Approximation Algorithms for Power-Aware Scheduling of Wireless Sensor Networks with Rate and Duty-Cycle Constraints*

Rajgopal Kannan[1] and Shuangqing Wei[2]

[1] Department of Computer Science
[2] Department of Electrical and Computer Engineering
Louisiana State University, Baton Rouge, LA 70803, USA
{rkanna1, swei}@lsu.edu
www.csc.lsu.edu/∼rkannan, www.ece.lsu.edu/∼swei

Abstract. We develop algorithms for finding the minimum energy transmission schedule for duty-cycle and rate constrained wireless sensor nodes transmitting over an interference channel. Since traditional optimization methods using Lagrange multipliers do not work well and are computationally expensive given the non-convex constraints, we develop fully polynomial approximation schemes (FPAS) for finding optimal schedules by considering restricted versions of the problem using multiple discrete power levels. We first show a simple dynamic programming solution that optimally solves the restricted problem. For two fixed transmit power levels (0 and P), we then develop a 2-factor approximation for finding the optimal fixed transmission power level per time slot, P_{opt}, that generates the optimal (minimum) energy schedule. This can then be used to develop a $(2, 1+\epsilon)$-FPAS that approximates the optimal power consumption and rate constraints to within factors of 2 and arbitrarily small $\epsilon > 0$, respectively. Finally, we develop an algorithm for computing the optimal number of discrete power levels per time slot ($O(1/\epsilon)$), and use this to design a $(1, 1+\epsilon)$-FPAS that consumes less energy than the optimal while violating each rate constraint by at most a $1+\epsilon$ factor.

1 Introduction

Energy-efficiency is a critical concern in many wireless networks, such as cellular networks, ad-hoc networks or wireless sensor networks (WSNs) that consist of large number of sensor nodes equipped with unreplenishable and limited power resources. Since wireless communication accounts for a significant portion of node energy consumption, network lifetime and utility are dependent on the design of energy-efficient communication schemes including low-power signaling and energy-efficient multiple access protocols.

Delay is also an important constraint in many wireless network applications, for example battlefield surveillance or target tracking in which data with finite

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lifetime-information must be delivered before a deadline. Delay constraints in wireless networks can also be examined in terms of node operation under periodic duty cycles, in which time is divided into active (awake) and inactive (asleep) periods. [1], [2, 3] establish the idea of duty cycles in WSNs as a practical means of conserving node energy. Minimizing transmission energy subject to latency constraints has been studied [4, 5]. Several approaches for maximizing information transmission over a shared channel subject to average power constraints have been proposed [6, 7, 8, 9, 10]. [11] addresses the issue of minimizing transmission power, subject to a given amount of information being successfully transmitted and derives power control multiple access (PCMA) algorithms for autonomous channel access.

We consider N sensor nodes transmitting to their destinations over a typical AWGN interference channel over a time period T. These nodes could represent reasonably close neighbors communicating as part of some MAC protocol. We assume that time T is divided into M slots of equal duration. Let P_{it} be the transmit power used by node i during time slot t, $1 \leq t \leq M$. Let R_{it} represent the achievable transmission rate for node i during time slot t over this N-node interference channel. Single user decoding is assumed at each receiver to decode the information from its own transmitter while treating the remaining information as Gaussian interference. Thus we have,

$$R_{it} = \frac{1}{2}\log_2\left(1 + \frac{\alpha_{ii}^t P_{it}}{\mathcal{N}_i^t + \sum_{j\neq i}\alpha_{ji}^t P_{jt}}\right), \quad 1 \leq i \leq N, \quad 1 \leq t \leq M \tag{1}$$

where α_{ji}^t represent the channel attenuation at i's receiver due to transmitter j, which captures the effects of path-loss, shadowing and frequency nonselective fading, and $\mathcal{N}_i^t$ represents the background interference (usually $\mathcal{N}_i^t = \mathcal{N}_0$), during time slot t. We assume these parameters remain fixed over a (short) time slot of duration T/M but can vary from slot to slot.

We are interested in the following scheduling and energy minimization problem (labeled MESP: minimum energy scheduling problem)

$$\begin{aligned} \min f : \sum_{i=1}^{N}\sum_{t=1}^{M} P_{it} \\ \text{s.t} \quad g : \sum_{t=1}^{M} A_{it}R_{it} \geq \tilde{R}_i \quad i = 1, 2, \ldots, N \\ A_{it} = \begin{cases} 0 \text{ if node } i \text{ is idle} \\ 1 \text{ otherwise} \end{cases} \\ \sum_{t=1}^{M} A_{it} \leq \mu_i \quad i = 1, 2, \ldots, N \end{aligned} \tag{2}$$

The objective function in MESP is to determine the schedule which minimizes the total energy. Since all slots are assumed to be of fixed duration, this is equivalent to minimizing the total transmitted power. Each node must maintain an average rate constraint $\tilde{R}_i$ over the M slots. Further, we assume that nodes operate under duty-cycles where time T is divided into active and idle time slots, wireless sensor networks for example, operate under such constraints [2, 1]. The duty-cycle constraint of node i is given by μ_i: the maximum number of time slots it can remain active, $1 \leq \mu_i \leq M$, $i = 1, 2, \ldots N$. $A_{it} \in \{0, 1\}$ depending on whether the node is idle or active during slot t, $1 \leq t \leq M$. Note that in this formulation of MESP, we do not have any overall power budget constraint (only duty-cycle constraints for limiting node activity) and we are looking to minimize the total power/energy over the universe of available power values. Individual/overall power budget constraints can be incorporated in our algorithm, if desired.

It can be seen that the rate constraints above are non-convex in the power variables P_{it}, even for the restricted version of MESP with two users ($N = 2$). Unfortunately this implies that traditional analytical optimization methods such as Lagrange multipliers [12] will not work well, since convexity of the constraints is a necessary condition for obtaining the global minimum using the Lagrangean $H = f + \lambda_k g_k$ (where g_k are the constraints), and computing $\bigtriangledown_{P_{it}, \lambda_k} = 0$. Moreover finding the global minimum through exhaustive search of all possible solutions of $\partial h / \partial P_{it} = 0$ is likely to be computationally expensive. Alternately computing the optimal dual $\max_\lambda \min_x h()$ introduces a duality gap which vanishes only under certain conditions on the number of constraints and parameters N and M [12, 13].

In this paper, we develop approximation algorithms for finding the optimal rate and duty-cycle constrained energy schedule by considering restricted versions of the problem using discrete power levels. From the algorithmic perspective, the MESP problem is NP-hard and related to the generalized assignment problem [14]. We develop fully polynomial approximation schemes (FPAS) for MESP using ideas related to bin-packing and the knapsack problem [14, 15]. We first show a simple dynamic programming solution (of exponential complexity in M) that optimally solves the restricted problem. For two fixed transmit power levels (0 and P), we then develop a 2-factor approximation for finding the optimal fixed transmit power level per time slot, P_{opt}, that generates the optimal (minimum) energy schedule. This can then be used to develop a $(2, 1+\epsilon)$-FPAS that approximates the optimal power consumption and rate constraints to within factors of 2 and arbitrarily small $\epsilon > 0$, respectively. Finally, we develop an algorithm for computing the optimal number of discrete power levels per time slot ($O(1/\epsilon)$), and use this to design a $(1, 1+\epsilon)$-FPAS that consumes less energy than the optimal while violating each rate constraint by at most a $1+\epsilon$ factor.

2 Basic Dynamic Programming Solution

First, we consider a simple relaxation of the minimum energy scheduling problem using two discrete transmit power levels. In this restricted version of the problem, a node is allowed to be either idle or transmit with a given (fixed) power P during its active slot. We illustrate our schemes using two nodes ($N = 2$) over M time slots. As mentioned above, even the restricted two node case is not amenable to traditional optimization methods. Later in section 6, we extend the dynamic program and approximations are extended to the N-node, M time slot case.

The restricted optimization problem is described by:

$$\min \sum_{i=1}^{2} \sum_{t=1}^{M} P_{it}$$

$$\begin{aligned} \text{s.t } \sum_{t=1}^{M} R_{it} &\geq \tilde{R}_i, \quad i = 1, 2 \\ P_{it} \in \{0, P\}, \quad i = 1, 2; \quad t &= 1, \ldots, M \\ A_{it} &= \begin{cases} 0 \text{ if } P_{it} = 0 \\ 1 \text{ otherwise} \end{cases} \\ \sum_{t=1}^{M} A_{it} &\leq \mu_i, \quad i = 1, 2 \end{aligned} \tag{3}$$

(4)

We assume that $\mu_1 + \mu_2 \geq M$, i.e the two nodes have to interleave during some of the slots. A more restricted version of 4 with $\alpha^t_{ji} = \alpha_{ji}$ independent of t is analyzed in [16].

Let $\bar{R}^{kP,a,b}_{i,j} = \{<R_1, R_2>\}$ represent the set of rate vector (rate pairs) corresponding to cumulative transmission rates for user 1 and user 2 from time slots i through j, $1 \leq i \leq j \leq M$, while using a total power (node 1 + node 2) of kP. For notational simplicity, if $i = j$, we drop one of the redundant subscripts in the rate vector. In the above definition, $R_l = \sum_{t=i}^{j} R_{lt}$, where R_{lt}, $l = 1, 2$, is the achievable rate for node l during time slot t, depending on the actions of the other node i.e active/asleep. The number of active slots for user 1 and 2 in this period is denoted by a and b, respectively, where $0 \leq a, b \leq j-i+1$. Since a node uses fixed power P during an active slot, $a+b = k$, in this case. Thus for a given time slot t, we have four different rate vectors specified by,

$$\begin{aligned} \bar{R}^{0,0,0}_t &= <0, 0> \\ \bar{R}^{P,0,1}_t &= <0, \frac{1}{2}\log_2\left(1 + \alpha^t_{22}P/\mathcal{N}^t_2\right)> \\ \bar{R}^{P,1,0}_t &= <\frac{1}{2}\log_2\left(1 + \alpha^t_{11}P/\mathcal{N}^t_1\right), 0> \\ \bar{R}^{2P,1,1}_t &= <\frac{1}{2}\log_2\left(1 + \frac{\alpha^t_{11}P}{\mathcal{N}^t_1 + \alpha^t_{21}P}\right), \frac{1}{2}\log_2\left(1 + \frac{\alpha^t_{22}P}{\mathcal{N}^t_2 + \alpha^t_{12}P}\right)> \end{aligned} \tag{5}$$

The restricted version of the problem consists of finding a transmission schedule of minimum total energy in which active nodes transmit at a fixed power during each active time slot while also satisfying the given duty-cycle and rate constraints. For fixed power level P, the optimal schedule is easily specified by the following dynamic program which maintains the current best-solution for each total power level and duty-cycle value. The boundary conditions are given by the rate vectors in Eq. 5. The recursive formula for each power level kP and duty-cycles a, b, $1 \leq k \leq 2M$, $0 \leq a \leq \mu_1$, $0 \leq b \leq \mu_2$ is

$$\bar{R}_{i,j}^{kP,a,b} = \text{vectormax}\Big\{ \bar{R}_{i,j-1}^{kP,a,b} \bigcup \Big(\bar{R}_{i,j-1}^{(k-1)P,a-1,b} + \bar{R}_{j}^{P,1,0} \Big) \bigcup \Big(\bar{R}_{i,j-1}^{(k-1)P,a,b-1} + \bar{R}_{j}^{P,0,1} \Big) \bigcup \Big(\bar{R}_{i,j-1}^{(k-2)P,a-1,b-1} + \bar{R}_{j}^{2P,1,1} \Big) \Big\} \tag{6}$$

where the rate vectors in each union operation above are computed using pairwise addition of the individual vectors. The vectormax operation eliminates all dominated rate pairs from a set of rate pairs, i.e. $\forall \{ <R_1, R_2>, <R_3, R_4> \} \in \bar{R}_{i,j}^{kP,a,b}$ either $R_1 > R_3$ and $R_2 \leq R_4$ or vice versa. Using the recursive function, the table of rate vector values is evaluated in increasing order of time slots from 1 to M. There are $O(MP\mu_1\mu_2)$ rate vectors and the set of feasible schedules correspond to those rate vectors $\geq \ <\tilde{R}_1, \tilde{R}_2>$ under the usual meaning of vector comparison. The optimal schedule for a given transmit power level P is the one whose rate vector satisfies

$$\bar{R}_{opt}^{P} = \underset{k=1,2\ldots,2M}{\text{argmin}} \Big\{ \exists \ <R_1, R_2> \in \ \bar{R}_{1,M}^{kP,\mu_1,\mu_2} \ | \ <R_1, R_2> \ \geq \ <\tilde{R}_1, \tilde{R}_2> \Big\} \tag{7}$$

In practice, it is likely that many of the vectors in $\bar{R}_{i,j}^{kP,a,b}$ would be dominated and hence eliminated by the vectormax operation. However in the worst-case, even after the vectormax operation, the size of $\bar{R}_{i,j}^{kP,a,b}$ can quadruple with each additional slot. Thus the above dynamic program is clearly exponential in terms of the slot parameter M, even though each slot contains only four rate vectors. This motivates us to consider a $(1+\epsilon, 1+\epsilon)$ FPAS for the problem, as described in Section 5.

3 2-Approximate Minimum Energy Schedule

Let $\mathcal{A}^P$ denote the (exponential time) dynamic programming algorithm for finding the optimal schedule under duty-cycle constraints and using only two fixed transmit power levels of 0 or P per slot. We note it is possible under $\mathcal{A}^P$ that $\forall k, \ \ \bar{R}_{1,M}^{kP,\mu_1,\mu_2} \ < \ <\tilde{R}_1, \tilde{R}_2>$. Thus $\bar{R}_{opt}^{P} = \phi$ and no feasible schedule exists for the given transmit power value P. In this case, we wish to find the optimal feasible transmit power $P = P_{opt}$ for which a feasible schedule exists under $\mathcal{A}^P$ and that uses minimum possible energy $E_{\mathcal{A}}^{P_{opt}}$ among all such feasible powers. In this section, we describe a 2-approximation for finding $E_{\mathcal{A}}^{P_{opt}}$. Subsequently (in Section 5), we develop an FPAS using $O(1/\epsilon)$ power levels, that approximates P_{opt} and the corresponding minimal energy schedule to within an ϵ-factor.

Let $E_{\mathcal{A}}^{P}$ denote the total energy of the schedule produced by $\mathcal{A}^{P}$. Let P_a and P_b, where $P_a > P_b$, represent two different transmit power levels. Consider two instances of the scheduling problem. In the first instance, each node can either transmit at power P_a or be idle during each slot. Likewise, with power P_b in the second instance.

Claim. For each $<R_1, R_2> \in \bar{R}_{i,j}^{kP_b,a,b}$ *there is a rate pair* $<R_3, R_4> \in \bar{R}_{i,j}^{kP_a,a,b}$ *such that* $<R_1, R_2> < <R_3, R_4>$*.*

Proof. From Eq. 5 it can be seen that for any slot t, we have $\bar{R}_t^{kP_a,a,b} > \bar{R}_t^{kP_b,a,b}, k = 1, 2, a = 0, 1, b = 0, 1$. The proof follows in a straightforward manner by induction.

Let P_{min} be the minimum (fixed) transmit power level per active slot for which a feasible schedule exists. Without loss of generality, we assume $P_{min} \geq 1$.

Theorem 1. $\lceil P_{min} \rceil$ *can be found in* $O(\lceil \log_2 P_{min} \rceil)$ *calls to the dynamic programming algorithm* $\mathcal{A}^{P}$*.*

Proof. Initialize $P = 1$[1]. While $\bar{R}_{opt}^{P} = \phi$, set $P = 2P$ and run algorithm $\mathcal{A}^{P}$. By Claim 1, the values of the rate vectors increase with P and hence the process will terminate with $\bar{R}_{opt}^{P} \neq \phi$. Let P_m be the terminating value of P which is found in $\lceil \log_2 P_{min} \rceil$ calls. $\lceil P_{min} \rceil$ can then be obtained through binary search in the interval $[P_m/2, P_m]$ with $O(\log_2(P_m/2))$ further calls to $\mathcal{A}^{P}$.

Note that Claim 1 for rate vectors cannot be translated to total energy values i.e $P_a > P_b$ does not imply $E_{\mathcal{A}}^{P_a} > E_{\mathcal{A}}^{P_b}$. $E_{\mathcal{A}}^{P}$ is not convex and can have multiple local minima for $P_a > P_{min}$. Thus to obtain a 2-approximation of the global minimum energy schedule, we first need to restrict the space of feasible transmit powers by finding an upper bound P_{max} such that $E_{\mathcal{A}}^{P_{opt}} < E_{\mathcal{A}}^{P}$ for all $P > P_{max}$.

A simple upper bound is $P_{max} = \left(\frac{\mu_1+\mu_2}{2}\right) P_{min} \leq MP_{min}$. Note that $E_{\mathcal{A}}^{P_{opt}} \leq E_{\mathcal{A}}^{P_{min}} \leq P_{min}(\mu_1+\mu_2)$. Since each node is active during at least one slot, $E_{\mathcal{A}}^{P} > E_{\mathcal{A}}^{P_{min}}$ for all $P > P_{max}$. Further, since $P_{opt} \in [P_{min}, P_{max}]$, we note that P_{opt} can be found by searching in an interval of size bounded by $O(MP_{min})$.

We can obtain a smaller bound on P_{max} (and hence the search space for P_{opt}) by using the following lemma: Let S_1^P, S_2^P and S_3^P be the set of time slots occupied by node 1 only, node 2 only and both nodes, under the schedule created by $\mathcal{A}^{P}$. Let $R_{i,S_i^P}^{P}$ denote the total rate obtained by node i over S_i^P, $i = 1, 2$. Let $S_{i,s}^P \subset S_i^P$ represent the set of $\lfloor |S_i^P|/2 \rfloor$ time slots with the *smallest* rates $\log_2(1 + \alpha_{ii}^t P/\mathcal{N}_i^t)/2$ among the slots in S_i^P. Similarly, let $S_{3,s}^P(i) \subset S_3^P$ denote the set of $\lfloor |S_3^P|/4 \rfloor$ slots with the smallest rates calculated as $\log_2(1+\alpha_{ii}^t P/\mathcal{N}_i^t)/2$ among the slots in S_3^P and let $R_{i,S_{3,s}^P(i)}^{P}$ denote the corresponding total rate over these slots. A sufficient condition for finding P_{max} is then given by:

[1] Note that a better initial value can be obtained by using $P = \min(P_1', P_2')/M$ from Eq 10 in the next section.

Lemma 1. *$P \le P_{max} < 2P$ if $S_3^P \cap S_3^{2P} \ne \emptyset$, $R^P_{i,S^P_{3,s}(i)} \ge (\lceil |S_3^P|/4\rceil)/2$ and $R_{i,S^P_{i,s}} \ge (\lceil |S_i^P|/2\rceil)/2$, $i = 1, 2$.*

For a detailed proof, please refer to [17]. The last rate condition of the lemma is derived from the fact that doubling the power over any set S of solo slots can increase the achieved rate by less than $|S|/2$. Thus if the worst half-set of slots ($S^P_{i,s}$) has a total rate at least $|S_i^P|/4$, $i = 1, 2$, then doubling the power over the best half-set of slots (thereby expending the same energy) cannot achieve the same rate as before. The second rate condition is derived using the fact that doubling the power still leads to overlapping slots. The first condition states that if overlapping slots persist even after doubling the transmit power, and simultaneously the second rate condition is also satisfied with respect to the worst $S^P_{3,s}(i)/4$ slots (pretending that each node i is transmitting without interference from the other in these slots), then no amount of further increases in transmit power can decrease the overall energy. Thus $P_{max} < 2P$.

We use the above bound on P_{max} to obtain a 2-approximation for $E_{\mathcal{A}}^{P_{opt}}$, the energy of the optimal (minimum energy) schedule as follows:

Theorem 2. *Let*

$$P^* = \operatorname*{argmin}_{P=2^t P_{min},\quad t=0,1\ldots,\lceil \log_2 \frac{P_{max}}{P_{min}}\rceil} E^P_{\mathcal{A}}.$$

Then $E_{\mathcal{A}}^{P^*}$ is a 2-approximation to $E_{\mathcal{A}}^{P_{opt}}$, the minimum energy schedule generated by the optimal transmit power P_{opt}. The algorithm for finding $E_{\mathcal{A}}^{P^*}$ uses $\lceil \log_2 \frac{P_{max}}{P_{min}}\rceil = o(\log_2 M)$ calls to $\mathcal{A}^P$.

Proof. We run the $\mathcal{A}^P$ algorithm starting with $P = P_{min}$ and doubling P with each iteration until we reach a P_{max} as defined by lemma 1. The total energy can oscillate between $E_{\mathcal{A}}^{P_min}$ and $E_{\mathcal{A}}^{P_max}$ as we sequentially double the power. For any solution using power P_a, $P < P_a < 2P$, the number of active slots $t_{P_a} = |S_1^{P_a}| + |S_2^{P_a}| + 2|S_3^{P_a}|$ cannot increase between t_P and t_{2P} i.e $t_P \ge t_{P_a} \ge t_{2P}$ (using claim 1). Thus $E_{\mathcal{A}}^{P_a} \ge (1/2) \min E_{\mathcal{A}}^P, E_{\mathcal{A}}^{2P}$. Let P^* be the power yielding the minimum energy among the iterations and choose $E_{\mathcal{A}}^{P^*}$ as the output of our algorithm. By the above arguments, $E_{\mathcal{A}}^{P^*} \le 2E_{\mathcal{A}}^{P_{opt}}$ and therefore this algorithm is a 2-approximation. Since $P_{max} = o(MP_{min}$, the number of iterations is $o(\log_2 M)$.

4 Minimum Energy Schedule with Multiple Power Levels

We now consider the scheduling problem with multiple discretized power levels, where each node can choose from a set of power levels per time slot. As shown below, if the power levels are chosen appropriately, the cost of the resulting minimum energy schedule approximates the cost of the optimal schedule to within an ϵ-factor.

For the optimization problem with multiple power levels, let P and L_t denote the maximum allowable transmit power and the number of discrete power levels available per time slot, respectively, with values as defined below. For this problem, the constraint 3 of Eq. 4 is replaced with

$$P_{it} \in \{P_l\},\ l = 0, 1, \ldots L_t;\ \ 0 = P_0 \leq P_l \leq P_{L_t} = P;\ \ i = 1, 2;\ \ t = 1, \ldots, M. \quad (8)$$

Note that the corresponding optimal version of the minimum energy scheduling problem contains the constraint

$$0 \leq P_{it} \leq P, \qquad i = 1, 2;\quad t = 1, \ldots, M \quad (9)$$

Let $\mathcal{A}^{P^*}$ denote the optimal algorithm for the above restricted version of MESP with per slot maximum power constraints (Eq. 9), i.e nodes select an optimal power value $0 \leq P^*_{it} \leq P$ in each slot, to satisfy their rate and duty-cycle constraints. Let R^*_{it} denote the corresponding optimal rate achieved per time slot, $i = 1, 2$, $t = 1, 2, \ldots M$. Finally, let $P^* = \sum\sum P^*_{it}$ and $R^*_i = \sum_t R^*_{it}$ denote the overall optimal power and rate allocations. In general, an (α, β) approximation of the optimal minimum energy scheduling problem is one which provides a feasible schedule with total power $\hat{P} \leq \alpha P^*$ and each rate constraint violated by at most a β-factor i.e $\beta\hat{R}_i \geq R^*_i$, for each node i. Note that $R^*_i \geq \tilde{R}_i$ and hence $\beta\hat{R}_i \geq \tilde{R}_i$. Given some $\epsilon > 0$, we first show the construction of a more computationally expensive $(1+\epsilon, 1+\epsilon)$-approximation in order to illustrate our approach and then describe a more efficient $(1, 1+\epsilon)$-approximation to the optimal.

Let $P' = P'_1 + P'_2$, where P'_i is the solution to the problem

$$\begin{aligned} \min P'_i &= \sum_{t=1}^{M} P_{it}, \quad i = 1, 2 \\ \text{s.t } \sum_{t=1}^{M} \frac{1}{2} \log_2\left(1 + \frac{\alpha^t_{ii} P_{it}}{\mathcal{N}^t_i}\right) &\geq \tilde{R}_i, \quad i = 1, 2 \\ P_{it} \geq 0 \ \ i &= 1, 2; t = 1, .., M \\ \sum_{t=1}^{M} A_{it} &\leq \mu_i, \quad i = 1, 2 \\ A_{it} &= \begin{cases} 0 \text{ if } P_{it} = 0 \\ 1 \text{ otherwise} \end{cases} \end{aligned} \quad (10)$$

P'_j is the solution to the problem of zero-interference scheduling of node j with variable (non-discrete) power levels and can be found using standard Lagrange multiplier techniques [12]. Thus P' is a lower bound for the minimum energy scheduling problem using discrete power levels. Now define

$q = \min_{i,t} \left\{ \frac{P'}{M}, \frac{\alpha_{ii}^t}{\alpha_{ji}^t} \left(2^{\epsilon \tilde{R}_i/M} - 1 \right) \right\}$, $i, j = 1, 2$, $1 \le t \le M$. Let k be the largest solution to the equation $kq = 2 \ln kP$ such that

$$e/P < k \le \frac{2(2^{\epsilon \tilde{R}_i/M} - 1)}{q \left(1 + \epsilon - 2^{\epsilon \tilde{R}_i/M}\right)} \tag{11}$$

else set $k = 0$. For the given $\epsilon > 0$, choose $\delta_1 = \frac{\epsilon q}{2+kq}$. If $k = 0$, let $r_0 = \lfloor \frac{2P}{q\epsilon} \rfloor$, otherwise $r_0 = \lceil \frac{2+kq)}{\epsilon kq} \rceil$. Let $s_0 = \lfloor \ln_{1+k\delta_1} P/r_0\delta_1 \rfloor$.

Allocate power to nodes in each time slot by dividing the total available power P into the following $L_t = r_0 + s_0 + 2$ discrete power levels.

$$\begin{aligned} P_r &= \begin{cases} r\delta_1, & 0 \le r \le r_0 \\ (1 + k\delta_1)^{r-r_0} P_{r_0}, & r_0 + 1 \le r \le r_0 + s_0 \end{cases} \\ P_{r_0+s_0+1} &= P \end{aligned} \tag{12}$$

Lemma 2. *For given max power level P and constraints $\tilde{R}_i$, the number of discrete power levels per slot L_t is $O(\frac{1}{q\epsilon})$.*

Proof. Note that we are allocating power levels by dividing the range of available power into two types of intervals: the first r_0 intervals of fixed size δ_1 and remaining intervals of geometrically increasing size. Since geometric intervals are small in the beginning, the total number of power levels would be much larger using only geometrically increasing intervals. Therefore we use intervals of fixed size initially and choose integer r_0 such that the size of the first geometric interval, $k\delta_1^2 r_0$ is the same as the size of the previous fixed interval δ_1. The overall objective is to find optimal values of k and δ_1 that minimize the total number of power levels, yet allow us to closely approximate the overall energy consumption and rate constraints. From the energy approximation requirements (as shown below), we will get the constraint $\delta_1 = q\epsilon/(2 + kq)$. Hence $k\delta_1 < \epsilon$ and thus for small ϵ, the total number of levels $L_t = r_0 + s_0 = 1/(k\delta_1) + \ln_{1+k\delta_1} kP$ can be approximated by $\frac{1+\ln kP}{k\delta_1} = (1/\epsilon)(1 + \ln kP)(1 + 2/(kq))$. Thus the objective is to find k that minimizes L_t. The solution to this minimization is $\ln kP = kq/2$ subject to $\ln kP > 1$. If k does not satisfy these conditions then $\delta_1 = q\epsilon/2$ and the number of power levels is $\lceil \frac{2P}{q\epsilon} \rceil$.

The remaining constraints on k as specified in Eq. 11, are obtained from the rate approximation requirements shown below.

Theorem 3. *For small $\epsilon > 0$, let $\mathcal{A}^{\hat{P}}$ denote the modified version of the (exponential) dynamic programming algorithm $\mathcal{A}^P$ in which each node can select from discrete power levels per time slot as specified by Eq. 12, subject to overall duty-cycle and rate constraints $\tilde{R}_i(1-\epsilon)$. Then $\mathcal{A}^{\hat{P}}$ is a $(1 + \epsilon, 1 + \epsilon)$-approximation of $\mathcal{A}^{P^*}$.*

Proof. Divide the set of time slots $T = \{1, 2, \ldots, M\}$ into disjoint sets T_{11} and T_{12} (resp. T_{21} and T_{22}) such that

$$\begin{aligned} t \in T_{11}(\text{resp. } T_{21}) \text{ if } P_{1t}^*(\text{resp. } P_{2t}^*) \in [0, r_0\delta_1] \\ t \in T_{12}(\text{resp. } T_{22}) \text{ if } P_{1t}^*(\text{resp. } P_{2t}^*) \in (r_0\delta_1, P] \end{aligned} \tag{13}$$

Let $\hat{P}_{it}$ and $\hat{R}_{it}$ denote the (discrete) power levels and rate allocations per node per time slot under $\mathcal{A}^{\hat{P}}$. Since $\mathcal{A}^{\hat{P}}$ considers combinations of power levels over M slots, the errors in power levels and rate allocations per slot (either absolute or relative) must be bounded from above. Consider the solution in $\mathcal{A}^{\hat{P}}$ that simply rounds up the optimal power level in each slot to the nearest (larger) discrete power level. For this solution, the absolute error is bounded by $\hat{P}_{it} - P_{it}^* < \delta_1, \quad t \in T_{i1}$, and the relative error by $\hat{P}_{it} < (1 + k\delta_1)P_{it}^*, \quad t \in T_{i2}$, $i = 1, 2$. Therefore we have

$$\begin{aligned} \hat{P} &= \sum_i \sum_{t \in T_{i1}} \hat{P}_{it} + \sum_i \sum_{t \in T_{i2}} \hat{P}_{it} \\ &\leq P^* + \frac{q\epsilon\left(|T_{11}| + |T_{21}|\right)}{2 + kq} + \frac{kq\epsilon}{2 + kq} \sum_i \sum_{t \in T_{i2}} P_{it}^* \\ &\leq P^* + \frac{2Mq\epsilon}{2 + kq} + \frac{\epsilon kq}{2 + kq} P^* \end{aligned} \tag{14}$$

The overall relative error in energy P_{err}, of this solution $\hat{P}$ is defined as

$$P_{err} = \frac{\hat{P} - P^*}{P^*} \tag{15}$$

Therefore we can bound the relative error as

$$P_{err} = \frac{2\epsilon}{2 + kq} \cdot \frac{Mq}{P^*} + \frac{\epsilon kq}{kq + 2} \leq \epsilon \tag{16}$$

since $q \leq P'/M \leq P^*/M$ as P' is a lower bound for the optimal energy value P^*. Hence this particular solution of algorithm $\mathcal{A}^{\hat{P}}$ approximates the optimal energy value of the minimum energy schedule to within an ϵ factor.

To complete the proof, we just need to show that the above power allocation is also a feasible solution in terms of the rate constraints i.e the overall rates achieved by $\mathcal{A}^{\hat{P}}$ also approximate each rate constraint to within an ϵ factor. First consider the achieved rate $\hat{R}_{1t}$, for the case $t \in T_{21}$.

$$\begin{aligned} \hat{R}_{1t} &\geq \frac{1}{2} \log_2 \left(1 + \frac{\alpha_{11}^t P_{1t}^*}{\mathcal{N}_1^t + \alpha_{21}^t (P_{2t}^* + \delta_1)}\right) \\ &\geq \frac{1}{2} \log_2 \left(1 + \frac{\alpha_{11}^t P_{1t}^*}{\mathcal{N}_1^t + \alpha_{21}^t P_{2t}^*} \cdot \frac{1}{1 + \frac{\alpha_{21}^t \delta_1}{\mathcal{N}_1^t + \alpha_{21}^t P_{2t}^*}}\right) \end{aligned}$$

$$\geq R^*_{1t} - \frac{1}{2}\log_2\left(1 + \frac{\delta_1}{P^*_{2t} + \frac{\mathcal{N}^t_1}{\alpha^t_{11}} \cdot \frac{\alpha^t_{11}}{\alpha^t_{21}}}\alpha^t_{21})\right) \tag{17}$$

Using the fact that $P^*_{2t} \geq 0$, and the background noise $\mathcal{N}^t_1/\alpha^t_{11} \geq 1$ for each time slot $t \in T_{11}$, we can bound the absolute R_1 rate error $= R^*_1 - \hat{R}_1$ over all such time slots by

$$\frac{M}{2}\log_2\left(1 + \max_t\left(\frac{\alpha^t_{21}}{\alpha^t_{11}}\right)\delta_1\right) \leq \frac{\epsilon\tilde{R}_1}{2}$$

by using the fact that $\delta_1 \leq \epsilon q \leq \min_t\left(\frac{\alpha^t_{11}}{\alpha^t_{21}}\right)\epsilon\left(2^{2\epsilon\tilde{R}_1/M} - 1\right)$.

Next, for $t \in T_{22}$ (when $k > 0$), we get

$$\begin{aligned}\hat{R}_{1t} &= \frac{1}{2}\log_2\left(1 + \frac{\alpha^t_{11}\hat{P}_{1t}}{\mathcal{N}^t_1 + \alpha^t_{21}\hat{P}_{2t}}\right)\\ &\geq \frac{1}{2}\log_2\left(1 + \frac{\alpha^t_{11}P^*_{1t}}{\mathcal{N}^t_1 + \alpha^t_{21}P^*_{2t}(1+k\delta_1)}\right)\\ &\geq \frac{1}{2}\log_2\left(1 + \frac{1}{1+k\delta_1} \cdot \frac{\alpha^t_{11}P^*_{1t}}{\frac{\mathcal{N}^t_1}{1+k\delta_1} + \alpha^t_{21}P^*_{2t}}\right)\end{aligned}$$

Since $k\delta_1 \geq 0$, this implies

$$\begin{aligned}\hat{R}_{1t} &\geq \frac{1}{2}\log_2\left(1 + \frac{\alpha^t_{11}P^*_{1t}}{\mathcal{N}^t_1 + \alpha^t_{21}P^*_{2t}}\right) - \frac{1}{2}\log_2(1+k\delta_1)\\ &= R^*_{1t} - \frac{1}{2}\log_2(1+k\delta_1)\end{aligned} \tag{18}$$

Hence the total error in R_1 over all the time slots when $t \in T_{22}$ is at most $(M/2)\log_2(1+k\delta_1) \leq \epsilon\tilde{R}_1/2$ using the upper bound on k as specified in Eq. 11. Combining the two cases, the total absolute error in $R_1 = \tilde{R}_1 - \hat{R}_1 \leq \epsilon\tilde{R}_1$ and thus the relative error in R_1 is bounded by ϵ i.e $\hat{R}_1 \geq \tilde{R}_1(1-\epsilon)$. The analysis is identical for rate R_2. Since algorithm $\mathcal{A}^{\hat{P}}$ uses $\tilde{R}_i(1-\epsilon)$ as the rate constraint for user i, therefore the choice of power levels described above is a feasible choice and hence the algorithm is a $(1+\epsilon, 1+\epsilon)$ approximation.

For the algorithm above, note that the number of discrete power levels per slot L_t, is a function of the channel quality parameters $\alpha^t_{ji}/\alpha^t_{ii}$. While the α's are exponentially distributed random variables with typically small means [18], the ratios can still be quite large, thereby increasing the number of levels. Therefore we consider a more optimal scheme where the rate and energy approximations are obtained independent of channel quality parameters.

Let $\tilde{R}_m = \min(\tilde{R}_1, \tilde{R}_2)$ and $k_1 = (M\log_2(1+P) - 2\tilde{R}_m)/\log_2\left(\frac{1+P}{1+1/k}\right)$. Define $\delta_1 > 0$ and $k > 0$ as the solutions to

$$\min \frac{1}{k\delta_1} + \ln_{1+k\delta_1} kP$$

$$\text{s.t } k_1\delta_1 + M\log_2(1+k\delta_1) = 2\epsilon\tilde{R}_m$$
$$k > \frac{1}{2^{2\tilde{R}_m/M}-1} \tag{19}$$

δ_1 and k can be obtained using standard constrained minimization techniques such as Lagrange multipliers [12]. However if no solution exists above, then δ_1 and k are the solutions obtained by replacing the constraints in Eq. 19 above by the constraint

$$\delta_1 + \log_2(1+k\delta_1) = \frac{2\epsilon\tilde{R}_m}{M} \tag{20}$$

If no solution still exists, then $\delta_1 = \epsilon\tilde{R}_m/M$ and $k = (2^{\epsilon\tilde{R}_m/M}-1)/\delta_1$. Now divide the available power per time slot into discrete power levels as specified by Eq. 12 using the δ_1 and k values above.

Theorem 4. *For $\epsilon > 0$, let $\mathcal{A}^{\overline{P}}$ denote the (exponential) dynamic programming algorithm for finding a minimal energy schedule using the discrete power levels defined above, subject to overall duty-cycle and rate constraints $\tilde{R}_i(1-\epsilon)$. Then $\mathcal{A}^{\overline{P}}$ is a $(1, 1+\epsilon)$-approximation of $\mathcal{A}^{P^*}$.*

Proof. For each slot t, round down the optimal power level choice P^*_{it} to the nearest discrete power level, represented by $\overline{P}_{it}$ and let $\overline{R}_{it}$ denote the corresponding achieved rate per slot. As before, divide the M time slots into sets T_{ij}, $i,j = 1,2$, based on the value of P^*_{it}. We show below that $\overline{P}_{it}$ represents a feasible allocation of power levels under the rate constraints $\tilde{R}_i/(1-\epsilon)$. Hence $\mathcal{A}^{\overline{P}}$ is a $(1, 1+\epsilon)$-approximation since the total energy consumption of $\mathcal{A}^{\overline{P}}$ is at most $\sum\sum\overline{P}_{it} \leq \sum\sum P^*_{it}$.

First, for $t \in T_{12}$, using $\overline{P}_{1t} \geq P^*_{1t}/(1+k\delta_1)$ and $\overline{P}_{2t} \leq P^*_{2t}$, we get

$$\begin{aligned}\overline{R}_{1t} &\geq \frac{1}{2}\log_2\left(1 + \frac{\alpha^t_{11}P^*_{1t}}{(1+k\delta_1)(\mathcal{N}^t_1 + \alpha^t_{21}\overline{P}_{2t})}\right)\\ &\geq R^*_{1t} - \frac{1}{2}\log_2(1+k\delta_1)\end{aligned} \tag{21}$$

Thus the absolute error in R_{1t} per time slot for this case is $\leq \frac{1}{2}\log_2(1+k\delta_1)$.

Next, for $t \in T_{11}$, define the total interference, $\overline{I}_{1t} = (\mathcal{N}^t_1 + \alpha^t_{21}\overline{P}_{2t})/\alpha^t_{11}$, and likewise I^*_{1t}, where $I^*_{1t} \geq \overline{I}_{1t} \geq 1$ (minimum total interference ≥ 1). Therefore we have,

$$R^*_{1t} - \overline{R}_{1t} \leq \frac{1}{2}\log_2\left(1 + \frac{P^*_{1t}}{\overline{I}_{1t}}\right) - \frac{1}{2}\log_2\left(1 + \frac{\overline{P}_{1t}}{\overline{I}_{1t}}\right)$$

Using the fact that $\ln x - \ln y < x - y$ for $x > y > 1$, we get $R^*_{1t} - \overline{R}_{1t} < (P^*_{1t} - \overline{P}_{1t})/2 \leq \delta_1/2$. Thus the absolute error in R_{1t} per time slot for this case is $\leq \delta_1/2$.

Combining the two cases, we can bound the overall rate error over M time slots as

$$T_{err} = \frac{|T_{11}|\delta_1}{2} + \frac{|T_{12}|\log_2(1 + k\delta_1)}{2} \tag{22}$$

For $\mathcal{A}^{\overline{P}}$ to be a $(1, 1 + \epsilon)$ algorithm, we must have $T_{err} \leq \epsilon\tilde{R}_1$. To finish the proof, note that the maximum R_1 rate we can obtain under this algorithm in any $t \in T_{12}$ is $\frac{1}{2}\log_2(1+P)$ and $\frac{1}{2}\log_2(1+r_0\delta_1) = \frac{1}{2}\log_2(1+1/k)$ in any $t \in T_{11}$. The maximum value of $|T_{12}|$ is M. (Clearly $\log_2(1+P)$ should be $\geq 2\tilde{R}_1(1-\epsilon)/M$, otherwise $\mathcal{A}^{\overline{P}}$ does not have a solution). However the maximum value of $|T_{11}|$ is $|T_{11}| \leq (M\log_2(1 + P) - 2\tilde{R}_1)/\log_2\left(\frac{1+P}{1+1/k}\right)$ if $\log_2(1 + 1/k) < 2\tilde{R}_1/M$ else $|T_{11}| \leq M$. When $|T_{11}|$ takes the first value, the total number of power levels per slot is minimized by choosing δ_1 and k as in Eq. 19, whereas in the second case it is minimized by Eq. 20. If both cases do not yield a solution then we set the two error components $\delta_1 = \log_2(1 + k\delta_1) = \epsilon\tilde{R}_m/M$ which makes the relative error over M slots $\leq \epsilon$ as desired.

Finally, we note that the worst-case values of k and $k\delta_1$ are $O(\epsilon\tilde{R}_m/M)$ and therefore

Theorem 5. *Given rate constraints $\tilde{R}_i$ and max power P, the number of discrete power levels per slot is $O(\frac{1}{\epsilon})$.*

Note that the time complexity of $\mathcal{A}^{\overline{P}}$ is still exponential. Using the fact that the number of power levels per slot required to closely approximate rate and energy constraints is $O(\frac{1}{\epsilon})$, we develop an FPAS in the next Section.

5 An FPAS for Rate Constraints

We now describe a simple Fully Polynomial Approximation Scheme that solves the minimum energy scheduling problem by using a β-relaxation on the rate constraints for some arbitrary constant $\beta > 0$. For clarity, we describe the FPAS using two power levels 0 and P per time slot. The algorithm for the multiple power level case is a simple extension as described later.

The FPAS solves the same restricted problem of Eq. 4 with only each rate constraint replaced by

$$\sum_{t=1}^{M} R_{it} \geq (1 - \beta)\tilde{R}_i \quad i = 1, 2 \tag{23}$$

For any $\delta > 0$, define the following

Definition 1. *A rate vector $<R_1, R_2>$ δ-dominates another vector $<R_3, R_4>$ iff either $R_3(1-\delta) \leq R_1 \leq R_3$ and $R_2 \geq R_4$ or $R_3 \leq R_1(1-\delta)$ and $R_4(1-\delta) \leq R_2$. For $R_1 \geq \tilde{R}_1$, the δ-dominant vector is the one with $\max R_2$ among all such vectors.*

Note that dominance (under standard vector comparison) implies δ-dominance but not vice-versa.

Definition 2. *Let $\bar{R}$ be a set of rate vectors. Define the operation vectormaxdelta($\bar{R}$) as one that eliminates all δ-dominated and dominated vectors from $\bar{R}$.*

Operation vectormaxdelta is equivalent to dividing the two-dimensional vector space into horizontal and vertical strips, each of whose left endpoint is $(1-\delta)$ times its right endpoint and choosing at most one vector per strip. A simple algorithm for implementing vectormaxdelta($\bar{R}$) is as follows. Assume $\bar{R}$ has been sorted by R_1 values. First obtain the δ-dominant vector for $R_1 \geq \tilde{R}_1$ if such R_1's exist. Then find the δ-dominant vectors successively in the strips defined by R_1 intervals $(\tilde{R}_1(1-\delta), \tilde{R}_1]$, $(\tilde{R}_1(1-\delta)^2, \tilde{R}_1(1-\delta)]$ $(\tilde{R}_1(1-\delta)^3, \tilde{R}_1(1-\delta)^2]$ and so on. Dominated vectors are eliminated simultaneously. Since $\bar{R}$ has been sorted by R_1, this can be done in one pass through $\bar{R}$, in decreasing order of R_1 values.

Choose $\delta = \frac{\beta}{2M}$. Let $\mathcal{A}_\beta^P$ denote the following dynamic programming algorithm for the fixed power minimum energy scheduling problem. The boundary conditions (i.e rate vectors for each slot t) are the same as before in Eq. 5. The main recursive step in the algorithm is derived by replacing the vectormax operation with vectormaxdelta. Let $\hat{R}_{i,j}^{kP,a,b}$ represent the set of δ-dominating rate pairs corresponding to cumulative transmission rates for user 1 and user 2 from time slots i through j, $1 \leq i \leq j \leq M$, while using a total power of kP, $1 \leq k \leq 2M$.

$$\hat{R}_{i,j}^{kP,a,b} = \text{vectormaxdelta}\Big\{ \hat{R}_{i,j-1}^{kP,a,b} \bigcup \left(\hat{R}_{i,j-1}^{(k-1)P,a-1,b} + \hat{R}_j^{P,1,0} \right) \bigcup \left(\hat{R}_{i,j-1}^{(k-1)P,a,b-1} + \hat{R}_j^{P,0,1} \right) \bigcup \left(\hat{R}_{i,j-1}^{(k-2)P,a-1,b-1} + \hat{R}_j^{2P,1,1} \right) \Big\} \quad (24)$$

The terminating condition for the algorithm occurs when the rate vectors are $\geq \tilde{R}_i(1-\beta)$, $i = 1, 2$. The optimal schedule corresponds to the minimum total power rate vector that satisfies the terminating condition.

Theorem 6. *$\mathcal{A}_\beta^P$ is a FPAS for the minimum energy scheduling problem with two fixed transmit power choices 0 or P per slot.*

Proof. First we show that the running time of $\mathcal{A}_\beta^P$ is polynomial in $1/\beta$. The number of δ-dominant vectors in $\hat{R}_{i,j-1}^{kP,a,b}$ is bounded by

$$1 + \ln_{1+\delta} \tilde{R}_1 = 1 + \frac{\ln \tilde{R}_1}{\ln(1+\delta)} = O\left(\frac{M}{\beta} \cdot \ln \tilde{R}_1 \right)$$

since we keep only one vector for each $1-\delta$-factor interval. and using $1/(1-\delta) = 1+\delta$. The running time for the creation of each $\hat{R}_{i,j}^{kP,a,b}$ is also polynomial since it includes sorting followed by the vectormaxdelta operation. There are $O(MP\mu_1\mu_2)$ such rate vector sets, each of size polynomial in $1/\beta$ and hence the overall running time is also polynomial in $1/\beta$.

Next we need to show that algorithm $\mathcal{A}_\beta^P$ provides a β-approximation of the rate constraints. Let $<R_1, R_2> \in \bar{R}_{1,j}^{kP,a,b}$ be an arbitrary non-dominated vector from the exponential time algorithm $\mathcal{A}^P$ up to time slot j. We can show by induction that $\exists <R_3, R_4> \in \hat{R}_{1,j}^{kP,a,b}$ such that $R_3 \geq R_1(1-\delta)^j$ and $R_4 \geq R_2(1-\delta)^j$. The 'parent' of $<R_1, R_2>$ (the vector that produced $<R_1, R_2>$ in stage $j{-}1$) is approximated within $(1-\delta)^{j-1}$ by the induction hypothesis. After combining with the vectors of stage j and implementing vectormaxdelta, at most a further $(1-\delta)$-factor error in R_1 and R_2 is introduced. Thus the total error in each dimension is bounded by $(1-\delta)^j$ after j slots. Therefore every rate vector in $\bar{R}_{1,M}^{kP,\mu_1,\mu_2}$ is approximated to within $(1-\delta)^M$ by a rate vector from algorithm $\mathcal{A}_\beta^P$. Using $\delta = \beta/2M$, we can see that there exist 'approximate' rate vectors $<R_3, R_4> \in \hat{R}_{1,M}^{kP,\mu_1,\mu_2}$ such that $R_3 \geq R_1(1-\beta)$ and $R_4 \geq R_2(1-\beta)$ for all 'actual' rate vectors $<R_1, R_2> \in \bar{R}_{1,M}^{kP,\mu_1,\mu_2}$. Hence $\mathcal{A}_\beta^P$ is a β-approximation.

Algorithm $\mathcal{A}_\beta^P$ above can be easily modified to incorporate multiple power levels per slot. For any small $\alpha > 0$, choose $\epsilon = \beta = \alpha/2$ and then set δ_1 and k as per Eq. 19 with L_t power levels per user per slot. Eq. 5 is modified to reflect $(L_t)^2 = O(1/\alpha^2)$ (from Theorem 5) total rate vectors per time slot t, corresponding to all combinations of power levels. Define a new algorithm $\mathcal{A}_\beta^{P_{L_t}}$ in which the vectormaxdelta operation applies to combinations of these $(L_t)^2$ rate vectors. The total number of table entries (for rate vectors) in the modified dynamic program is now increased to $(L_t)^2 MP\mu_1\mu_2$. However by applying the vectormaxdelta operation, the size of each rate vector set remains the same size, $O(1/\beta)$, as before.

Theorem 7. *For any $\alpha > 0$ and $\epsilon = \beta = \alpha/2$, $\mathcal{A}_\beta^{P_{L_t}}$ is a $(1, 1+\alpha)$-Fully Polynomial Approximation Scheme for the minimum energy scheduling problem with L_t power levels per slot.*

Proof. By choosing multiple power levels as defined above, each rate vector is no more than a $1-\epsilon = (1-\alpha/2)$-factor away from the ideal rate vector for that stage. For each such vector, the vectormax operation selects another which is at most another $1-\alpha/2$-factor away. Thus at the end of algorithm $\mathcal{A}_\beta^{P_{L_t}}$, the rate constraints are violated by at most a factor of $(1-\alpha/2)^2 < (1-\alpha)$. For given M, P, μ_1 and μ_2, the total number of table entries and related operations is $O(1/\alpha^2)$ and hence $\mathcal{A}_\beta^{P_{L_t}}$ is a $(1, 1+\alpha)$ FPAS.

Finally, we note that the 2-factor approximation of Section 3 that finds a minimal energy schedule corresponding to optimal transmit power P_{opt} can be improved by using $\mathcal{A}_\beta^{P_{L_t}}$ instead of the exponential $\mathcal{A}^P$. We increase P by a factor of $1 + k\delta_1 = 1 + \alpha$ in each iteration rather than doubling as in Theorem 2. Unlike the two fixed transmit powers case, $E^{P(1+k\delta_1)}_{\mathcal{A}_\beta^{P_{L_t}}} \leq E^{P}_{\mathcal{A}_\beta^{P_{L_t}}}$ since the former contains the schedule of the latter as a subset. The other arguments of Theorem 2 remain valid and by outputting the lowest energy value from the iterations, we obtain a $(1+\alpha, 1+\alpha)$-approximation algorithm that finds the optimal maximum transmit power level P_{opt} and the corresponding minimum energy schedule in $O(\log_{1+k\delta_1} P_{opt}/P_{min}) = O(\frac{1}{\alpha} \cdot \frac{P_{opt}}{P_{min}}) = O(\frac{M}{\alpha})$ iterations (since $P_{opt} \leq P_{max} \leq MP_{min}$), where each iteration takes time $O(1/\alpha^2)$. Since this is the solution to the unrestricted MESP problem of Eq. 2, we have

Theorem 8. *There is a $(1+\alpha, 1+\alpha)$ FPAS for solving the unrestricted MESP problem.*

6 Multiple Node Case

Even with N nodes, the number of discrete power levels L_t, required to approximate each nodes rate and overall energy within a $(1+\alpha)$-factor, remains the same as defined by Eq. 12 and Eq. 19 since the arguments of Theorem 4 apply even with interference from multiple nodes. Hence each node can select from $O(1/\alpha)$ power levels per slot. Even with only 2 power levels, the number of rate vectors per slot is 2^N, and in general $O(1/\alpha)^N$. However, we can extend the preceding algorithm to the multiple node case by defining δ-dominance for N-tuple rate vectors. If the number of users is treated as a fixed constant N, this extended algorithm is still an FPAS since 1) the number of rate vectors per slot t is polynomial in $1/\alpha$ and 2) the size of each table entry (corresponding to the rate vector set upto the j^{th} slot) is $O\left(\left(\frac{M \ln \tilde{R}_m}{\alpha})^{N-1}\right)\right)$, where $\tilde{R}_m = \min_i\{\tilde{R}_i\}$, since the number of δ-dominant vectors in the smallest dimension is $O((M \ln \tilde{R}_m)/\beta)$ and we are considering dominant vectors over an N-dimensional hypercube of vector elements.

7 Conclusions

We have considered the problem of finding a minimum energy transmission schedule for duty-cycle and rate constrained wireless sensor nodes. Since traditional optimization methods using Lagrange multipliers are computationally expensive given the non-convex constraints, we develop fully polynomial time approximation schemes by considering restricted versions of the problem using discrete power levels. We derive an $(1+\epsilon, 1+\epsilon)$-FPAS for MESP that approximates the optimal energy consumption and rate constraints to within an $1+\epsilon$-factor.

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