Strategic Versus Collaborative Power Control in Relay Fading Channels

Shuangqing Wei
Department of Electrical and Computer Eng.
Louisiana State University
Baton Rouge, LA 70803
Email: swei@ece.lsu.edu

Rajgopal Kannan
Department of Computer Science
Louisiana State University
Baton Rouge, LA 70803
Email: rkannan@bit.csc.lsu.edu

***Abstract*— Relaying is often advocated for improving system performance by enhancing spatial diversity in wireless networks. Relay nodes make contributions to improving the source-destination link quality by sacrificing their own energy. In this paper, we address the issue of energy tradeoff made by relay nodes between transmitting their own data and forwarding other nodes' information in fading channels. Assuming channel state information (CSI) on fading amplitudes is perfectly known to both transmitters and receivers, we propose two power control and relaying policies. One is based on a *strategic* motivation, where each node functions as a relay and minimizes its own energy expenditure while meeting the outage probability requirement of all nodes. The second approach is based on *complete collaboration*, where the total energy consumption across all nodes is minimized. Numerical results demonstrate a significant impact of CSI on energy saving in relaying as compared with the relaying scheme without power control. In most cases, collaborative relaying dominates over the non-cooperative strategic one in the sense that the former not only minimizes total energy but also reduces individual energy expenditure of all nodes. Our results also show that when inter-user channel quality is not good, power control without relaying is preferable. This implies that energy saved through increasing diversity gain is often countered by energy spent in relaying the other node's information, and thus relaying should not be enforced blindly to serve the purpose of increasing diversity gain.**

I. Introduction

Recent years have witnessed tremendously growing interests in cooperative diversity and its applications to wireless ad hoc networks and sensor nets [1], [2], [3], [4]. As noted in [5], majority of the research work on cooperative diversity are based on the assumption of no channel state information. Channel state information (CSI) plays an important role in communication systems [6]. There is large amount of work on exploiting CSI to various extents in system design including a line of work lately on using CSI in relay fading channels to improve reliability and energy efficiency [7], [8], [9].

The preceding works share one common feature in that all of them assume a set of relay nodes is already selected and the remaining issue is to determine power allocations across all transmitting nodes without considering the data originated from relay nodes themselves. No consideration is given toward relay's own needs other than its function as relaying. The question not addressed there can be described as follows.

Given all nodes have their own data to transmit and their individual quality of service requirement, e.g. outage probability P_{out} as a good approximation for frame error rate (FER) [6], each node divides its entire energy budget into two parts. One is transmitting its own data, the other part is devoted to relaying other nodes' information. As a partner relationship is established between two nodes such that each of them helps the other in forwarding/relaying information, we are interested in a fundamental question regarding the power allocation policy to be followed by each node in order to save its energy to the largest extent, while meeting the outage probability constraints and complying with the obligation as a relay. This question is actually tied to the partner selection issue in wireless networks using cooperative diversity, which has been partially addressed in recent works [10] and [9]. These works, however, assume fixed transmission power for all users even at the presence of CSI [9].

In our paper, assuming perfect CSI available to all nodes, we develop two power control and relaying policies. One is based on a *strategic* motivation of each node which intends to minimize its own transmission energy while meeting the outage probability requirement and demands from the other node in relaying. The second policy is based on an objective of *complete collaboration* in which nodes aim at minimizing the summation of all energy consumptions across all nodes. Power control algorithms are developed under each policy and results are compared with those approaches assuming either power control without relay or relay without power control.

II. System Model and Problem Statements

A. System Model

We consider a simple model in which there are two nodes N_1 and N_2 transmitting to a common receiver N_D with help from each other. Narrow-band quasi-static fading channel is assumed, where channel fading coefficients remain fixed during the transmission of a whole packet, but are independent from node to node. The complex channel coefficient $h_{i,j}$ captures the effects of both pathloss and the quasi-static fading on transmissions from node N_i to node N_j, where $i \in \{1, 2\}$, and $j \in \{2, 1, D\}$. Statistically, $h_{i,j}$ are modeled as zero mean, mutually independent proper complex Gaussian random variables with variances: $E|h_{1,D}|^2 = 2\sigma^2_{1,D}$, $E|h_{2,D}|^2 = 2\sigma^2_{2,D}$, $E|h_{1,2}|^2 = 2\sigma^2_{1,2}$, and $E|h_{2,1}|^2 = 2\sigma^2_{2,1}$. We assume a non-causal system model in which $|h_{i,j}|$ are available to all transmitters and receivers at the beginning of transmissions. In a quasi-static fading channel, this can be realized by sending training sequences such that receivers send back to transmitters the estimated fading variables [11].

Consider a time-division (TD) multiple access scheme in which an entire time period is divided into 4 slots [3, Fig. 2]. A repetition coding-based decode-and-forward strategy (R-DF)is assumed at N_j, $j = 1, 2$ where relay node transmits the same codeword as what source sends if its decoding is successful. The cooperative communication protocol can be described as follows: Based on the available CSI, N_1 can determine whether relaying from N_2 is needed or not, as explained in the power control algorithms below. If such collaboration is sought, N_1 transmits as a source to N_D in the first slot and then in the second slot N_2 forwards its decoded messages to the destination. If N_2 is not asked for relaying, N_1 transmits in the first 2 slots of on its own. Over the last two slots, N_1 and N_2 exchange their roles as a source and relay.

The mathematical characterization of the whole process is:

$$Y_{1,D}[k] = h_{1,D}S_1[k]+W_{1,D}[k], Y_{1,2}[k] = h_{1,2}S_1[k]+W_{1,2}[k]$$

for $k \in [0, N/4]$; and $Y_{2,R}[k] = h_{2,D}\tilde{S}_1[k] + W_{2,R}[k]$ for $k \in (N/4, N/2]$, if relay N_2 is needed and decoding is successful. Over the next two slots, similar models can be set up for node 2 based on symmetry over $k \in (N/2, N]$.

The figure N is the total number of degrees of freedom available over the entire transmission period, and noise processes $W_{i,j}$ are independent complex white Gaussian noise with two-sided power spectral density $\mathcal{N}_0 = 1$. For R-DF schemes, $\tilde{S}_j[k]$ are scaled versions of the transmitted Gaussian codewords $S_j[k]$. Given CSI on $|h_{i,j}|$, transmission powers over various periods are denoted as: $E|S_1[k]|^2 = P_{1,D}$, $k \in [0, N/4]$ and $E|\tilde{S}_1[k]|^2 = P_{2,R}$, $k \in (N/4, N/2]$ if N_2 is need and decoding is successful; $E|S_1[k]|^2 = P_{1,D}$, $k \in [0, N/2]$ and $E|\tilde{S}_1[k]|^2 = 0$, $k \in [0, N/2]$, if N_2 is not needed. $E|S_2[k]|^2 = P_{2,D}$, $k \in (N/2, \frac{3}{4}N]$ and $E|\tilde{S}_2[k]|^2 = P_{1,R}$, $k \in (\frac{3}{4}N, N]$ if N_1 is needed and decoding is successful; $E|S_2[k]|^2 = P_{2,D}$, $k \in (N/2, N]$ and $E|\tilde{S}_2[k]|^2 = 0$, $k \in (N/2, N]$ if N_1 is not needed.

B. Problem Statements

The goal is to find optimal power control strategies $P_{j,D}$ and $P_{j,R}$ to minimize some energy cost functions under constraints of the outage probability. Let $P_{i,out}$ denote an upper-bound on the outage probability of the communication link between N_i and N_D, $i = 1, 2$. To compute an outage probability, we need to first determine the mutual information between each source node and its destination. For node 1, we have

$$I_1 = \begin{cases} \log_2 \left[1 + P_{1,D}|h_{1,D}|^2\right] \\ \frac{1}{2}\log_2 \left[1 + P_{2,R}|h_{2,D}|^2 + P_{1,D}|h_{1,D}|^2\right] \end{cases} \quad (1)$$

where the first case is true when N_2 does not forward and the second case holds when N_2 performs R-DF. In (1), the situation of N_2 not forwarding emerges when the channel between N_1 and N_2 is in outage or it is part of the proposed power control policy even when there is no outage. The outage occurs when $I_{1,2} = \frac{1}{2}\log\left[1 + P_{1,D}|h_{12}|^2\right] < R_1$, where R_1 is the source transmission rate of node N_1 and $I_{1,2}$ is the mutual information between N_1 and N_2. The factor $1/2$ is introduced because it only takes source $1/2$ degrees of freedom of the direct transmission without relay. Following similar arguments, we obtain the overall mutual information I_2 between N_2 and N_D by changing the role of node 1 and 2 in (1).

Under the constraint that each source node has an outage probability no greater than $P_{j,out}$, i.e. $\Pr[I_j < R_j] \le P_{j,out}$, our objective is to investigate power control policies under which either the individual energy expenditure is minimized where we consider every node as a selfish agent aiming at saving its own energy, or the total energy of these two nodes is minimized in a complete collaborative manner. The two problems can be formulated as below:

Strategic approach:

$$\min E\left[P_{j,D} + P_{j,R}\right], \text{ subject to } \Pr\left[I_j < R_j\right] \le P_{j,out} \quad (2)$$

for $j = 1, 2$. And **Collaborative approach:**

$$\min \sum_{j=1}^{2} E\left[P_{j,D} + P_{j,R}\right], \text{ subject to } \Pr\left[I_k < R_k\right] \le P_{k,out}, \quad (3)$$

for $k = 1, 2$.

III. Solution to a Point-to-point Link Power Control Problem

To solve the two power control problems posed in (2) and (3), we need to first provide solutions to a closely related power control problem for a point-to-point channel without relays as formulated below:

$$\text{Minimize: } E\left[\gamma(\boldsymbol{\alpha})\right], \text{ s.t.} \Pr\left[I\left(\boldsymbol{\alpha}, \gamma(\boldsymbol{\alpha})\right) < R\right] \le P_0, \quad (4)$$

where $I\left(\boldsymbol{\alpha}, \gamma(\boldsymbol{\alpha})\right)$ denotes the mutual information of a quasistatic fading channel with fading coefficient $\boldsymbol{\alpha}$ known perfectly at both transmitter and receiver. Power allocation function deployed by the transmitter is $\gamma(\boldsymbol{\alpha})$. In [11], a dual problem of (4) was solved in which an optimal power allocation function is found to minimize the outage probability under a constraint on transmission power. We follow the footsteps of [11] and obtain the solution to problem (4).

Let $\gamma_{mod}(\boldsymbol{\alpha})$ denote the solution to an optimization problem:

$$\begin{cases} \text{Minimize} & \gamma(\boldsymbol{\alpha}) \\ \text{Subject to} & I\left(\boldsymbol{\alpha}, \gamma(\boldsymbol{\alpha})\right) \ge R \end{cases} \quad (5)$$

For $s \in \mathbb{R}^+$, we define regions: $\mathcal{R}(s) = \{\boldsymbol{\alpha} : \gamma_{mod}\left(\boldsymbol{\alpha}\right) < s\}$ and $\overline{\mathcal{R}}(s) = \{\boldsymbol{\alpha} : \gamma_{mod}\left(\boldsymbol{\alpha}\right) \le s\}$. Then, we define two probabilities $\mathcal{Q}(s) = \Pr\left[\boldsymbol{\alpha} \in \mathcal{R}(s)\right]$ and $\overline{\mathcal{Q}}(s) = \Pr\left[\boldsymbol{\alpha} \in \overline{\mathcal{R}}(s)\right]$, and the power threshold s^* by $s^* = \sup\{s : \mathcal{Q}(s) < 1 - P_0\}$. Finally, we introduce a non-negative number $w^* = \left(1 - P_0 - \mathcal{Q}(s^*)\right) / \left(\overline{\mathcal{Q}}(s^*) - \mathcal{Q}(s^*)\right)$.

Theorem 1: The solution to problem (4) is given by

$$\hat{\gamma}(\boldsymbol{\alpha}) = \begin{cases} \gamma_{mod}(\boldsymbol{\alpha}), & \text{if } \boldsymbol{\alpha} \in \mathcal{R}(s^*) \\ 0, & \text{if } \boldsymbol{\alpha} \notin \mathcal{R}(s^*) \end{cases} \quad (6)$$

while if $\boldsymbol{\alpha} \in \mathcal{B}(s^*)$, which is the boundary surface of $\overline{\mathcal{R}}(s)$, then $\hat{\gamma}(\boldsymbol{\alpha}) = \gamma_{mod}(\boldsymbol{\alpha})$ with probability w^* and $\hat{\gamma}(\boldsymbol{\alpha}) = 0$ with probability $1 - w^*$.

Proof: The proof is very similar to that in [11] and omitted here (see [12]). ∎

As manifested in Theorem 1, the key to solving the problem (4) is to determine $\gamma_{mod}(\boldsymbol{\alpha})$ and s^*. Next, we are going to exploit Theorem 1 to solve the strategic and collaborative power control problems formulated in Section II-B.

IV. Non-Cooperative Strategic Relaying Game

As described in [1], [3], the primary objective of employing cooperative diversity schemes in wireless networks is to improve diversity gain by sharing information among different nodes and taking advantage of independent channels from each node to one common receiver. The energy expenditures of each node can be split into two portions. One portion is devoted to transmitting its own data to the receiver with power $P_{i,D}$, $i = 1, 2$, the other portion is spent on forwarding/relaying other nodes' packets with power $P_{i,R}$. The strategic cooperative diversity is based on an assumption about nodes' selfish behavior, namely, each of them would like to save his own power to the largest extent while requesting cooperation from other node for forwarding his packets. The aim of strategic power control for cooperative diversity scheme is to look for certain tradeoff between these two sources of power expenditures.

We consider the power control problem in (2) as a two-player non-cooperative strategic power game: $[\{N_i\}, \{C_i\}, \{\mu_i\}]$, where $C_i = \{P_{i,D}, P_{i,R}\}$ and $\mu_i = -E[P_{i,D} + P_{i,R}]$ are the power allocation strategy and utility function of node $N_i, i = 1, 2$, respectively. Denote $C_{-2} = C_1$ and $C_{-1} = C_2$. The objective of player i is to maximize μ_i, i.e. to minimize its total power expenditure, under constraints of outage probabilities $\Pr[I_i < R_i] \le P_{i,out}$. Strategy C_i^* is defined as the best response of node i to a given strategy C_{-i}.

WLOG, let us first study node 1's best response to a given power allocation vector $[P_{2,D}, P_{2,R}]$. As proved in Section III, given a power function vector of node 2, the minimizing power vector for node 1 can be determined using Theorem 1, which means we need to find $P_{1,D}$ and $P_{1,R}$ to solve the related constraint optimization problem as described in Eq. (5).

We can observe from Eq. (5) and Eq. (1) that to satisfy $I_1 \ge R_1$, the minimum $P_{1,D}$ is uniquely determined as a function of $P_{2,R}$

$$P_{1,D} = \frac{2^{2R_1} - 1 - P_{2,R}|h_{2,D}|^2}{|h_{1,D}|^2}, \quad (7)$$

when node 2 is involved in forwarding node 1's packets over the interval $t \in (N/4, N/2]$ with transmission power $P_{2,R}$. For a R-DF scheme, the above equation for $P_{1,D}$ only holds when node 2 is able to decode the information transmitted by node 1 successfully over $t \in [0, N/4]$, which requires $I_{1,2} \ge R_1$, i.e. $\frac{1}{2}\log\left(1 + P_{1,D}|h_{1,2}|^2\right) \ge R_1$ leads to $P_{1,D} \ge \frac{2^{2R_1}-1}{|h_{1,2}|^2}$.

On the other hand, node 1 has an option of transmitting on its own without node 2's help. In this case I_1 is computed using the first equation in Eq. (1) yielding

$$P_{1,D} \ge \frac{2^{R_1} - 1}{|h_{1,D}|^2} \triangleq \tilde{P}_{1,D} \quad (8)$$

Consider a time interval of period $N/4$ as one unit, the energy spent by node 1 without node 2's help is therefore $2\frac{2^{R_1}-1}{|h_{1,D}|^2}$ as its transmission spans over an interval of length $N/2$, while that expenditure changes to at least $\frac{2^{2R_1}-1}{|h_{1,2}|^2}$ when node 2 is involved. Therefore, we have

$$\begin{aligned} P_{1,D} &= \tilde{P}_{1,D},\ P_{2,R} = 0, \text{ if } |h_{1,2}|^2 \le \frac{2^{R_1}+1}{2}|h_{1,D}|^2 \\ P_{1,D} &= \frac{2^{2R_1} - 1 - P_{2,R}|h_{2,D}|^2}{|h_{1,D}|^2} \ge \frac{2^{2R_1}-1}{|h_{1,2}|^2}, \text{ O.T.} \end{aligned} \quad (9)$$

where the first inequality holds when $2\tilde{P}_{1,D} < \frac{2^{2R_1}-1}{|h_{1,2}|^2}$. Similarly, we can determine the relationship between $P_{2,D}$, $P_{1,R}$ and channel coefficients $\{h_{21}, h_{2,D}, h_{1,D}\}$ by reversing indexes 1 and 2 in (9).

There are apparently four cases on combinations of relative strength of ratios $\left\{|h_{1,2}|^2/|h_{1,D}|^2, |h_{2,1}|^2/|h_{2,D}|^2\right\}$: Case I, $\frac{|h_{1,2}|^2}{|h_{1,D}|^2} \le \frac{2^{R_1}+1}{2}$ and $\frac{|h_{2,1}|^2}{|h_{2,D}|^2} \le \frac{2^{R_2}+1}{2}$; Case II, $\frac{|h_{1,2}|^2}{|h_{1,D}|^2} \le \frac{2^{R_1}+1}{2}$ and $\frac{|h_{2,1}|^2}{|h_{2,D}|^2} > \frac{2^{R_2}+1}{2}$; Case III, $\frac{|h_{1,2}|^2}{|h_{1,D}|^2} > \frac{2^{R_1}+1}{2}$ and $\frac{|h_{2,1}|^2}{|h_{2,D}|^2} \le \frac{2^{R_2}+1}{2}$; Case IV, $\frac{|h_{1,2}|^2}{|h_{1,D}|^2} > \frac{2^{R_1}+1}{2}$ and $\frac{|h_{2,1}|^2}{|h_{2,D}|^2} > \frac{2^{R_2}+1}{2}$.

In Case I, none of two nodes needs forwarding and hence the power allocations are $P_{i,D} = \tilde{P}_{i,D}$ and $P_{i,R} = 0$ for $i = 1, 2$. In all other cases, as long as node i needs node j's help in relaying, we have an issue in determining $P_{i,R}$ given $P_{j,D}$ for they satisfy the following linear condition:

$$P_{i,D}|h_{i,D}|^2 + P_{j,R}|h_{j,D}|^2 = 2^{2R_i} - 1, \text{ for } P_{i,D} \ge \hat{P}_{i,D}, \quad (10)$$

where $\hat{P}_{i,D} \triangleq \frac{2^{2R_i}-1}{|h_{i,j}|^2}$ for $i, j = 1, 2$ and $i \ne j$. One key assumption in this paper on relaying principle is each node forwards its partner's information whenever it is needed provided the decoding is successful. This principle has been widely assumed in existing literature on cooperative diversity schemes. From Eq. (10), it can be seen this principle implies that node 1 transmits at its minimum $P_{1,D} = \hat{P}_{1,D}$ in case III and IV, whereas $P_{2,R}$ can be computed using the linear equation (10) under this $\hat{P}_{1,D}$. In cases II and IV, $P_{2,D}$ and $P_{1,R}$ can be calculated in a similar manner. Equilibrium of power functions are thus summarized as below:

$$\text{Case I}: P_{i,D} = \tilde{P}_{i,D}, P_{i,R} = 0,\ i = 1, 2 \quad (11)$$

$$\text{Case II}: P_{1,D} = \tilde{P}_{1,D},\ P_{2,R} = 0,\ P_{2,D} = \hat{P}_{2,D}, P_{1,R} = \hat{P}_{1,R}$$

$$\text{Case III}: P_{1,D} = \hat{P}_{1,D}, P_{2,R} = \hat{P}_{2,R}\, P_{2,D} = \tilde{P}_{2,D},\ P_{1,R} = 0$$

$$\text{Case IV}: P_{i,D} = \hat{P}_{i,D}, P_{j,R} = \hat{P}_{j,R}, i, j = 1, 2, i \ne j \quad (12)$$

where $\hat{P}_{2,R} \triangleq \frac{2^{2R_1}-1}{|h_{2,D}|^2}\left(1-\frac{|h_{1,D}|^2}{|h_{1,2}|^2}\right)$ and $\hat{P}_{1,R} \triangleq \frac{2^{2R_2}-1}{|h_{1,D}|^2}\left(1-\frac{|h_{2,D}|^2}{|h_{2,1}|^2}\right)$.

Next, we will show in case IV when both nodes are involved in relaying, there exists some situation in which none of two nodes saves energy from relaying.

Lemma 1: Denote $a_2 = (2^{R_2}-1)\left(1-\frac{2^{R_2}+1}{2}\frac{|h_{2,D}|^2}{|h_{2,1}|^2}\right)$, $a_1 = \frac{2^{2R_1}-1}{2}\left(1-\frac{|h_{1,D}|^2}{|h_{1,2}|^2}\right)$, $b_2 = (2^{R_1}-1)\left(1-\frac{2^{R_1}+1}{2}\frac{|h_{1,D}|^2}{|h_{1,2}|^2}\right)$ and $b_1 = \frac{2^{2R_2}-1}{2}\left(1-\frac{|h_{2,D}|^2}{|h_{2,1}|^2}\right)$.

When $a_2 < a_1$ and $b_2 < b_1$, no one benefits from relaying, i.e. $\hat{P}_{i,D}+\hat{P}_{i,R} > 2\tilde{P}_{i,D}$ for $i=1,2$; when $a_2 \geq a_1$, relaying helps node 2 but not node 1; when $b_2 \geq b_1$, relaying helps node 1 not node 2. The power functions in Case IV are therefore the same as in Case I if $a_2 < a_1$ and $b_2 < b_1$, and will remain the same as specified in Eq. (12), otherwise.

Proof: The proof is straightforward by comparing $\hat{P}_{i,D}+\hat{P}_{i,R}$ with $2\tilde{P}_{i,D}$ for $i=1,2$ and noticing that a_j and b_j always satisfy $b_1 > a_2$ and $a_1 > b_2$. Consequently, it is impossible for $a_2 \geq a_1$ and $b_2 \geq b_1$ to hold simultaneously. ■

From Theorem 1, there are additional parameters s_1^* and s_2^* to determine in order to meet the outage probability requirement for each node. These two thresholds are solutions to following two non-linear equations:

$$\Pr\{E_1 < s_1^*\} = 1-P_{1,out}, \text{ and } \Pr\{E_2 < s_2^*\} = 1-P_{2,out}, \quad (13)$$

where E_1 and E_2 can be determined subject to different cases as specified in Eq. (11) and Lemma 1. These two nonlinear equations do not yield to analytical solutions for $s_i^*, i=1,2$. Numerical approach will be taken to calculate s_i^* and average energy expenditure for each node.

V. Collaborative Relaying

Non-cooperative strategic approach is driven by the selfish behavior of each user in the sense of saving its own energy while taking advantage of other users' help to the largest extent. In this section, we will investigate an alternative in which users are fully collaborating with each other by minimizing the total energy as formulated in Eq. (3).

When total energy is an objective function, the optimization problem of Eq. (3) can be essentially decomposed into two separate problems:

$$\min E\left[P_{i,D}+P_{j,R}\right], \text{ subject to } \Pr\left[I_i < R_i\right] \leq P_{i,out}, \quad (14)$$

for $i,j=1,2$ and $i \neq j$. WLOG, we provide the solution to the first problem for $i=1, j=2$, and the solution to the case $i=2, j=1$ follows in the same way.

Based on Theorem 1, we need to first obtain the minimizing power allocation of $P_{1,D}$ and $P_{2,R}$ for problem:

$$\min P_{1,D}+P_{2,R}, \text{ subject to } I_1 \geq R_1 \quad (15)$$

and then determine $s_1^{(J,*)}$ satisfying

$$\Pr\left[P_{1,D}^J + P_{2,R}^J \leq s_1^{(J,*)}\right] = 1-P_{1,out} \quad (16)$$

where $P_{1,D}^J$ and $P_{2,R}^J$ are non-zero solutions to problem (15).

Theorem 2: The optimal power allocation vector $[P_{1,D}^J, P_{2,R}^J]$ for solving problem (15) depends on channel strength ratios captured by $|h_{1,D}|/|h_{2,D}|$ and $|h_{1,D}|/|h_{1,2}|$. The resulting solutions are: $\min[P_{1,D}+P_{2,R}] = \hat{P}_{1,D}+\hat{P}_{2,R}$ with $P_{1,D}^J = \hat{P}_{1,D}$, $P_{2,R}^J = \hat{P}_{2,R}$ if $h_{i,j}$ are in the set $A_1 = \left\{h_{i,j} \middle| \frac{|h_{1,D}|^2}{|h_{1,2}|^2} < \frac{2}{2^{R_1}+1} \text{ and } \frac{|h_{1,D}|^2}{|h_{1,2}|^2} + \frac{|h_{1,D}|^2}{|h_{2,D}|^2}\left(1-\frac{|h_{1,D}|^2}{|h_{1,2}|^2}\right) \leq \frac{2}{2^{R_1}+1}\right\}$; otherwise if $h_{i,j} \in A_2 = A_1^c$, $\min[P_{1,D}+P_{2,R}] = \tilde{P}_{1,D}$ with $P_{1,D}^J = \tilde{P}_{1,D}$, $P_{2,R}^J = 0$.

Proof: For brevity, the proof is put in [12]. ■

VI. Power Control without Relaying and Relaying without Power Control

To reveal energy savings through power control and relaying, we will compare schemes proposed in Section IV and Section V with two other possible approaches. One is cooperative diversity scheme without power control at an absence of CSI on $|h_{i,j}|$. The other one is power control without relaying as studied. The purpose of this comparison is to illustrate the impact of relaying, as well as CSI on energy consumption.

For relaying with fixed power, [4] derived outage probability of R-DF schemes. Therefore, transmission powers of two users under given outage probabilities can be determined from those two non-linear equations resulting from two outage probability expressions.

While for the case when each node employs power control strategy to transmit its data without relaying, the outage probability is

$$P_{i,out} = \Pr\left\{\log_2\left(1+P_{i,Nr}|h_{i,D}|^2\right) < R_i\right\}, i=1,2 \quad (17)$$

where $P_{i,Nr}(|h_{i,D}|), i=1,2$ can be determined using Theorem 1,

$$P_{i,Nr} = \begin{cases} \frac{2^{R_i}-1}{|h_{i,D}|^2} & \text{if } 2\frac{2^{R_i}-1}{|h_{i,D}|^2} < s_{(i,Nr)}^* \\ 0 & \text{Otherwise} \end{cases}, \quad (18)$$

Thus, the threshold $s_{(i,Nr)}^*$ is the solution to $\Pr\left[2\frac{2^{R_i}-1}{|h_{i,D}|^2} < s_{(i,Nr)}^*\right] = 1-P_{i,out}$. Solving it gives us

$$s_{(i,Nr)}^* = \frac{-2\lambda_i(2^{R_i}-1)}{\ln(1-P_{i,out})}. \quad (19)$$

Combining (18) and (19), we obtain the average energy spent by each node

$$E\left(P_{i,Nr}\right) = 2\lambda_i\left(2^{R_i}-1\right) \cdot \mathrm{E}_1\left[-\ln(1-P_{i,out})\right], \quad (20)$$

where $i=1,2$ and $\mathrm{E}_1(z)$ is a special function defined by an exponential integral: $\mathrm{E}_1(z) = \int_z^\infty e^{-t}t^{-1}\,dt$ for $z>0$ [13].

VII. Numerical Results

In numerical results, data rates $R_i, i=1,2$ and outage probabilities $P_{i,out}, i=1,2$ are put into a vector $[R_1, R_2, P_{1,out}, P_{2,out}] = [1, 2, 0.1, 0.08]$. For each set of fading variances $\left\{\sigma_{i,j}^2\right\}$, we calculate an average energy vector (E_1, E_2) for each scheme and tabulate them in Table I and Table II in which we use the following acronyms to represent each case we have studied so far: RSRP: Repetition-coding-based Strategic Relaying with Power control, RCRP: Repetition-coding-based Collaborative Relaying with Power control, RRWP: Repetition-coding-based Relaying Without Power control, and NRP: No Relaying with Power control.

In Table I and Table II , cases $\{1,2,3,4,5\}$ demonstrate that when inter-node channel quality is not as good as that of channels between each node and its destination, both RSRP and RCRP approaches are dominated by the NRP approach. Here for a given case i and j, if we have $E_k(i) \leq E_k(j), k = 1,2$, case i is said to dominate over case j. In [10], it is shown that the decision in favor of employing relaying depends only on the averaged SNR of channels between users and their destination when only channel statistic is available. However, it is clearly seen from our results that relaying does not benefit any node when inter-node channel is not strong enough. Consequently, power control without relaying is preferable.

Cases 6 through 9 are for scenarios when inter-node quality is greatly improved compared with cases 1 through 5 and one of the two nodes has weak channel to its destination. Under RCRP, the user having weaker channel to its destination benefits from relaying compared with NRP at the expense of another node spending more than it costs under NRP. For example, in case 6, node 1 experiences bad channel to destination. Joint power control with relaying reduces its energy consumption from 8.88 under NRP to 1.49, while increasing node 2's energy from 6.63 to 10.25.

Energy consumptions for node 2 having bigger load and smaller outage probability requirement than node 1 is significantly larger under RRWP than in other cases. Also, we notice when inter-user channel is relatively weak, RRWP is dominated by both RSRP and RCRP. This illustrates the benefit of power control based on complete knowledge of CSI. In addition, for all cases except cases 3 and 5, RSRP is dominated by RCRP, which implies that collaborative relaying not only minimizes total energy but also benefits all individual nodes.

As a summary, numerical results tabulated above provide overwhelming evidence for taking advantage of CSI, as well as relaying to the largest extent in the sense of performing completely collaborative power control. Therefore in wireless networks, once forwarding and relaying is adopted across various nodes, exchanging of CSI becomes crucial, and collaborative energy minimization rather than the non-cooperative strategic approach should be pursued. Results also suggest that non-cooperative strategic relaying approach is not recommended since it cannot factor in the possibility of asymmetry of channel conditions and data loads. fairness issue in game theory. As future work, we are considering relaying games based on cooperative/bargaining strategies.

Index	$\left(\sigma^2_{1,2}, \sigma^2_{2,1}, \sigma^2_{1,D}, \sigma^2_{2,D}\right)$	RSRP (E_1, E_2)	RCRP (E_1, E_2)
1	$(0.5, 0.5, 0.5, 0.5)$	$(11.03, 20.53)$	$(6.85, 19.07)$
2	$(1, 0.5, 2, 0.5)$	$(1.42, 15.94)$	$(1.19, 15.65)$
3	$(0.5, 0.1, 0.5, 0.5)$	$(4.66, 25.84)$	$(4.77, 24.62)$
4	$(0.1, 0.1, 0.5, 0.5)$	$(7.10, 23.93)$	$(7.10, 23.90)$
5	$(0.1, 0.2, 1, 0.5)$	$(2.06, 22.71)$	$(1.99, 22.86)$
6	$(2, 1, 0.2, 0.9)$	$(23.62, 11.66)$	$(1.49, 10.25)$
7	$(2, 1, 0.9, 0.2)$	$(18.60, 56.90)$	$(14.13, 19.16)$
8	$(1, 2, 0.2, 1.5)$	$(46.11, 4.25)$	$(3.02, 4.04)$
9	$(5, 2, 0.2, 1.5)$	$(39.36, 4.49)$	$(0.24, 3.89)$

TABLE I

Index	$\left(\sigma^2_{1,2}, \sigma^2_{2,1}, \sigma^2_{1,D}, \sigma^2_{2,D}\right)$	RRWP (E_1, E_2)	NRP (E_1, E_2)
1	$(0.5, 0.5, 0.5, 0.5)$	$(15.04, 181.31)$	$(3.55, 11.93)$
2	$(1, 0.5, 2, 0.5)$	$(10.65, 14.92)$	$(0.89, 11.93)$
3	$(0.5, 0.1, 0.5, 0.5)$	$(12.56, 223.52)$	$(3.55, 11.93)$
4	$(0.1, 0.1, 0.5, 0.5)$	$(25.17, 162.68)$	$(3.55, 11.93)$
5	$(0.1, 0.2, 1, 0.5)$	$(17.69, 113.69)$	$(1.78, 11.93)$
6	$(2, 1, 0.2, 0.9)$	$(10.74, 161.42)$	$(8.88, 6.63)$
7	$(2, 1, 0.9, 0.2)$	$(5.88, 533.52)$	$(1.97, 29.83)$
8	$(1, 2, 0.2, 1.5)$	$(16.43, 87.07)$	$(8.88, 3.98)$
9	$(5, 2, 0.2, 1.5)$	$(5.68, 105.73)$	$(8.88, 3.98)$

TABLE II

NUMERICAL RESULTS FOR DIFFERENT TYPES OF POWER CONTROL/RELAYING SCHEMES

REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-part I: system description," *IEEE Trans. Commun*, vol. 51, pp. 1927–1938, Nov. 2003.

[2] T. Hunter and A. Nosratinia, "Coded cooperation under slow fading, fast fading, and power control," in *Proc. Asilomar Conference on Signals, Systems, and Computers*, Nov. 2002.

[3] J. Laneman, D. Tse, and G. Wornel, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[4] J. Laneman and G. Wornell, "Distributed space-time coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inform. Theor*, vol. 49, pp. 2415–2425, Oct. 2003.

[5] D. Gunduz and E. Erkip, "Opportunistic cooperation and power control strategies for delay-limited capacity," in *CISS*, The Johns Hopkins University,Baltimore, March 2005.

[6] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, May 2005.

[7] D. Gunduz and E. Erkip, "Outage minimization by opportunistic cooperation," in *WirelessCom, Symposium on Information Theory*, Maui, Hawaii, June 2005.

[8] M. Chen, S. Serbetli, and A. Yener, "Distributed power allocation for parallel relay networks," in *IEEE Global Telecommunications Conference*, 2005.

[9] T. E. Hunter and A. Nosratinia, "Coded cooperation in multi-user wireless networks," *IEEE Transactions on Wireless Communications*, 2004, submitted.

[10] Z. Lin, E. Erkip, and A. Stefanov, "Cooperative regions and partner choice in coded cooperative systems," *IEEE Transactions on Communications.*, 2006, to appear.

[11] G. Caire, G. Taricco, and E. Biglieri, "Optimum power control over fading channels." *IEEE Transactions on Information Theory*, vol. 45, no. 5, pp. 1468–1489, 1999.

[12] S. Wei and R. Kannan, "Strategic versus collaborative power control in relay fading channels," *Technical Report*, Jan. 2006, Department of ECE, LSU.

[13] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series and Products*, 6th ed. New York: Academic Press, 1980.