Gaussian Jamming in Block-Fading Channels under Long Term Power Constraints

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***Abstract*— We formulate a Gaussian uncorrelated jamming problem in block fading channels under long term power constraints. Source aims at minimizing the outage probability of its transmission under the presence of a malicious jammer, while the jammer attempts to maximize the corresponding outage probability under its average power constraint. Optimal power control strategies for both source and jammer are obtained for minimax and maxmin problems, respectively, for any arbitrary finite number of blocks in block fading channels. Our results demonstrate the non-existence of Nash-equilibria of this two-person zero-sum game.**

I. Introduction

The problem of jamming in wireless networks started to attract interest in the '80s [1], [2], and was focused on simple, point-to point communication systems, affected by intelligent jammer. The jammer was assumed to have access to either a noise-distorted version of the transmitter's output [1], or the transmitter's input message [2]. The mean-squared error was considered as a performance indicator.

A saddlepoint for the jamming game of [1] consists of an amplifying transmitter and a jammer that performs a linear transformation of its available version of the transmitter's output signal. Similar results were obtained in [3], for correlated jammers suffering from phase/time jitters at acquisition or at transmission. Channel capacity was used as performance indicator. Extensions to more complex, multi-user channels with fading, were derived in [4], [5], [6], [7] and [8].

The general tendency seems to be in favor of an assumption that jammer has access to either the transmitter's output or input and consequently is able to produce correlated jamming signals. Uncorrelated jammers are often studied only as a particular case. We, however, argue that correlation assumption is sometimes inappropriate because of the effect of causality.

In addition, most recent works adopt ergodic capacity as a common objective function over which transmitter and jammer fight against each other [7], [5], which is not a suitable metric if delay constraint is considered.

In this paper, we take a look at a constant-rate wireless system with power and delay constraints, which is depicted in Figure 1. This system model is similar to the one used in [7]. However, the major difference is that we investigate jamming in delay constrained block fading channels, and therefore adopt outage probability [9] as an objective over which jammer and transmitter fight against each other. In addition, in our formulation the jammer is assumed to possess no knowledge about the output of or the codebook employed by the transmitter.

Our problem is formulated as a two-player, zero-sum game, where only pure strategies (no randomized strategies) are considered. The power constraint in a block fading channel can be manifested in terms of either short term or long term [9]. In this paper, we only present the long-term case.

We have already solved the $M = 1$ case in [10]. In this paper, we provide the extension to the more general case of any finite $M > 1$. The extension is far from trivial. Not only do the KKT conditions not yield a closed form solution, but the methods used in [10] are no longer valid. Moreover, the new problem of allocating power between blocks in a frame arises.

II. Channel Model and Notations

The channel model is depicted in Figure 1. Each codeword spans a concatenation of M blocks, each of which has N channel uses. As assumed in [9], we let $N \to \infty$ in order to average out the impact of Gaussian noise.

Over a given frame, the transmitter (Tx) allocates power P_m to block m, $0 \leq m \leq M-1$, while the jammer (Jx) invests power J_m in jamming the same block with the worst possible jamming signal which is uncorrelated with the transmitter's output, white and Gaussian distributed [11].

The channel squared fading coefficient h_m is constant over the length of one block. The vector $\mathbf{h} = [h_0, h_1, \ldots, h_{M-1}]$ of channel coefficients over a whole frame is assumed to be perfectly known to transmitter, receiver and jammer before transmission begins. This condition does not imply non-causality if we think of modeling a multicarrier system [9].

Then the mutual information over a subchannel m when transmitter uses Gaussian codebook is given by $I(h_m, P_m, J_m) = 1/2 \log(1 + \frac{h_m P_m}{\sigma_N^2 + J_m})$, where σ_N^2 is the variance of the AWGN.

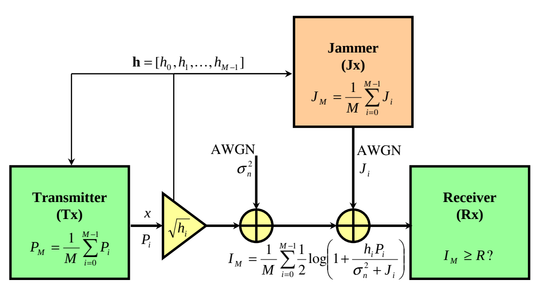

Fig. 1. Channel model

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In the sequel, the following denotations will be repeatedly used: (1) Power allocated by transmitter to a frame: $P_M = \frac{1}{M}\sum_{m=0}^{M-1} P_m$; (2) Power allocated by jammer to a frame: $J_M = \frac{1}{M}\sum_{m=0}^{M-1} J_m$; (3) Instantaneous mutual information for a frame:$I_M = \frac{1}{M}\sum_{m=0}^{M-1} I(h_m, P_m, J_m)$.

Also note that P_M is a function of the channel realization $\mathbf{h}$, so we often write $P_M(\mathbf{h})$ when this relation needs to be pointed out. This can be interpreted as the function giving the power distribution between frames. We use $P_M(h)$ and $J_M(h)$ to denote intra-frame power allocation for the case $M = 1$, since in this case a frame only contains one block.

The notation $P(h)$ and $J(h)$ will denote the functions giving the power distribution within a frame (between blocks).

The probability of outage will be the cost/reward function. It is defined as [9] $P_{out} = Pr(I_M < R)$, with R denoting the fixed rate of the system.

The power required for a player to achieve its objective (reliable communication for transmitter, and outage for jammer) over some frame, given a fixed opponent's behavior, will be denoted as "required power". Depending on its optimal strategy, this power may or may not be matched by the player.

The long-term power constrained jamming game can be described as:

$$\text{Tx}\begin{cases} \text{Minimize} & Pr(I_M(\mathbf{h}, P(h), J(h)) < R) \\ \text{Subject to} & E[P_M(\mathbf{h})] \le \mathcal{P} \end{cases} \tag{1}$$

$$\text{Jx}\begin{cases} \text{Maximize} & Pr(I_M(\mathbf{h}, P(h), J(h)) < R) \\ \text{Subject to} & E[J_M(\mathbf{h})] \le \mathcal{J} \end{cases} \tag{2}$$

where expectation is with respect to the vector of channel coefficients $\mathbf{h} = (h_0, h_1, \ldots, h_{M-1}) \in \mathbb{R}_+^M$, and $\mathcal{P}$ and $\mathcal{J}$ are the upper-bounds on average transmission power of the source and jammer, respectively.

The two players need to make decisions on whether or not to transmit over a frame, given a fixed opponent's strategy. In the sequel, we look at both maximin and minimax solutions to the above game and determine the associated power control functions $P_M(h)$ and $J_M(h)$.

The maximin solution is defined as the set of optimal strategies when jammer plays first. No matter what strategy jammer uses, transmitter will take advantage of its weaknesses, and aim at a minimum of the outage probability. The best option for jammer is to maximize the minimum achievable by the transmitter. On the other hand, the minimax solution is defined as the set of optimal strategies when the transmitter plays first. Jammer aims at a maximum outage probability. The best option for transmitter is to minimize the maximum achievable by jammer. In [10] we found these solutions for the case $M = 1$.

Let m denote the probability measure introduced by the probability density function (p.d.f.) of $\mathbf{h}$, i.e., for a set $\mathscr{A} \subseteq \mathbb{R}_+^M$, we have $m(\mathscr{A}) = \int_{\mathscr{A}} f(\mathbf{h}) d\mathbf{h}$.

III. Power Allocation Between Blocks within a Frame for $M > 1$

If the number of blocks M in each frame is larger than 1, the game between transmitter and jammer has two levels. The first (coarser) level is about power allocation between frames, and has the probability of outage as cost/reward function. The case of $M = 1$ is only concerned with this level. The second (finer) level is that of power allocation between blocks within a frame.

The probability of outage is determined by the m-measure of the set over which the transmitter is not present or transmission is jammed. This set is established in the first level of power control. In this subsection, we study the second level strategies.

In the maximin case (when jammer plays first), assume that the jammer has already allocated power J_M to a given frame. Depending on the channel realization, the value of J_M, and its power constraints, the transmitter decides whether it wants to achieve reliable communication over that frame. If it decides to transmit, it needs to spend as little power as possible (transmitter will be able to use the saved power for achieving reliable communication over another set of positive m-measure, and thus to decrease the probability of outage). Therefore, the transmitter's objective is to minimize the power P_M spent for achieving reliable communication. The transmitter will adopt this strategy whether the jammer is present over the frame, or not. The jammer's objective is then to allocate J_M between the blocks such that the required P_M is maximized. Similar arguments apply for the minimax scenario.

The two problems can be formulated as:
Problem 1 (for the maximin solution)

$$\max_{\{J_m\}} \left[\min_{\{P_m\}} P_M = \frac{1}{M}\sum_{m=0}^{M-1} P_m, \text{ s.t. } I_M(\{P_m\}, \{J_m\}) \ge R \right]$$
$$\text{s.t.} \frac{1}{M}\sum_{m=0}^{M-1} J_m \le J_M; \tag{3}$$

Problem 2 (for the minimax solution)

$$\max_{\{P_m\}} \left[\min_{\{J_m\}} J_M = \frac{1}{M}\sum_{m=0}^{M-1} J_m, \text{ s.t. } I_M(\{P_m\}, \{J_m\}) \le R \right]$$
$$\text{s.t.} \frac{1}{M}\sum_{m=0}^{M-1} P_m \le P_M. \tag{4}$$

Note that the first level power allocation strategies cannot be derived before the second level strategies are available. Due to the linearity of the cost function and convexity of the constraints, the solutions of the above optimization problems can be sought by solving the KKT conditions.

The following proposition provides a result that we shall use in the sequel. Due to the intuitive nature of the proof, as well as space limitations, we put the proof in [12].

Proposition 1: The optimal solution of either of the two problems above satisfies both constraints with equality.

Denote $x_m = J_m + \sigma_N^2$. Without loss of generality, throughout the sequel we assume that the ratios x_m/h_m, with $m = 0, 1, 2, \ldots, M-1$ are always ordered increasingly, i.e. $x_0/h_0 \le x_1/h_1 \le \ldots \le x_{M-1}/h_{M-1}$ [9].

Solution of Problem 1

The transmitter's problem can be written as:

$$\min_{P_m} \frac{1}{M}\sum_{m=0}^{M-1} P_m, \text{ s. t.} \sum_{m=0}^{M-1} \log\left(1+\frac{h_m P_m}{\sigma_N^2+J_m}\right) \geq 2RM. \quad (5)$$

With the notation $c = \exp(2RM)$, the KKT conditions yield:

$$P_n = \left[\frac{c^{(1/M')}\left(\prod_{m=0}^{M'-1} x_m\right)^{(1/M')}}{\left(\prod_{m=0}^{M-1} h_m\right)^{(1/M')}} - \frac{x_n}{h_n}\right]_+, \quad (6)$$

where $[z]_+ = \max\{z, 0\}$. We can now show that the solution is unique.

Proposition 2: Problem 1 has a unique solution.

Proof: P_M is a strictly concave function of the vector $\mathfrak{X} = (x_0, x_1, \ldots, x_{M-1})$ (for a detailed proof see [12]). The second part of *Problem 1* can be written as

$$\min_{x_m}(-P_M(\mathfrak{X}))\text{subject to} \sum_{m=0}^{M-1} x_m = M(J_M+\sigma_N^2), \quad (7)$$

which is a nonlinear minimization problem, with convex cost function and linear equality constraints, thus has a unique solution. ■

The jammer's problem can be written as:

$$\text{Find } \max_{x_m} \frac{M'}{M}\left(\frac{c}{\left(\prod_{m=0}^{M'-1} h_m\right)}\right)^{\frac{1}{M'}}\left(\prod_{m=0}^{M'-1} x_m\right)^{\frac{1}{M'}} - \\ -\frac{1}{M}\sum_{m=0}^{M'-1}\frac{x_m}{h_m} \quad (8)$$

$$\text{subject to } \frac{1}{M}\sum_{m=0}^{M-1} x_m = (J_M+\sigma_N^2) \quad (9)$$

$$\text{and } x_m \geq \sigma_N^2, \text{ for } m = 1, 2, \ldots, M-1. \quad (10)$$

The new KKT conditions are given by

$$\left(\prod_{m=0, m\neq n}^{M'-1} x_m\right) A \left(\prod_{m=0}^{M'-1} x_m\right)^{\frac{1-M'}{M'}} - \frac{1}{h_n} + \mu = 0, \quad (11)$$

along with (9) and (10). We used the notation $A = \left(\frac{c}{\left(\prod_{m=0}^{M'-1} h_m\right)}\right)^{(1/M')}$ for simplicity. The system given by (11), (9) and (10) can be solved by checking the condition in (10) while assuming the jammer is present on a number of blocks which decreases from M' to 1.

An interesting situation occurs when both transmitter and jammer are present over the same M' blocks. In this case, the function $P_M(J_M)$ is linear (see [10] for a proof).

However, if jammer and transmitter are present over different blocks linearity of $P_M(J_M)$ function no longer holds.

Solution of Problem 2

The minimax intra-frame power allocation problem can also be solved by writing the KKT conditions. However, we were not able to reach satisfying results by this method. Instead we use the above solution of *Problem 1* and show that for both problems, the power allocation should be the same.

Theorem 1: If $J_{M,1}$ is the value used for the second constraint in *Problem 1* above, and $P_{M,1}$ is the resulting solution, then solving *Problem 2* with $P_M = P_{M,1}$ yields the solution $J_M = J_{M,1}$. Moreover, the power distributions should be the same, in both problems.

Proof: Assume that the vectors $\mathfrak{P}^* = (P_0^*, P_1^*, \ldots, P_{M-1}^*)$ and $\mathfrak{J}^* = (J_0^*, J_1^*, \ldots, J_{M-1}^*)$ are a solution of *Problem 1*. Then $\frac{1}{M}\sum_{m=0}^{M-1} J_m = J_{M,1}$ and $\frac{1}{M}\sum_{m=0}^{M-1} P_m = P_{M,1}$.

Since $\mathfrak{P}^*$ and $\mathfrak{J}^*$ form a solution, by Proposition 1, they satisfy the first constraint in *Problem 1* with equality, and so they also satisfy the first constraint in *Problem 2*. Furthermore, setting the second constraint of *Problem 2* as $P_M = P_{M,1}$, we note that $\mathfrak{P}^*$ and $\mathfrak{J}^*$ are in the feasible set of this problem.

If we evaluate the cost function at this point, we get $J_M = J_{M,1}$.

Thus, keeping the power distribution given by $\mathfrak{P}^*$, in the second problem, we can only obtain $J_{M,2} \leq J_{M,1}$, by minimizing the cost function over $(J_0, J_1, \ldots, J_{M-1})$.

Now take any different power distribution $\mathfrak{P}' = (P_0', P_1', \ldots, P_{M-1}') \neq \mathfrak{P}^*$, satisfying $\frac{1}{M}\sum_{m=0}^{M-1} P_m' = P_{M,1}$. The pair of vectors $(\mathfrak{P}', \mathfrak{J}^*)$ cannot satisfy the first constraint in *Problem 1*, because if it did, it would make a second solution of this problem, and the solution of *Problem 1* is unique, as specified by Proposition 2 above.

Thus, this pair has to satisfy the first constraint in *Problem 2* (with strict inequality). We know this could not possibly be a solution of the second problem, since the first constraint is not tight, but it is a feasible point and, by evaluating the cost function at this point, we get $J_M = J_{M,1}$.

Thus, any power distribution of $P_{M,1}$ we pick, we should always obtain $J_M \leq J_{M,1}$ in *Problem 2*. But any solution of *Problem 2*, with $P_M = P_{M,1}$ is also a solution of *Problem 1*, and so we cannot have $J_M < J_{M,1}$.

Therefore, $\mathfrak{P}^*$ and $\mathfrak{J}^*$ are a solution of *Problem 2*. ■

We have shown that the second level optimal power allocation strategies for the maximin and minimax problems coincide. We need to characterize a particular channel realization in terms of this power allocation tehchnique. Considering the maximin problem, we can map each channel vector $\mathbf{h}$ to a unique curve in the plane $P_M(J_M)$. That is, for fixed $\mathbf{h}$, we increase the jammer's power over the frame from 0 to ∞, and compute the transmitter power $P_M(J_M, \mathbf{h})$ required for achieving reliable communication. We have already mentioned that $P_M(J_M)$ is a continuous, concave function.

In the remainder of this section we present the particular case of $M = 2$ as an example of intra-frame power allocation.

Particular case: M=2

The case of $M = 2$ is the simplest and most intuitive illustration of the second-level power control strategy. Since we already showed that the minimax and maximin solutions coincide, the following considerations refer to the maximin scenario only.

Particularizing (6) to $M = 2$, for $n \in \{1, 2\}$ and taking the average of the two quantities we get:

$$P_M = \begin{cases} \sqrt{c\frac{x_1}{h_1}\frac{x_2}{h_2}} - \frac{1}{2}\left(\frac{x_1}{h_1} + \frac{x_2}{h_2}\right), & \text{if } c\frac{x_1}{h_1} \geq \frac{x_2}{h_2} \\ (c-1)\frac{x_1}{h_1}, & \text{if } c\frac{x_1}{h_1} < \frac{x_2}{h_2}. \end{cases}$$

This is the minimum required transmitter power P_M, obtained by optimally distributing transmitter power between the two blocks.

If the transmitter is only present on the first block, i.e. $c\frac{x_1}{h_1} < \frac{x_2}{h_2}$, then P_M only depends on x_1/h_1. In order to maximize this quantity, the jammer should allocate all its power to the first block. Therefore, this situation is only possible if $x_2 = \sigma_N^2$. The jammer's strategy in this case is to decrease the ratio $r = (x_2/h_2)/(x_1/h_1)$ by increasing the denominator.

Next assume that the transmitter is present over both blocks (as a result of either channel conditions or of jammer allocating enough power over the first block). Using the ratio r defined as above, and the fact that $x_1 + x_2 = 2(J_M + \sigma_N^2)$, we obtain:

$$P_M = \frac{(J_M + \sigma_N^2)(2\sqrt{cr} - r - 1)}{h_2 r + h_1}, \text{ if } c\frac{x_1}{h_1} \geq \frac{x_2}{h_2}. \quad (12)$$

The ratio r that maximizes P_M is found by setting the derivative equal to zero, as:

$$r_{opt} = \left(\frac{\sqrt{(h_1 - h_2)^2 + 4h_1h_2c} - (h_1 - h_2)}{2h_2\sqrt{c}}\right)^2, \quad (13)$$

and is between 1 (for $h_1 = h_2$) and c (for $h_2 = 0$). Furthermore, $P_M(r)$ is strictly increasing for $r \in [1, r_{opt})$ and strictly decreasing for $r \in (r_{opt}, c]$.

This implies that the optimal jammer strategy is to allocate its power such that the ratio $r = (x_2/h_2)/(x_1/h_1)$ approaches the optimal ratio r_{opt}. If the optimal ratio is attained, jammer should further maintain the ratio.

If J_M increases from 0 to ∞, we can define the characteristic curve $P_M(J_M)$. For instance, if $(\sigma_N^2/h2)/(\sigma_N^2/h1) > c$ the transmitter transmits on the first block only. As jammer starts transmitting, it will concentrate its power over the first block, until the ratio $(\sigma_N^2/h2)/[(2J_M + \sigma_N^2)/h1]$ reaches r_{opt}. Note that in doing so, the ratio passes through c, and that is when the transmitter starts transmitting on both blocks. After optimal ratio is attained, as J_M increases the jammer keeps allocating power over both blocks, while keeping the ratio r constant and equal to r_{opt}. Similar strategies apply if $(\sigma_N^2/h2)/(\sigma_N^2/h1) \in [r_{opt}, c]$. Note that $h_1/h_2 \geq r_{opt}$ for any channel realization (h_1, h_2), and so the two scenarios above cover all possible situations.

IV. Inter-Frame Power Allocation for $M > 1$: Maximin Solution

In this subsection we present the first level optimal power allocation strategies for the maximin problem, in the general case $M \geq 1$. The jammer needs to find the best choice of the set $\mathscr{X} \subset \mathbb{R}_+^M$ of channel realizations over which it should be present, and the optimal way $J_M(h)$ to distribute its power over $\mathscr{X}$, such that when the transmitter employs its optimal strategy, the probability of outage is maximized.

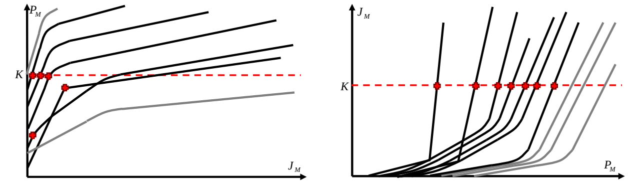

Fig. 2. First level power allocation techniques for the maximin (left) and minimax (right) solutions

The solution of this problem is presented in the following theorem.

Theorem 2: It is optimal for jammer to make $J_M(\mathbf{h})$ satisfy the power constraint with equality. The optimal jammer strategy for allocating power across frames is to increase the *required* transmission power, starting with those frames whose channel realizations exhibit the steepest instantaneous slope of the characteristic $P_M(J_M)$ curve. This increase should be done such that the *required* transmitter power over each channel realization where the jammer is present does not exceed a pre-defined level K.

A description of the technique is given in Figure 2.

The optimal value for K that maximizes the outage probability can be found numerically.

Proof: Let $\mathscr{S}, \mathscr{X} \subset \mathbb{R}_+^M$ denote the sets of channel realizations over which the transmitter and the jammer are present, respectively.

Let jammer pick a certain strategy $J_M(\mathbf{h})$. Since the transmitter's strategy is predictable, the jammer knows the set $\mathscr{S}$ as well as the maximum level of required power that will be matched by the transmitter. Denote this level by K. The *required* transmitter power should be equal to K over $\mathscr{X} \setminus \mathscr{S}$, since otherwise either the jammer (if larger) or the transmitter (if smaller) would be wasting power.

Assume there exist two sets $\mathscr{A}, \mathscr{B} \subset \mathscr{S} \bigcap \mathscr{X}$ of non-zero m-measure such that $\frac{dP_M(\mathbf{h}_1)}{dJ_M} > \frac{dP_M(\mathbf{h}_2)}{dJ_M}$ $\forall\, \mathbf{h}_1 \in \mathscr{A}$ and $\mathbf{h}_2 \in \mathscr{B}$, and such that the required P_M is less than K on $\mathscr{A}$ and $J_M > 0$ on $\mathscr{B}$.

Consider a small enough amount of jamming power δJ_M, such that, for any channel realization $\mathbf{h} \in \mathscr{A} \bigcup \mathscr{B}$, we can modify the jamming power by δJ_M without changing the slope of the $P_M(J_M)$ curve. Subtracting δJ_M from all frames in $\mathscr{B}$, the jammer obtains the excess power $\delta J_M m(\mathscr{B})$, which it can allocate uniformly over $\mathscr{A}$. This way, the jammer improves its strategy by forcing the transmitter to allocate more power to the set $\mathscr{A} \bigcup \mathscr{B}$, and hence increases the probability of outage.

Note that the optimal pre-defined constant K should be the limit of at lest one sequence of power levels $P_M(\mathbf{h})$ matched by the transmitter. ∎

V. Inter-Frame Power Allocation for $M > 1$: Minimax Solution

In Theorem 1, we showed that, for the minimax problem, the power allocation within a frame, as well as the relationship between the total powers used by transmitter and receiver over a particular frame, are identical to the maximin problem.

Hence, by rotating the $P_M(J_M)$ plane, we get the characteristic $J_M(P_M)$ curves for the minimax problem.

The main result of this section is presented in the following theorem.

Theorem 3: It is optimal for transmitter to make $P_M(\mathbf{h})$ satisfy the long-term power constraint with equality. The optimal transmitter power allocation across frames is to increase the *required* jamming power up to some pre-defined level K, starting with those frames on which the required transmitter power to achieve this goal is least.

A description of the technique is given in Figure 2.

The optimal value for K that minimizes the outage probability can be found numerically.

Proof: Let $\mathscr{S}$ and $\mathscr{X}$ denote the sets over which the transmitter and the jammer are present, respectively. Let transmitter pick a certain strategy $P_M(\mathbf{h})$. Since jammer's strategy is predictable, the transmitter knows the maximum level of required power that will be matched by jammer. Denote this level by K and note that the *required* jamming power over $\mathscr{S} \setminus \mathscr{X}$ should be equal to K (otherwise either the jammer - if smaller than K - or the transmitter - if larger - would be wasting power).

Assume there exist two sets $\mathscr{A}, \mathscr{B} \subset \mathscr{S} \bigcap \mathscr{X}$ of non-zero m-measure such that $\frac{K}{P_M(\mathbf{h}_1, K)} > \frac{K}{P_M(\mathbf{h}_2, K)}$ $\forall$ $\mathbf{h}_1 \in \mathscr{A}$ and $\mathbf{h}_2 \in \mathscr{B}$, and such that the required J_M is less than K on $\mathscr{A}$ and $J_M > 0$ on $\mathscr{B}$. Denote the original transmitter power allocations by $P^A_{M,0}(\mathbf{h})$ and $P^B_{M,0}(\mathbf{h})$ respectively.

We know that $J_M(P_M)$ is convex, and hence

$$\frac{K - J_{M,1}}{P_M(\mathbf{h}_1, K) - P_M(\mathbf{h}_1, J_{M,1})} \geq \frac{K}{P_M(\mathbf{h}_1, K)} > \\ > \frac{K}{P_M(\mathbf{h}_2, K)} \geq \frac{J^2_M}{P_M(\mathbf{h}_2, J_{M,2})} \\ \forall\ \mathbf{h}_1 \in \mathscr{A}, \mathbf{h}_2 \in \mathscr{B} \text{ and } J_{M,1}, J_{M,2} < K. \qquad (14)$$

If the transmitter cuts off transmission over a subset $\mathscr{B}' \subset \mathscr{B}$, it obtains the excess power $\int_{\mathscr{B}'} P_M(\mathbf{h}) dm(\mathbf{h})$, which it can allocate to a subset $\mathscr{A}' \subset \mathscr{A}$ such that the required J_M is equal to K over $\mathscr{A}'$, i.e.

$$\int_{\mathscr{B}'} P^B_{M,0}(\mathbf{h}) dm(\mathbf{h}) = \int_{\mathscr{A}'} \left[P_M(\mathbf{h}, K) - P^A_{M,0}(\mathbf{h})\right] dm(\mathbf{h}) \qquad (15)$$

Replacing $P_M(\mathbf{h}_1, J_{M,1})$ by $P^A_{M,0}(\mathbf{h})$ and $P_M(\mathbf{h}_2, J_{M,2})$ by $P^B_{M,0}(\mathbf{h})$ in (14), we see the transmitter improves its strategy by forcing the jammer to allocate more power to the set $\mathscr{A} \bigcup \mathscr{B}$, and hence decreases the probability of outage.

Note that since $\mathscr{B}' \subset \mathscr{S} \bigcap \mathscr{X}$, the set $\mathscr{B}'$ is in outage, regardless of whether the transmitter is present or not. Thus, transmitter does not increase P_{out} by cutting off transmission on $\mathscr{B}'$. ∎

VI. Numerical Results

We have computed the outage probabilities for both minimax and maximin problems when $M = 2$. The channel coefficients are assumed i.i.d. exponentially distributed with parameter $\lambda = 1/6$. Figure 3 shows the outage probability vs. the maximum allowable average transmitter power $\mathcal{P}$ for fixed $\mathcal{J} = 10$ when $R = 1$. For comparison purposes, we also provide results for the cases when $M = 1$ and when the jammer is not present ($\mathcal{J} = 0$).

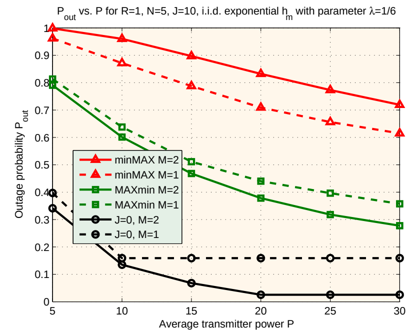

Fig. 3. Outage probability vs. $\mathcal{P}$ for $M = 1$ and $M = 2$ when $\mathcal{J} = 10$ - minimax and maximin cases - and when $\mathcal{J} = 0$

The numerical results demonstrate a sharp difference between minimax solutions and maxmin solutions, which implies the non-existence of Nash-equilibria of this two-person zero-sum game.

In addition, note that increasing M from 1 to 2 produces an increase in the outage probability for the minimax, and a decrease for the maximin. This can be explained by the fact that the first player is always at a disadvantage, and this disadvantage increases as the second player gains degrees of freedom.

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