilde{P}_{S,L}/\overline{M}_L, \ \tilde{J}_i^* = P_J/\overline{M}_L, \ i \in I_L(\mathbf{h}). \tag{22}$$

Proof: The proof of Theorem 2 hinges upon the fact $J_i^*(\mathbf{h})$ and $P_i^*(\mathbf{h})$ in (11) and (12) are Nash equilibrium points under the strategy profile (P_{sp}, J_{sp}) when $\{\mathbf{h}\}$ is completely known to both transmitter and jammer. In addition, the $J_i^*(\mathbf{h})$ and $P_i^*(\mathbf{h})$ result in the same ratio as that in (20) under $\tilde{P}_{i,L}^*$ and $\tilde{J}_{i,L}^*$ when $i \in I_L$, which implies that deploying $\tilde{P}_{i,L}^*$ and $\tilde{J}_{i,L}^*$ as specified above yields the same average rate as if the full CSI is known. The details of the proof are in [8]. ■

Remarks: Theorem 2 essentially unveils to us that due to the mutual restrictions between transmitter and jammer in this two-person zero sum game, we can achieve the same average sum rate at the equilibrium point when receiver sends back less amount of channel state information.

VI. Conclusion

In this paper, we have investigated issues related with sending back channel state information in parallel Gaussian fading channels to defend against Gaussian jamming attacks. With total sum rate as the pay-off function, our results reveal how legitimate transceivers should collaborate to jointly handle CSI in the presence of an intelligent jammer. Similar questions can be raised when we adopt other metric as the pay-off function, such as the error exponent, which might not have the same functional feature as the total sum rate and thus could lead to different conclusions. Interesting results as to other pay-off functions will be presented in our future work.

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