FE Review Course
Outline

1. Electrostatics
    a. Charge
    b. Voltage
    c. Current
    d. Resistance

2. Circuit Analysis Basics
    a. Resistor simplification
        i. Parallel
        ii. Series
    b. Source Equivalents
        i. Thevenin
        ii. Norton
    c. Node Analysis
    d. Loop Analysis

3. Transient Circuits
    a. RC Circuits
    b. RL Circuits

4. AC Circuits
        i. RMS
        ii. Phasor Transforms
        iii. AC impedance
        iv. AC Steady State analysis
5. Power
    a. DC Power
        i. Power supplied
        ii. Power Absorbed
    b. AC Power
        i. Complex power
        ii. power factor

6. Transformers
    a. Current and Voltage in an Ideal transformers
    b. Impedance seen at the input of an ideal transformer

7. Operational Amplifiers (OP-AMPS)
    a. Ideal OP-AMPS
    b. solving OP-AMP Circuits

8. Resonant Circuits
    a. Series Resonance
    b. Parallel Resonance
    c. Quality Factor
    d. Bandwidth

What you need to know:

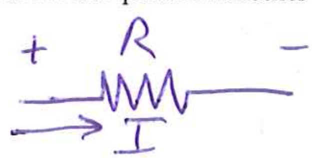
1. Electrostatics
   a. Charge
      i. Units: Coulombs (C) , 1 C is defined as the charge of $6.24\text{x}10^{16}$ electrons. The charge of an electron is $1.6\text{x}10^{-19}$ C.
      ii. The force of one charge on another charge: $\overrightarrow{\mathbf{F_{12}}} = \frac{Q_1 Q_2}{4\pi\epsilon r^2}\overrightarrow{a_{12}}$;

         Where:
         1. Q_i = the i the point charge
         2. $\overrightarrow{\mathbf{F_{12}}}$ = the force on charge 2 due to charge 1
         3. r = the distance between the two charges
         4. $\overrightarrow{a_{12}}$ = a unit vector directed from 1 to 2
         5. ϵ = the permittivity of the medium (how capable is the medium in which the charges exist of allowing these forces to exist)

   b. Voltage – The potential difference between two points, is the work done per unit charge required to move the charge between two points.

   c. Current – Rate of charge passing across a surface:
$$I = i(t) = \frac{dQ}{dt}$$

   d. Resistance –Measure of the ability of charge to move from one point to another for a given potential difference (voltage).
$$R = \frac{\rho L}{A}$$



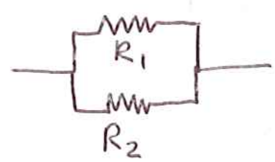
$V_R = IR$

   e. Ohm's Law V=IR or $v(t) = i(t)R$

2. Circuit Analysis
   a. Resistor Simplification

      i. Two resistors in series: $R_{TOT} = R_1 + R_2$

      ii. Two resistors in parallel: $R_{TOT} = \frac{R_1 R_2}{R_1 + R_2}$



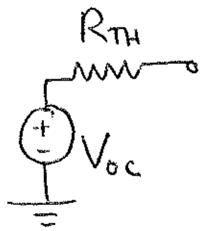
   b. Node Analysis – Using Kirchhoff's Current Law (KCL): $(\sum I = 0)$ at any node.

      And: $I_{AB} = \frac{V_A - V_B}{R_{AB}}$

   Write a system of equations to solve for unknown node voltages.

   c. Loop Analysis – Using Kirchhoff's Voltage Law (KVL): $(\sum V = 0)$ around any closed path.
   Write a system of equations to solve for the unknown currents in each branch.

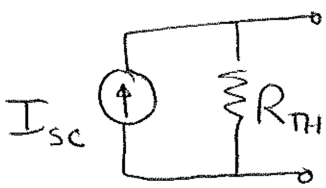
   d. Source equivalents – At any port a linear circuit can be simplified into an ideal source and a resistance
      i. Thevenin Equivalent Circuit – circuit is simplified into a series combination of an ideal voltage source and a resistance.
         1. Find the open circuit voltage (V_{oc}) at the port of interest



2. Find the equivalent resistance (R_{TH})at the port of interest
   Or Find the short circuit current (I_{sc}) at the port of interest

$$R_{TH} = \frac{V_{oc}}{I_{sc}}$$

ii. Norton Equivalent Circuit – circuit is simplified into a parallel combination of an ideal current source and a resistance



1. Find the short circuit current (I_{sc}) at the port of interest
2. Find the equivalent resistance (R_{TH})at the port of interest
   Or Find the open circuit voltage (V_{oc}) at the port if interest

$$R_{TH} = \frac{V_{oc}}{I_{sc}}$$

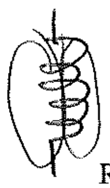
3. Transient Circuits – Circuits that contain capacitors and/or inductors as well as resistors. Voltages or Current sources are switched on or off at t = 0. Response is analyzed
   a. Capacitors: $C = \frac{\varepsilon A}{d}$; capacitance for a parallel plate capacitor

$$i(t) = C\frac{dv_c(t)}{dt}$$

open CkT to DC

   b. Inductors: $L = \frac{N\phi}{i(t)}$;

L SHORT TO DC



inductance for a coil with N turns and magnetic flux ϕ enclosed in the coil

$$v(t) = L\frac{di_L(t)}{dt}$$

Requires solving first order (C or L only) or second order (L and C) differential eqns.

4. AC Circuits – Steady state analysis of circuits with (periodic) sinusoidal voltage and current sources.

$\omega = 2\pi f$

   a. Capacitors replaced with frequency dependent impedance $Z_C = \frac{1}{j\omega C}$
   b. Inductors replaced with frequency dependent impedance $Z_L = j\omega L$
   c. Circuit Analysis is simplified from DIFFEQ's to (complex) algebraic techniques used in resistive circuit analysis

5. Power
   a. DC (Resistive) Circuits
      i. Real power P supplied by a source; P = VI
      ii. Power absorbed by a resistor; $P = \frac{V^2}{R} = I^2R$
      iii. Circuits with multiple sources may have some sources ABSORBING power. (Battery charging)
   b. AC (Complex) Circuits
      Complex power; S = P + jQ

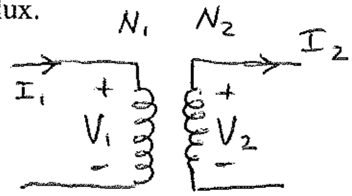
      Real Power $P = (^1/_2)V_{max}I_{max}cos\theta$
      $= V_{rms}I_{rms}cos\theta$

      Complex Power Q = $(^1/_2)V_{max}I_{max}sin\theta$

θ is the angle measured from V to I.

6. Transformers: Two coils in proximity sharing magnetic flux.

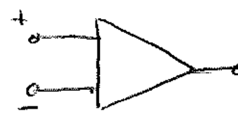
$n = \frac{N_1}{N_2}$; the ratio of the number of turns in the two coils



$$n = \frac{|V_1|}{|V_2|} = \frac{|I_2|}{|I_1|}$$

Beware of the Dots!

7. Operational Amplifiers



$\frac{V_{OUT}}{V_{IN}}$ = GAIN

TRANSFER $\begin{cases} \text{FCN} \\ \text{CHARACTERISTIC} \end{cases}$

For Ideal Op-Amps:
- No current flows into the input terminals
- The input terminals have the same voltage

8. Resonant Circuits – Parallel and Series LC circuits have a bandpass frequency response. This response is located at a center frequency (ω_o) and has a bandwidth (BW)

Resonant Frequency: $\omega_o = \frac{1}{\sqrt{LC}}$

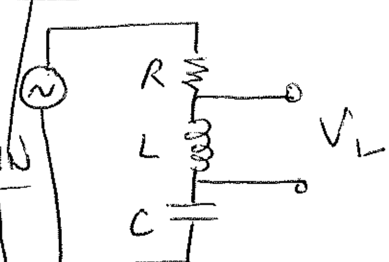
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Impedance at Resonance: Z = R

Series Resonance: $Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o RC}$

$Z_L + Z_C = 0$

$BW = \frac{\omega_o}{Q}$

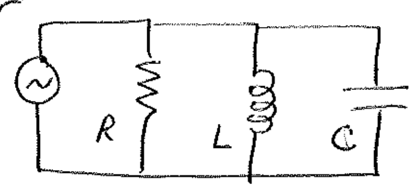


Parallel Resonance: $Q = \omega_o RC = \frac{R}{\omega_o L}$

$\infty = Z_L || Z_C$

Quality Factor

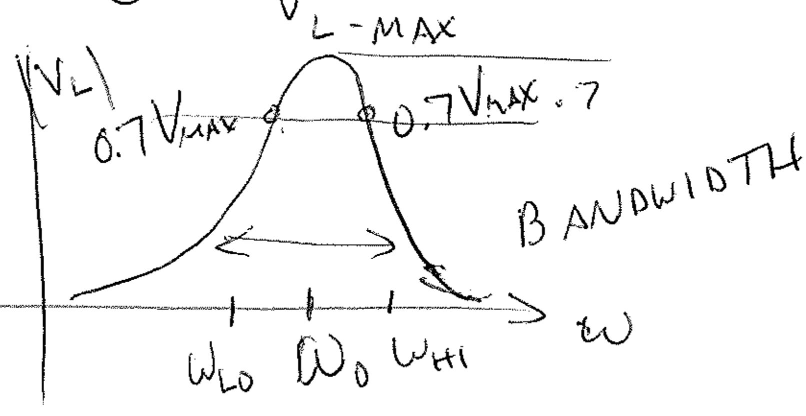
V_L as a fcn of ω



V_L @ DC

$V_L = I Z_L$

$V_L(\omega)\big|_{\omega=0} = I j\omega L = 0$



SIMPLIFYING RESISTIVE CIRCUITS.

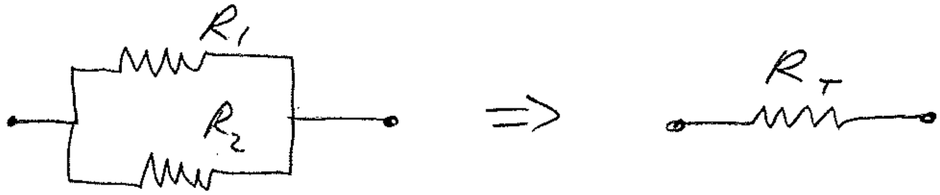
- RESISTOR SIMPLIFICATION
- THEVENIN EQUIVALENT CIRCUITS
- NORTON EQUIVALENT CIRCUITS.

RESISTORS IN SERIES

$$R_T = R_1 + R_2$$

$$R = R_1 + R_2$$

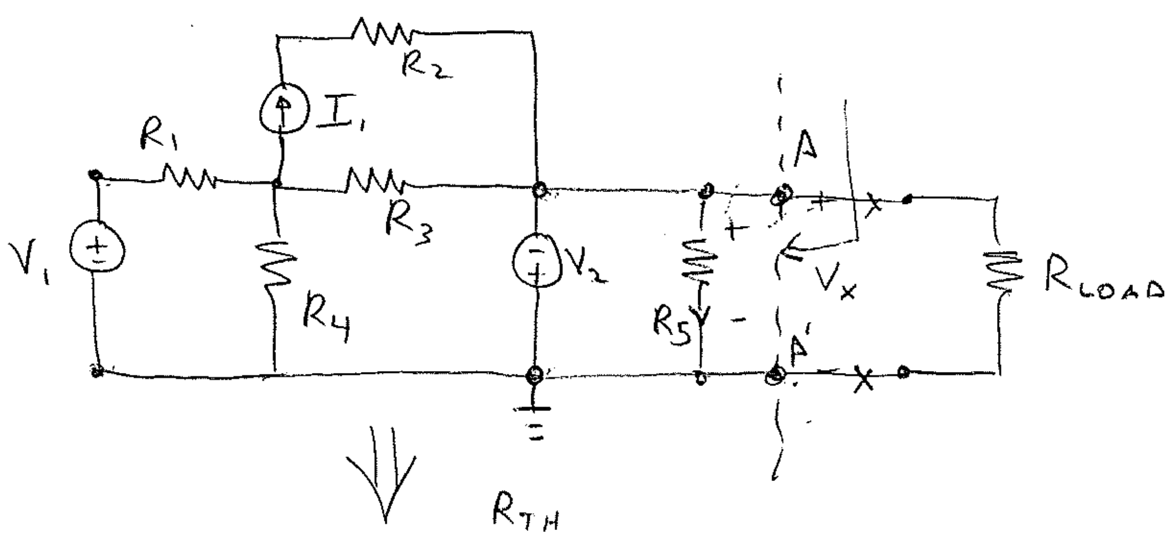
RESISTORS IN PARALLEL

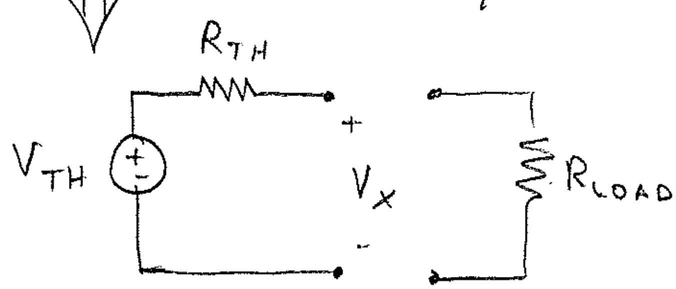


$$R_T = \frac{(R_1)(R_2)}{R_1 + R_2}$$

THEVENIN EQUIVALENT CIRCUIT

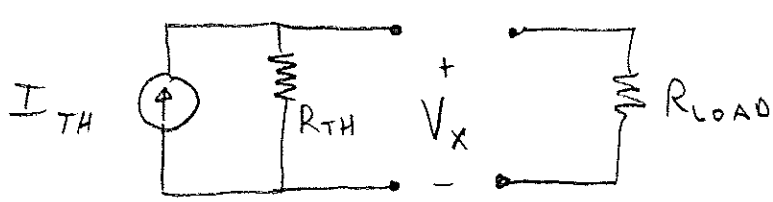
ANY RESISTIVE CIRCUIT, NO MATTER HOW COMPLEX, CAN BE REDUCED TO A SERIES VOLTAGE AND RESISTANCE, AT ANY TERMINAL IN THAT CIRCUIT.





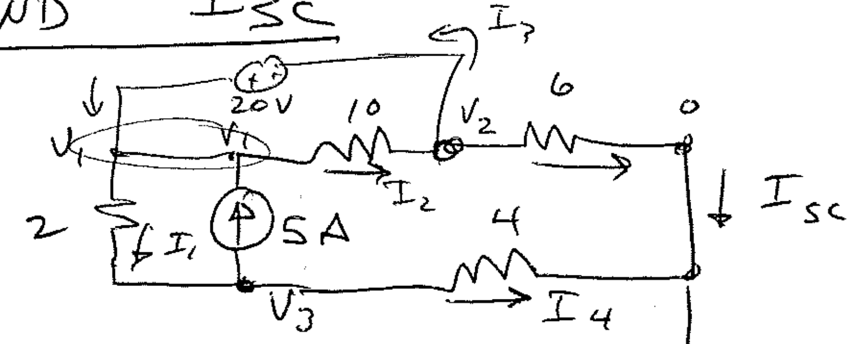
OR

NORTON EQUIVALENT.



$$V_x = I_{TH}\left(\frac{R_{TH} R_L}{R_{TH} + R_L}\right)$$

FIND I_{SC}



$V_2 = V_1 - 20$

$$I_{sc} = \frac{V_2}{6} = \frac{V_1 - 20}{6}$$

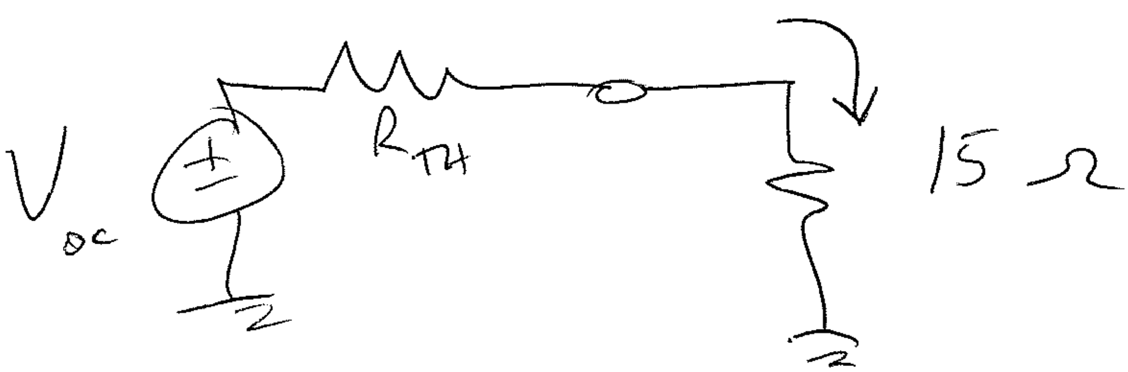
KCL $I_1 + I_2 = I_3 + 5A$

V_1 $\frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{10} =$

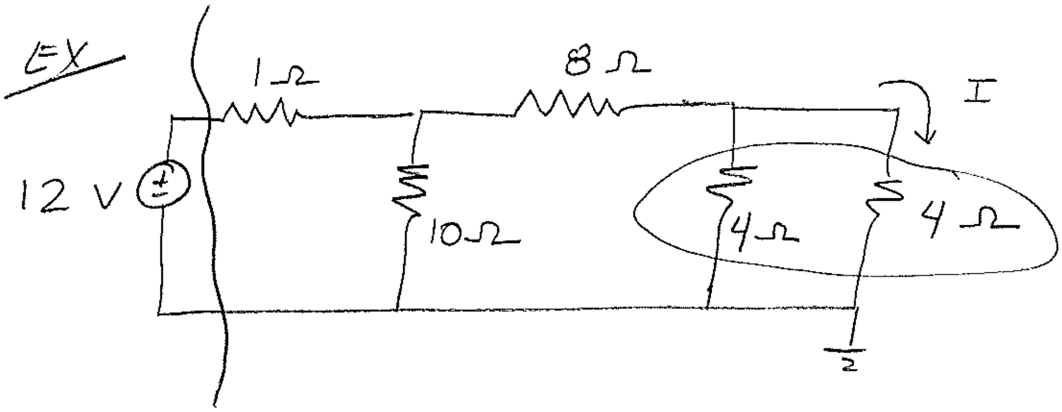
KCL @ V_2 $I_2 = I_3 + I_{SC}$

I_{SC}

$$R_{TH} = \frac{V_{oc}}{I_{SC}}$$

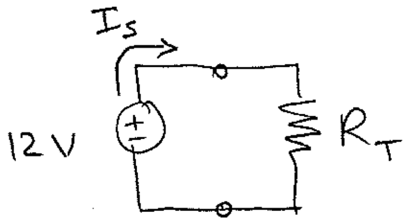


EX



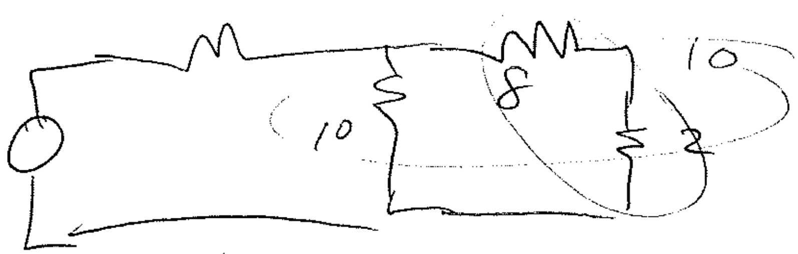
FIND I IN THIS CKT.

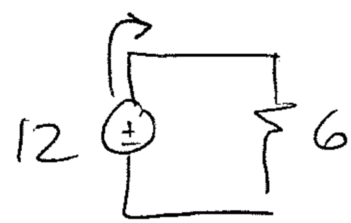
CAN USE LOOP EQUATIONS OR RESISTIVE SIMPLIFICATION



CURRENT FROM SOURCE I_S DELIVERED TO RESISTIVE CIRCUIT.

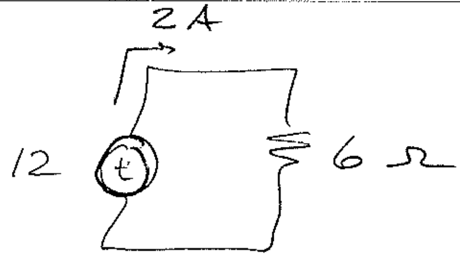
$$4\,\Omega \parallel 4\,\Omega = \frac{16}{8} = 2\,\Omega$$

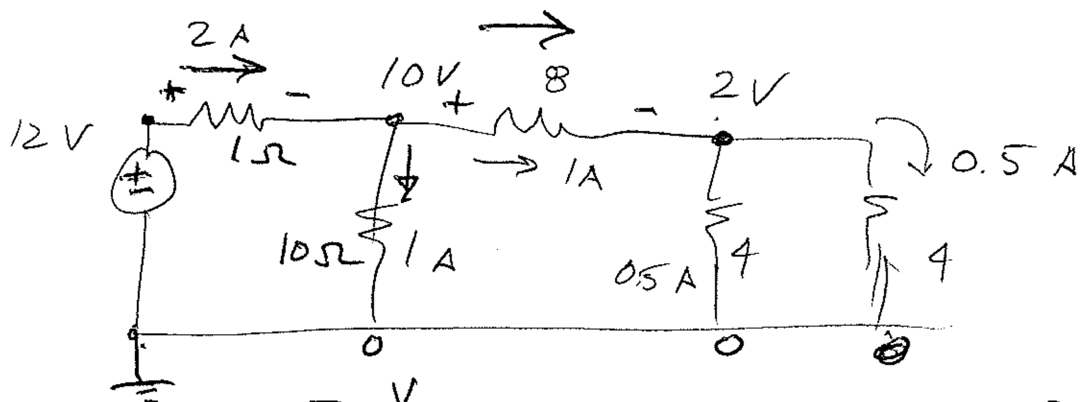




$V = IR$

$\boxed{I = 2\,A}$

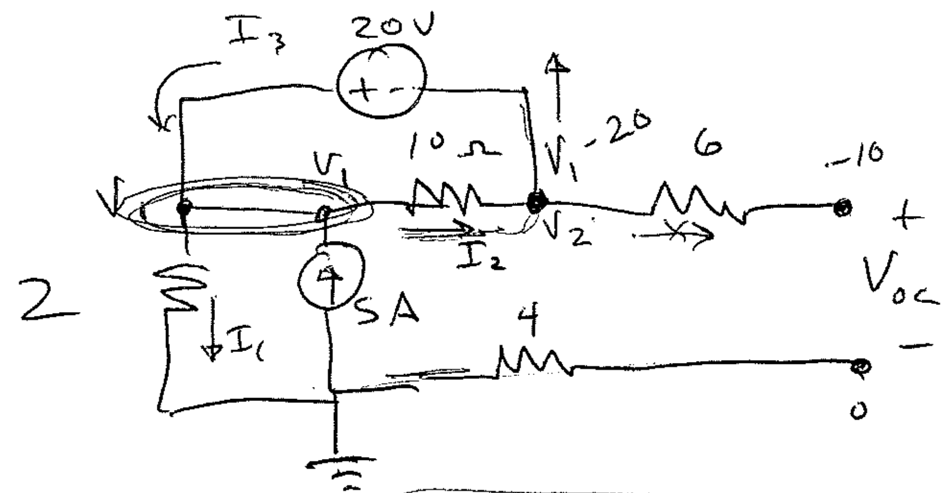




$I = \frac{V}{R}$

$I = \frac{V}{R} = \frac{2}{4} = \frac{1}{2}$ A

FIND V_{oc}



KCL @ V_1 $\boxed{\cancel{I_2} + I_1 = 5 + \cancel{I_3}}$

$$\frac{V_1 - (V_1 - 20)}{10} + \frac{V_1 - 0}{2} = 5 + I_2$$

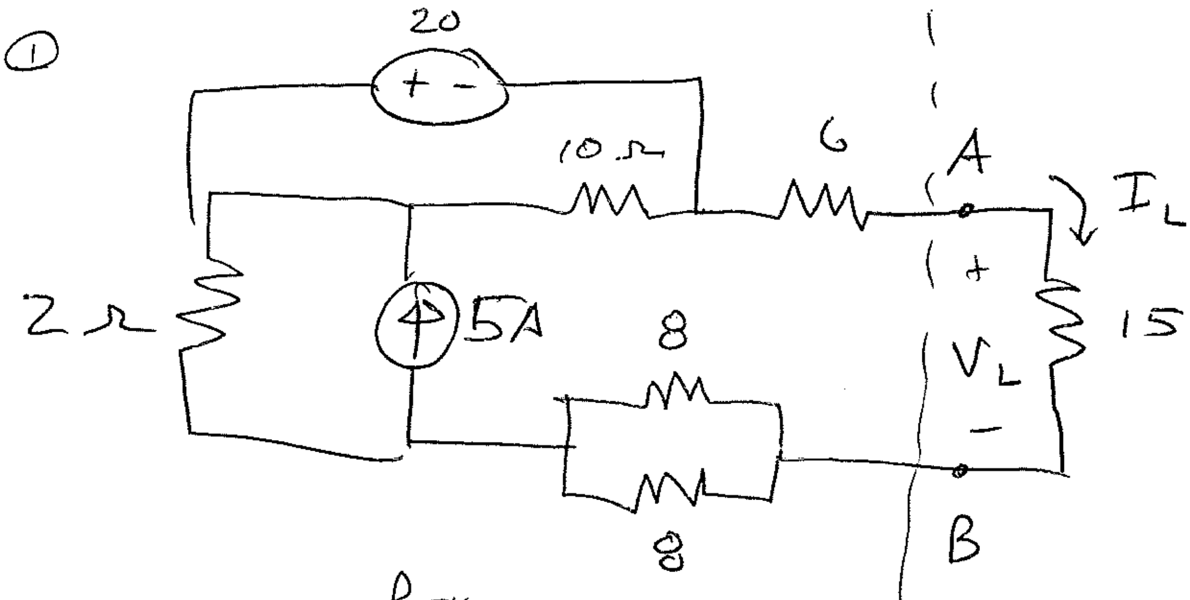
$$\boxed{\frac{V_1}{2} = 5}$$

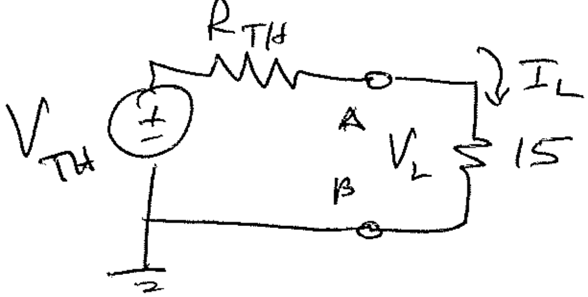
$V_1 = 10$ $\boxed{V_2 = -10}$

$$\boxed{V_{oc} = -10}$$

FINDING T.E.C

(1)

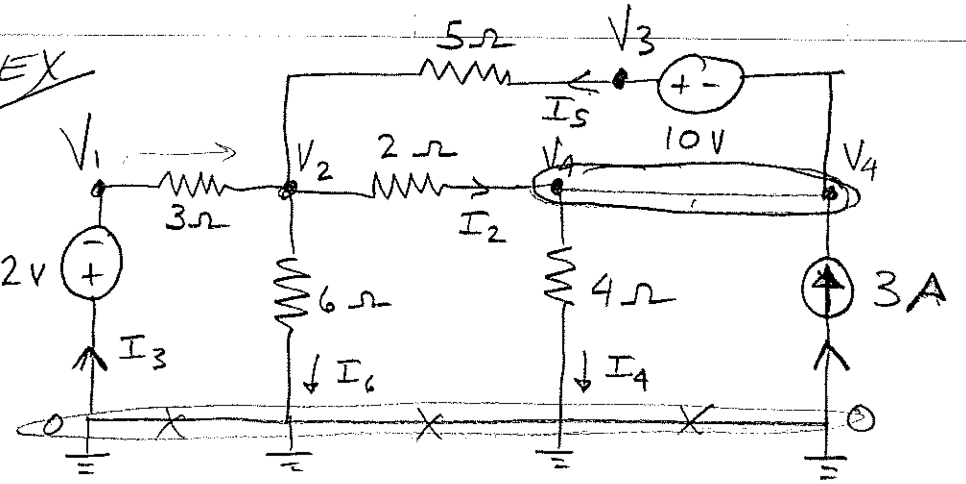




$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

$$V_L = 15\ I_L$$

EX



FIND ALL BRANCH CURRENTS AND NODE VOLTAGES.

STEP 1) LABEL ALL BRANCHES AND NODES

BRANCH CURRENTS $\quad I_3 , I_6 \quad I_4 \quad I_5 , I_2$

FIND VOLTAGES AT ALL NODES
THEN USE Ohm's LAW

$$I_2 = \frac{V_2 - V_4}{2\Omega} \qquad I_6 = \frac{V_2 - 0}{6}$$

$$\boxed{V_1 = -12\ V} \qquad \boxed{V_3 = 10 + V_4}$$

KCL @ NODE 2 $\quad I_3 + I_5 = I_6 + I_2$

$\sum I = 0$

$$I_3 = \frac{V_1 - V_2}{3} \qquad I_5 = \frac{V_3 - V_2}{5}$$

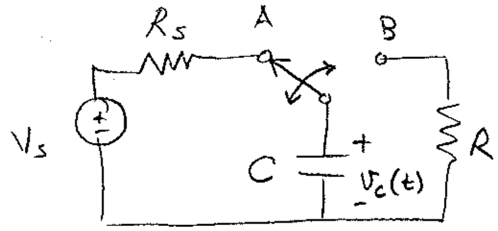
$$\frac{V_1 - V_2}{3} + \frac{V_3 - V_2}{5} = \frac{V_2}{6} + \frac{V_2 - V_4}{2}$$

KCL @ NODE 4

$$I_2 + 3 = I_4 + I_5 \qquad I_4 = \frac{V_4 - 0}{4}$$

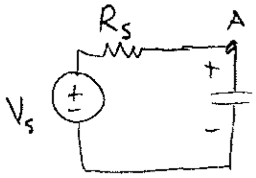
4 EQN. 4 UNKNOWNS CAN BE SOLVED SIMULTANEOUSLY

FIRST ORDER TRANSIENT CIRCUITS



1. VOLTAGE DOES NOT CHANGE INSTANTANEOUSLY ACROSS CAP
2. CAP IS O.C. TO DC CURRENT AS $t \to \infty$

POSITION A (FOR A LONG TIME)

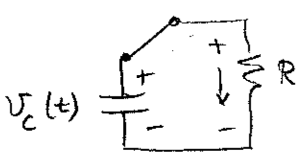


$V_c(t) = v_c(0^-)$

$$V_s - i_s R_s - v_c = 0$$

$$i_s = 0 \quad \therefore V_s = v_c$$

POSITION B



$$i_R = C\frac{dv_c}{dt}$$

$$v_c + C\frac{dv_c}{dt}R = 0$$

$$v_c + RC\frac{dv_c}{dt} = 0$$

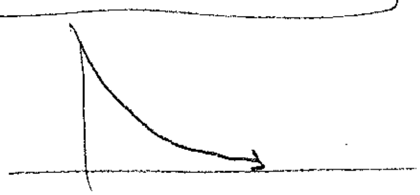
$$\frac{dv(t)}{dt} + \frac{1}{RC}v(t) = 0$$

$$v(t) = K_2 e^{-\left(\frac{1}{RC}\right)t}$$

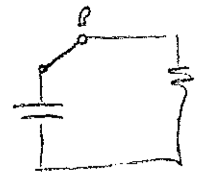
$$v(0^+) = v(0^-) = K_2 e^{-\left(\frac{1}{RC}\right)0} = K_2$$

$$\boxed{K_2 = V_s}$$

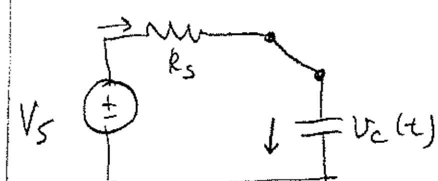
$$\boxed{v_c(t) = V_s e^{-t/RC}}$$



POSITION B



$$i_c = 0 \qquad v_c(0) = 0$$



$$i_R = \frac{V_s - v_c(t)}{R} = C\frac{dv_c}{dt} = i_c$$

$$\frac{dv_c(t)}{dt} + \frac{1}{R_sC}v_c(t) = \frac{+V_s}{R_sC}$$

$$v_c(t) = k_1 + k_2 e^{-t/\tau}$$

$$k_2\left(-\frac{1}{\tau}\right)e^{-t/\tau} + \frac{k_1}{RC} + \frac{k_2}{RC}e^{-t/\tau} = \frac{V_s}{RC}$$

$$\frac{k_1}{RC} = \frac{V_s}{RC} \qquad \boxed{V_s = K_1}$$

$$-\frac{k_2}{\tau}e^{-t/\tau} = \frac{-k_2}{RC}e^{-t/\tau}$$

$$\boxed{\tau = RC}$$

$$v(t) = V_s + K_2 e^{-t/RC}$$

$$v(0^-) = 0 = V_s + k_2$$

$$\boxed{K_2 = -V_s}$$

$$v_c(t) = V_s - V_s e^{-t/RC}$$

$$\frac{d\,x(t)}{dt} + a\,x(t) = f(t)$$

IF $x_p(t)$ IS a soln, AND $X_c(t)$ IS A SOLN

TO THE HOMOGENEOUS EQN:

$$\frac{d\,x(t)}{dt} + a\,x(t) = 0$$

THEN $x(t) = x_p(t) + x_c(t)$

LET $f(t) = A$ constant

$$\frac{d\,x_p(t)}{dt} + a\,x_p(t) = A \qquad x_p(t) = K_1$$

$$\frac{d\,x_c(t)}{dt} + a\,x_c(t) = 0$$

$$x_p(t) = K_1 = \frac{A}{a} \qquad x_c(t) = K_2 e^{-at}$$

$$\boxed{x(t) = K_1 + K_2 e^{-at}}$$

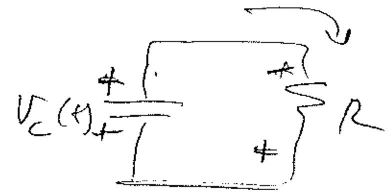
$$v(t) = k_1 + k_2 e^{-(\frac{1}{RC})t}$$

$$v(0^-) = v(0^+) = \boxed{V_s = k_1 + k_2}$$

$$v(t) = k_2 e^{-(\frac{1}{RC})t}$$

$$\frac{dv_C(t)}{dt} + \frac{v_C(t)}{RC} = 0$$

$$v_C(t) = k_2 e^{-t/RC}$$



$$v_C(t) + i_R R = 0$$

$$i_R = C\frac{dv_C}{dt}$$

$$v_C + RC\frac{dv_C}{dt} = 0$$

$$\frac{dv_C}{dt} + \frac{1}{RC}v_C = 0$$

$$v_C = k_2 e^{-\frac{1}{RC}t}$$

$$\frac{k_2}{RC}e^{-\frac{1}{RC}t} - \frac{k_2}{RC}e^{-\frac{1}{RC}t} = 0$$

USING LAPLACE TRANSFORMS

$$v_c(t) + CR\frac{dv_c}{dt} = 0 \qquad v(0^-) = V_s$$

$$V(s) + CR\left(s\,V(s) - v(0^-)\right) = 0$$

$$V(s)(1 + CRs) - CR\,v(0^-) = 0$$

$$V(s) = \frac{CR\,V_s}{1 + CRs} = \frac{V_s}{\frac{1}{RC} + s}$$

$$v(t) = V_s\,e^{-\frac{1}{RC}t}$$

$$\frac{1}{s+a} \Leftrightarrow e^{-at}$$

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{V_s}{RC}$$

$$sV(s) + \frac{V(s)}{RC} = \frac{V_s}{sRC}$$

$$V(s)\left(s + \frac{1}{RC}\right) = \frac{\frac{V_s}{RC}}{s(s + 1/RC)} = \frac{k_1}{s} + \frac{k_2}{s + 1/RC}$$

$$\frac{\frac{V_s}{RC}}{1/RC} = \boxed{k_1 = V_s}$$

$$\left.\frac{\frac{V_s}{RC}}{s}\right|_{s = -1/RC} = \boxed{k_2 = -V_s}$$

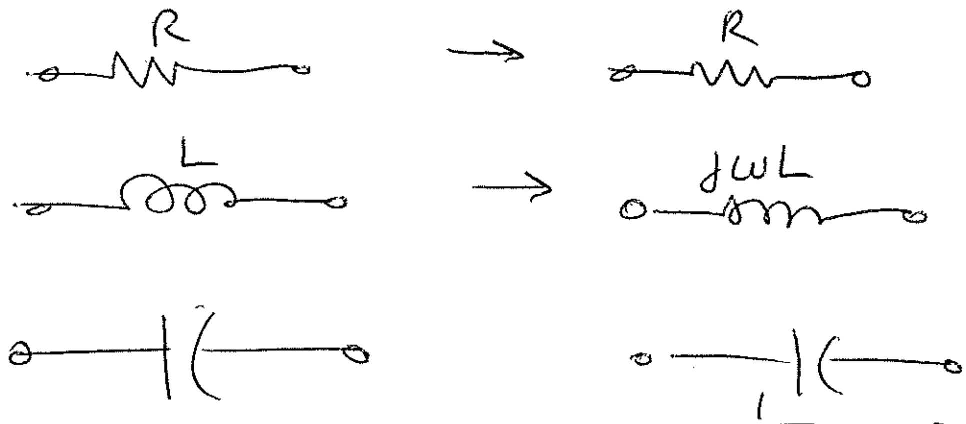
$$v(t) = V_s - V_s\,e^{-t/RC}$$

INIT. COND $= 0$

AC SS CKT ANALYSIS

AC CKT ANALYSIS

$$v(t) = A\cos(\omega t + \theta)$$



$$\frac{1}{j\omega C} = \frac{-j}{\omega C}$$

AC CIRCUIT ANALYSIS – STEADY STATE ANALYSIS

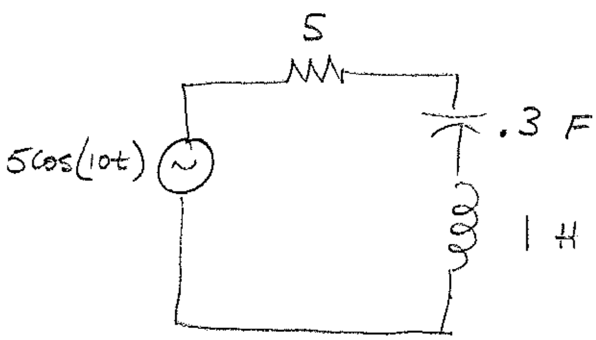
CONVERT TO PHASOR DOMAIN

$$v(t) = A\cos(\omega t + \theta) = \mathrm{Re}\left[A e^{j(\omega t + \theta)}\right]$$

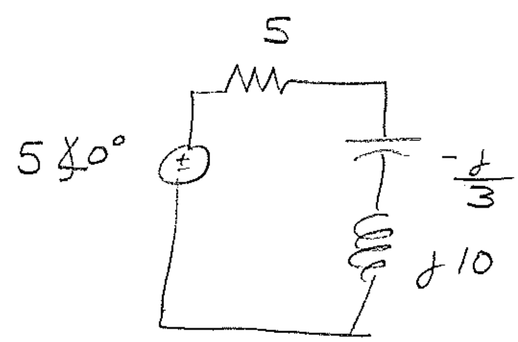
ALSO WRITTEN AS A PHASOR $\underline{V} = A \angle \theta$

CONVERT COMPONENTS

TIME DOMAIN		PHASOR
R	$\longrightarrow$	R
L	$\longrightarrow$	$j\omega L$
C	$\longrightarrow$	$\frac{1}{j\omega C}$



$\longrightarrow$



$$I = \frac{5\angle 0^\circ}{5 - j/3 + j10}$$

$$\underline{I} = \frac{5\angle 0^\circ}{5 + j(10 - 1/3)}$$

$$I = \frac{5\angle 0^\circ}{5 + j9.67} = \frac{5\angle 0^\circ}{10.89\angle 62.6^\circ} = \underline{0.459\angle -62.6^\circ}$$

$$\boxed{i(t) = 0.459\cos(10t - 62.6^\circ)}$$

SUMMARY OF COMPLEX NUMBERS

$$x + jy = re^{j\theta}$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$\frac{1}{e^{j\theta}} = e^{-j\theta} \quad \text{or} \quad \frac{1}{5\angle 30} = \underline{0.2\angle -30^\circ}$$

AC RMS

RMS → DC EQUIVALENT OF AN AC SIGNAL

$$V_{rms} = \left[\frac{1}{T} \int_0^T v^2(t)\, dt \right]^{1/2}$$

FOR SINUSOID

$$V_{RMS} = \frac{V_p}{\sqrt{2}}$$

TIME $\quad v(t) = V_p \sin(\omega t - \theta_v)$

PHASOR $\quad \vec{V} = \frac{V_p}{\sqrt{2}} \angle \theta_v$

8. A.C. Circuits

Whenever a voltage and current change in a circuit, inductance and capacitance must be accounted for. If the voltage is of the form $v = V_{max}\cos(\omega t + \theta)$ it must be converted to the effective or RMS value.

$V_{RMS} = [\frac{1}{period}\int_0^{period} v^2 dt]^{\frac{1}{2}}$ for a sine wave $V_{RMS} = \frac{V_{max} e^{j\theta}}{\sqrt{2}} = \frac{V_{max}}{\sqrt{2}} \angle\theta$

$\omega = 2\pi$ (frequency in HZ)

$\omega = \frac{2\pi}{T}$ where $T =$ period in seconds

ω is in radians per second

$e^{j\theta} = cos\theta + j sin\theta$

where $j = \sqrt{-1}$ and represents the y axis

The essential facts in A.C. circuits are:

R	L	C
$v_R = i_R R$ $i_R = \frac{V_R}{R}$	$v_l = L\frac{di}{dt}$ $i_L = \frac{1}{L}\int v_l dt$ i_L does not change instantaneously	$v_c = \frac{1}{c}\int i_c dt$ $i_c = c\frac{dv_c}{dt}$ v_c does not change instantaneously

R L C

When the voltage and currents are converted to RMS values.

	R	L	C	
	$V_R = I_R R$	$V_L = I_L(jX_L)$	$V_c = I_c(-jX_c)$	
		$= I_L X_L \angle 90 = I_L X_L e^{j90}$	$= I_c X_c \angle -90 = I_0 X_c e^{-j90}$	
V		where $X_L = \omega L$	$X_c = \frac{1}{wc}$	V
		I_L is at 0°	I_c is at 0°	
	$I_R = \frac{V_R}{R}$	$I_L = \frac{V_L \angle 0}{jx_L} = \frac{V_L}{x_L} \angle 90$	$I_c = \frac{V_c \angle 0}{-jx_c} = \frac{V_c}{X_c} \angle 90$	I
I	Current is in phase with voltage	I_L lags V_L by 90	I_c leads V_c by 90	

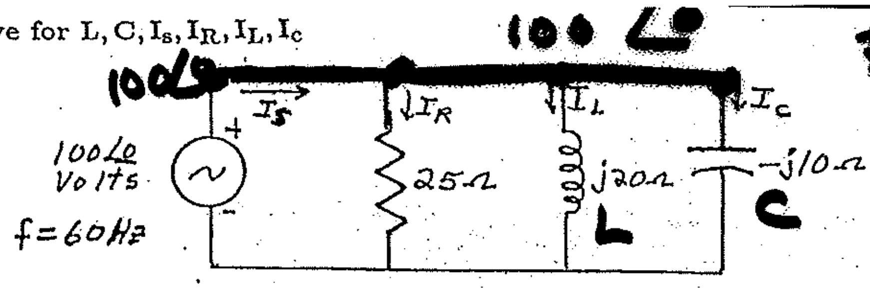
Impedance in $\Omega =$ Resistance in $\Omega \pm j$ Reactance in ohms

$Z \angle\theta = R + jx = R + j(X_L - X_c)$

$Z \angle\theta = Zcos\theta + jZsin\theta$

so $R = Zcos\theta$ $\quad X = Zsin\theta$

Solve for L, C, I_s, I_R, I_L, I_c



$Z_L = j\omega L$

$\tan^{-1}\frac{20}{0}$

$$Z_L = j20 = 20\angle 90$$

$$V_L = I_L Z_L$$

$$V_C = I_C Z_C$$

$$Z_C = -j10 = 10\angle -90$$

$$I_R = \frac{100\angle 0}{25} = 4\angle 0 = 4 \text{ AMPS}$$

$$i(t) = 4\sqrt{2}\cos\omega t$$

$$I_L = \frac{100\angle 0}{20\angle 90} = 5\angle -90$$

$$i_L(t) = 5\sqrt{2}\cos(\omega t - 90)$$

$$I_C = \frac{100\angle 0}{10\angle -90} = 10\angle 90$$

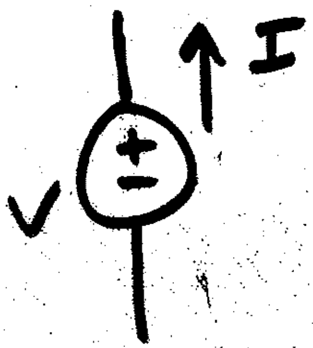
$$i_C(t) = 10\sqrt{2}\cos(\omega t + 90)$$

POWER

DC

$P = VI$

RESISTOR $\quad P_{ABS} = \frac{V^2}{R} = I^2 R$



DELIVERING

ABSORBING

COMPLEX

$S = P + jQ = VI^*$

$S = |S|\angle\theta_s \qquad = (|V|\angle\theta_v)(|I|\angle\theta_i)$

$S = |V||I|\angle\theta_v - \theta_i$

Power factor = pf

$\cos(\theta_v - \theta_i)$

What is the power in the components of the series circuit

What you need to know:

1. In A.C. circuits power is complex so that:

Apparent or complex power in Volt amperes = Power in watts $\pm j$ Reactive power in VARS (volt amperes reactive)

$S \angle\theta = P + jQ = s\cos\theta + js\sin\theta$

2. Resistance is the only thing that dissipates power in watts.

3. VARS can come only from reactance.

4. Inductance causes $+j$ VARS

5. Capacitance causes $-j$ VARS

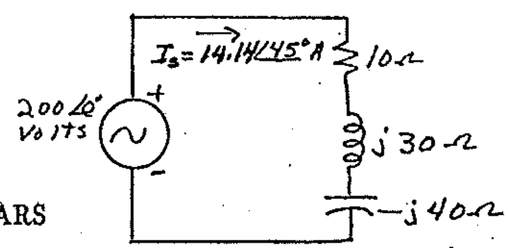
6. The power equations are:

$$\begin{array}{llll} \text{For } R & |P| = VI\cos\theta & P = |I_R^2|R & = \frac{|V_R^2|}{R} watts \\ \text{For } X_L & |Q_L| = VI\sin\theta & jQ_L = |I_L^2|(jX_L) & = \frac{|V_L^2|}{jX_L)*} VARS \\ \text{For } X_c & |Q_c| = VI\sin\theta & -jQ_c = |I_c^2|(-jX_c) & = \frac{|V_c^2|}{(-jX_c)*} VARS \end{array}$$

Note: The $*$ means take the conjugate of the quantity (change the sign of the j term).

$VA = VI^*$ where V is at 0 degrees

Solution:



1. $P = |I_R^2|R = 14.14^2(10) = 2000$ watts

2. $jQ_L = |I_L^2|jX_L = 14.14^2(j30) = j6000$ VARS

3. $jQ_L = |I_c|^2(-jX_c) = 14.14^2(-j40) = -j8000$ VARS

4. The generator is supplying $2000 + j6000 - j8000$ volt amperes $= 2000 - j2000 = 2828 \angle -45. VA$

5. We get the same result by using

$V_sI_s* = (200)(14.14 \angle +45)^* = (200)(14.14 \angle -45) = 2828 \angle -45$ Volt amperes

$= P - jQ = 2828cos45 \quad - j2828sin45 \quad = 2000 - j2000$

COMPLEX POWER

$v(t) = V_m \cos(\omega t + \theta_v)$

$i(t) = I_m \cos(\omega t + \theta_i)$

INSTANTANEOUS POWER $\quad p(t) = v(t)\, i(t) = I_m V_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$

$$p(t) = \frac{V_m I_m}{2}\Big[\underbrace{\cos(\theta_v - \theta_i)}_{\uparrow\ \text{TIME IND.}} + \underbrace{\cos(2\omega t + \theta_v + \theta_i)}_{\uparrow\ \text{TIME DEP.}}\Big]$$

AVERAGE POWER

$$P_{AV} = \frac{V_M I_M}{2} \cos(\theta_v - \theta_i) = \frac{V_M}{\sqrt{2}} \angle\theta_v \; \frac{I_M}{\sqrt{2}} \angle{-\theta_i}$$

$pf \equiv$ power factor $= \cos(\theta_v - \theta_i) \quad = V_{rms}\, I_{rms} (pf)$

DIFFERENCE IN PHASE ANGLE OF V and I

SINUSOIDAL R.M.S.

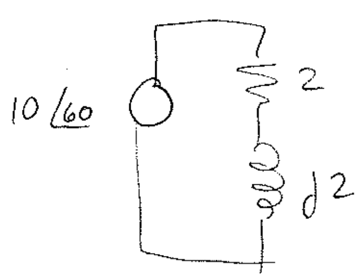
$V_{rms}\, I_{rms} \equiv$ APPARENT POWER

$-1 \le \cos(\theta_v - \theta_i) \le 1$

$0 < pf < 1$ lagging pf

$-1 < pf < 0$ leading pf

EX



FIND P_{ave}, pf

$V = 10 \angle 60$

$$I = \frac{10 \angle 60}{2 + j2} = 3.53 \angle 15°$$

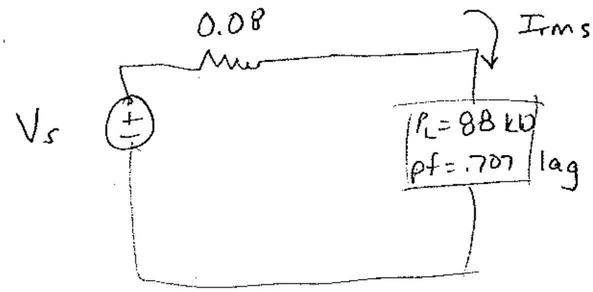
$$P_{AVE} = \frac{10\,(3.53)}{2} \cos(60 - 15) = 12.48 \text{ Watts}$$

$pf = \cos(45°) = .707$ lagging (INDUCTIVE LOAD)

EX. INDUSTRIAL LOAD USES 88 kW @ pf = .707 lagg.
FROM 480 Vrms LINE

a) FIND POWER NEEDED FROM POWER CO.

b) if pf = 0.90



$$P_L = 88 \text{ kW} = I_{rms} V_{rms} (pf)$$

$$88 \text{ kW} = I_{rms} (480)(.707)$$

$$\boxed{I_{rms} = 259.3 \text{ A}}$$

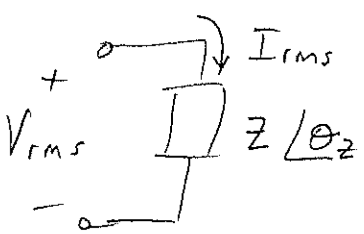
$$P_s = I_{rms}^2 (R) + 88 \text{ kW}$$

$$\boxed{P_s = 93.38 \text{ kW}}$$

b) $pf = 0.90$ $\quad I_{rms} = 203.7$ A

$$\boxed{P_s = 91.32 \text{ kW}}$$

COMPLEX POWER



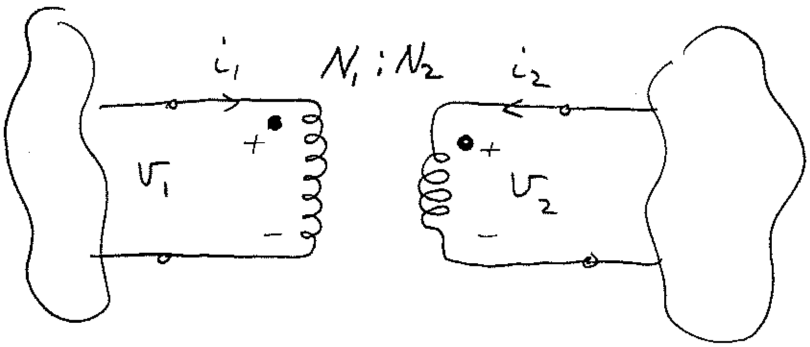
$$S = V_{rms} I_{rms}^{*} = V_{rms} I_{rms} \angle \theta_v - \theta_i$$

$$= \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{\text{real power}} + j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{\text{reactive quadrature}}$$

$$S = P + jQ$$

$$P = Re(S) = V_{rms} I_{rms}$$

IDEAL TRANSFORMERS



$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \qquad \frac{i_1}{i_2} = -\frac{N_2}{N_1}$$

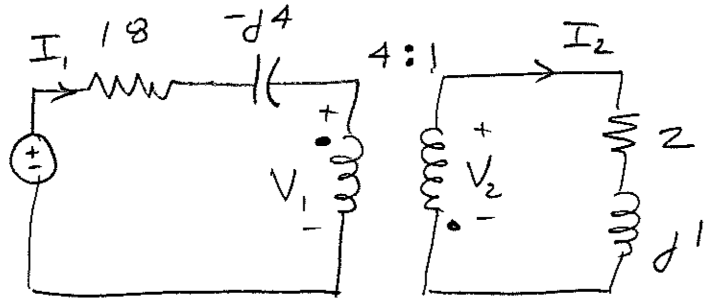
$$v_1 i_1 + v_2 i_2 = 0$$

NO POWER LOST IN AN IDEAL XFORMER

$$n = \frac{N_2}{N_1} := \text{TURNS RATIO}$$

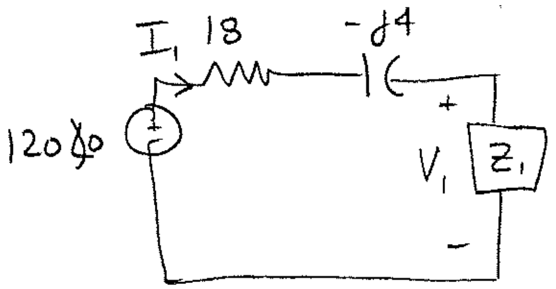
$$V_1 = \frac{V_2}{n} \qquad I_1 = n\, I_2 \qquad Z_1 = \frac{Z_L}{n^2}$$

Ex



$V_1 = \frac{-V_2}{n}$ $\qquad I_1 = -n I_2$ $\qquad n = \frac{1}{4}$

$$Z_1 = \frac{Z_L}{n^2} = 4^2 Z_L$$



$4^2(2+j1)$
$= 32 + j16$

$$I_1 = \frac{120\angle 0}{18 - j4 + 32 + 16j} = 2.33\angle -13.5^\circ$$

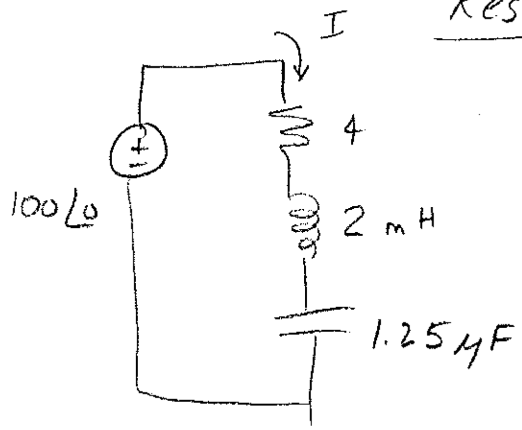
$$V_1 = I_1 Z_1 = (2.33\angle -13.5^\circ)(32 + j16)$$
$$= 83.49\angle 13.07^\circ$$

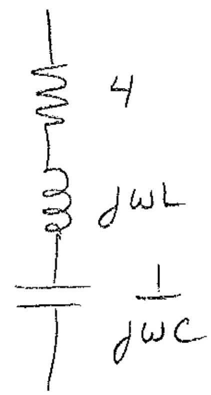
$$V_2 = -V_1 n = \tfrac{1}{4}\, 83.49\angle 13.07^\circ = 20.87\angle 193.07^\circ$$

$$I_2 = -\frac{I_1}{n} = 4\,(2.33\angle 13.5^\circ) = 9.33\angle 166.50^\circ$$

SERIES

Resonance Ckts





$$Z_T = R + j\omega L - j\frac{1}{\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$|V_L| = |V_C|$ BUT ARE 180° OUT OF PHASE

@ $\omega L = \frac{1}{\omega C}$

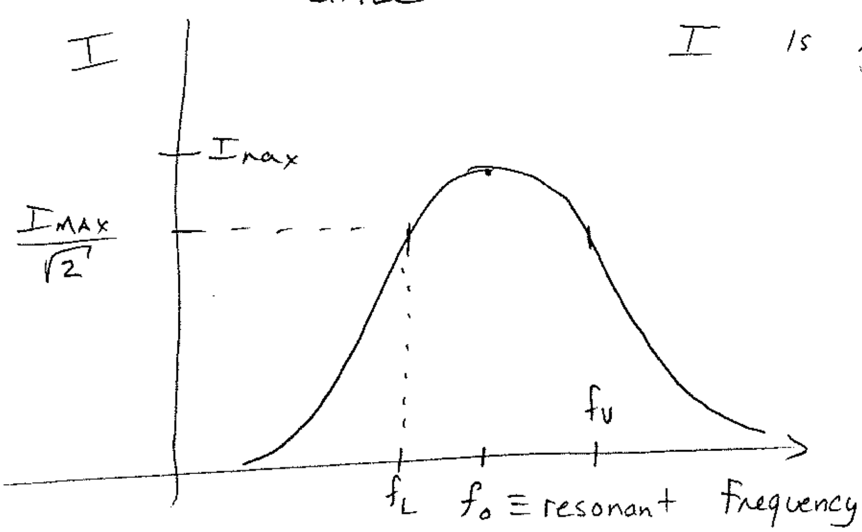
$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \implies Z_T = R$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$|Z_T|$ IS MINIMUM

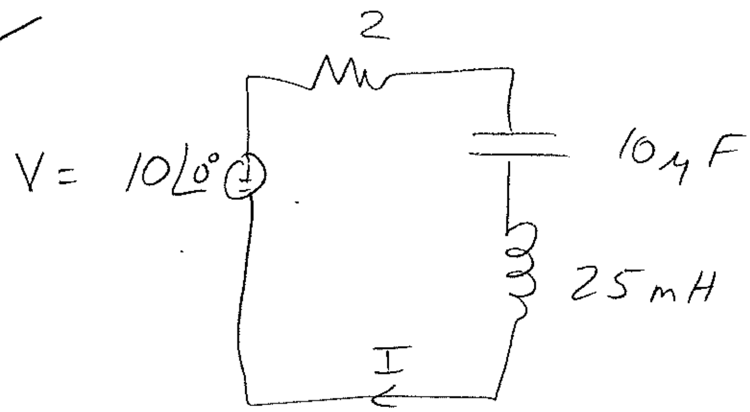
I IS MAXIMUM



$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$ Quality factor

$$BW = \frac{\omega_0}{Q}$$ BANDWIDTH $= f_U - f_L$

EX



FIND ω_o, Q, V_C, V_L, I @ ω_o

$$Z_T = R + j\omega L - \frac{1}{j\omega C}$$

$$\omega_o = \frac{1}{\sqrt{LC}} = 2000$$

@ ω_o $\quad I = \dfrac{V}{R}$ $\quad$ B/c $\quad j\omega L - \dfrac{1}{j\omega C} = 0$

$$= 5\angle 0°$$

$$V_R = I(2) = 10\angle 0°$$

$$V_L = j\omega L\, I = 250\angle 90°$$

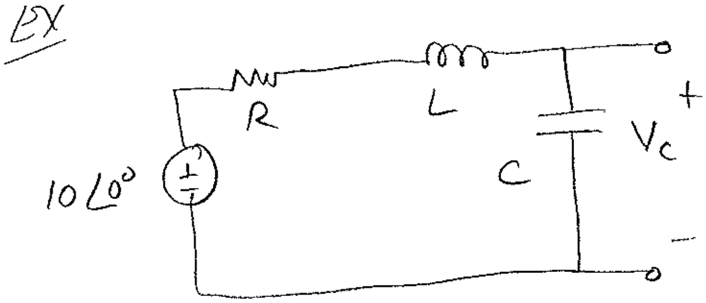
$$V_C = \frac{1}{j\omega C}(I) = 250\angle -90°$$

$$Q = \frac{\omega_o L}{R} = 25$$

NOTICE: $\quad |V_L| = \omega_o L|I| = \dfrac{\omega_o L}{R}|V_s| = Q|V_s|$

$$|V_C| = \frac{1}{\omega_o C}|I| = \frac{1}{\omega_o C R}|V_s| = Q|V_s|$$

EX



CKT MUST HAVE:

$f_0 = 1000$ Hz

$L = 0.02$ H

$R = 0.63$

CHOOSE C FIND V_c @ Resonance.

Q

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad C = 1.27\,\mu F \qquad \omega_0 = 6275 \text{ rad/s}$$

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi(1000)(0.02\ H)}{0.63} = 200$$

AT Resonance:

$$j\omega L - \frac{1}{j\omega C} = 0$$

$$Z_c = \frac{1}{j\omega C} = \frac{1}{2\pi f C}$$

$$Z_T = R$$

$$I = \frac{10\angle 0^\circ}{0.63} = 15.87\angle 0^\circ$$

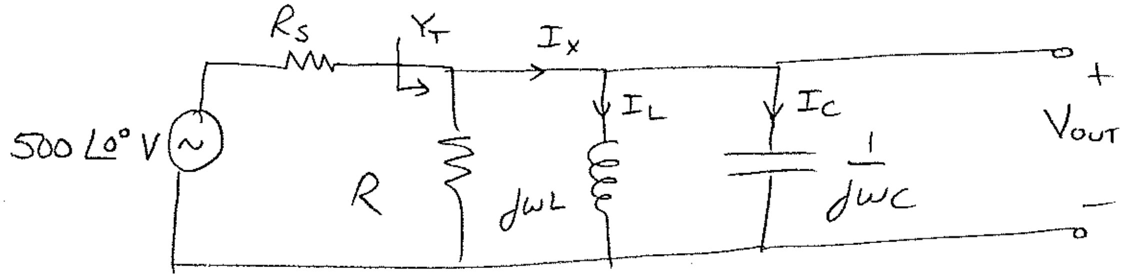
$$V_c = I\,Z_c = 6.28\angle 0^\circ \left(\frac{1}{2\pi(1000)(1.27\,\mu F)}\right) \angle -90$$

$$= 15.87\angle 0^\circ (125.3\angle -90)$$

$$V_c = \frac{V_s}{R}\left(\frac{1}{2\pi f C}\right) = 2000$$

$$\left(\frac{1}{2\pi f C}\right)$$

Parallel Resonance



$$Y_T = \frac{1}{R} + \frac{1}{jwL} + jwC$$

$$Y_T = \frac{1}{R} + j\left(wC - \frac{1}{wL}\right)$$

$$@\ \omega C = \frac{1}{\omega L}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi LC} \Rightarrow$$

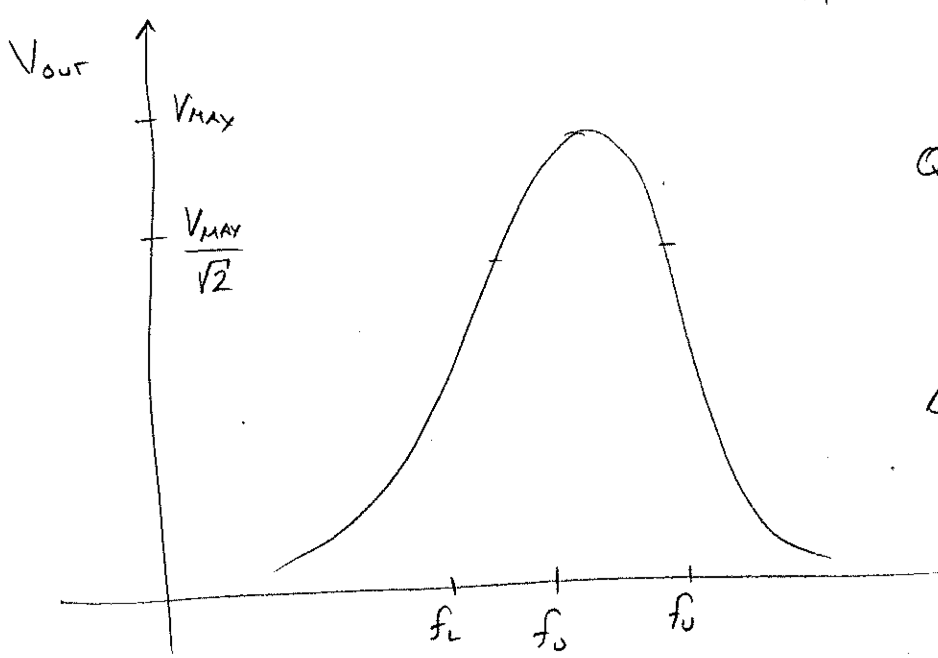
Y_T IS MINIMUM

Z_T IS MAXIMUM

V_{OUT} IS MAXIMUM

$|I_L| = |I_C|$ BUT ARE 180° OUT OF PHASE

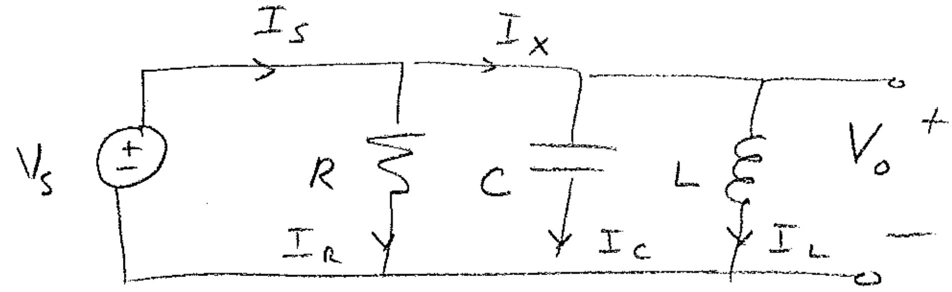
$\therefore I_X = 0$



$$Q = \frac{R}{\omega_0 L} = \omega_0 RC$$

$$BW = \frac{\omega_0}{Q}$$

$$BW = f_U - f_L$$



$V_S = 120\angle 0°$ $\quad R = 100 \quad C = 600\mu F \quad L = 120 mH$

FIND ALL I, V_o @ Resonance

$$\omega_o = \frac{1}{\sqrt{LC}} = 117.85 \text{ rad/s}$$

$$Y_C = j\omega_o C = j70.7 \times 10^{-3}$$

$$Y_L = \frac{1}{j\omega_o L} = -j70.7 \times 10^{-3}$$

$$I_R = \frac{V_S}{R} = \frac{120\angle 0°}{100} = 1.2\angle 0°$$

$$I_C = V_S Y_C = 8.49\angle 90°$$

$$I_L = V_S Y_L = 8.49\angle -90°$$

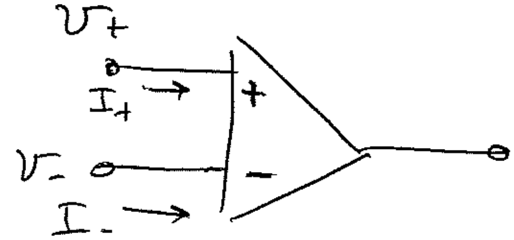
$$I_S = I_R + I_C + I_L = 1.2\angle 0° = I_S.$$

$V_o = V_S$ (OF COURSE)

OP-AMPS

1. ONLY USE NODE ANALYSIS
2. "+" AND "–" SIGNS ARE ARBITRARY
3. NO CURRENT FLOWS INTO AN OP-AMP
4. THERE IS 0 VOLTS BETWEEN INPUT TERMINALS

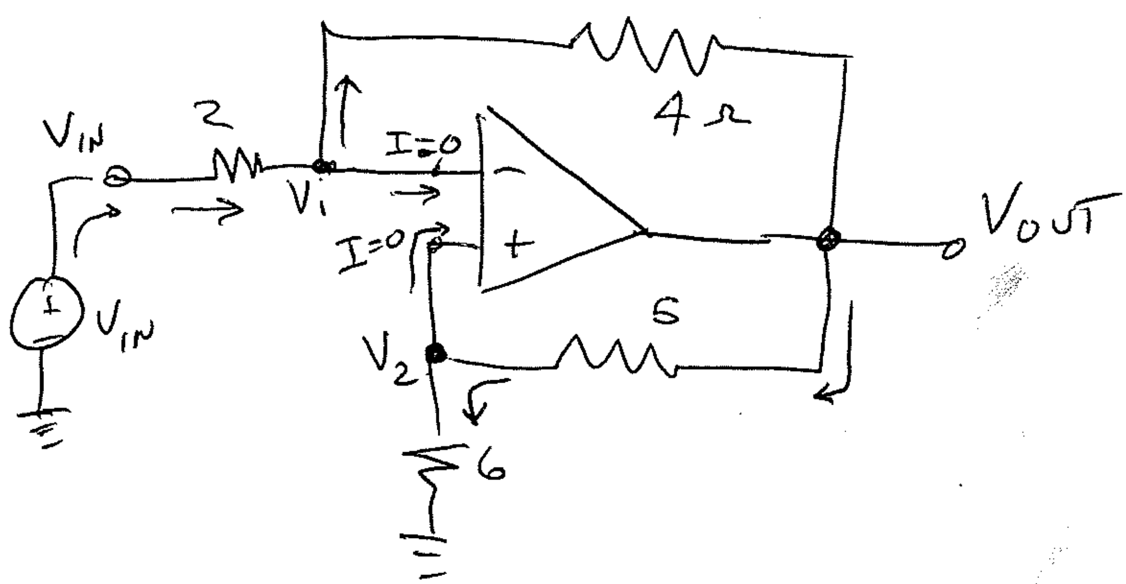
OP AMPS - IDEAL



NO CURRENT FLOWS INTO OP-AMP $I_+ = I_- = 0$

VOLTAGES AT v_+ AND v_- ARE SAME.

VIRTUAL SHORT

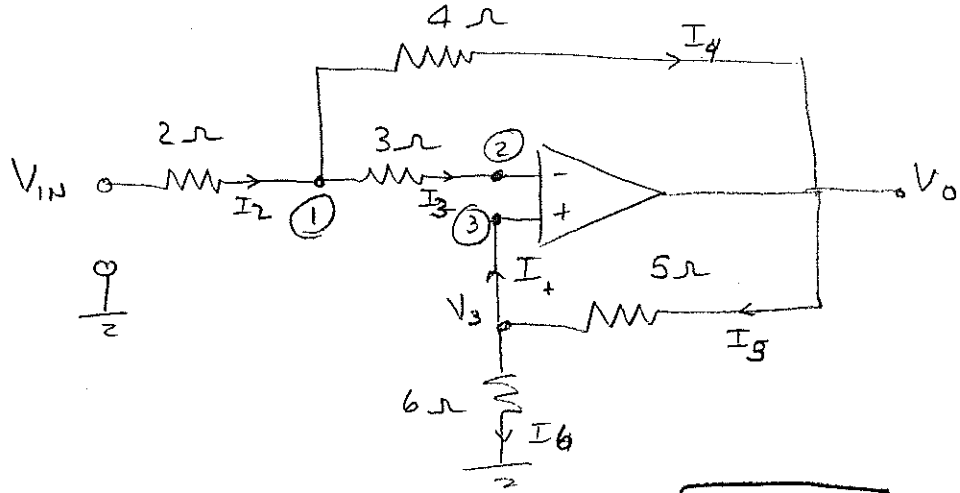


$V_1 = V_2$

@ NODE 1 $\quad \dfrac{V_{IN} - V_1}{2} = \dfrac{V_1 - V_{OUT}}{4} \qquad V_1 = f(V_{IN}, V_{OUT})$

@ NODE 2 $\quad \dfrac{V_{OUT} - V_2}{5} = \dfrac{V_2}{6}$

$\dfrac{V_{OUT}}{V_{IN}} \Rightarrow$ GAIN $\quad$ TRANSFER CHARACTERISTIC.



FIND $\frac{V_o}{V_{IN}}$:

USING IDEAL PROPERTIES:

$I_3 = 0$ $\quad I_2 = I_4$

$I_+ = 0$ $\quad I_5 = I_6$

$V_3 = V_1$

$\boxed{\text{NODE } 3}$: $I_5 = I_6$

$$\frac{V_o - V_3}{5} = \frac{V_3 - 0}{6}$$

$$6V_o - 6V_3 = 5V_3$$

$$6V_o = 11V_3$$

$$\boxed{V_3 = \frac{6}{11} V_o}$$

ALSO BY VOLTAGE DIVISION

$$V_3 = V_o\left(\frac{6}{5+6}\right) = V_1$$

$\boxed{\text{NODE 1}}$

$$\frac{V_{IN} - V_1}{2} = \frac{V_1 - V_o}{4} \rightarrow \frac{V_{IN} - \frac{6}{11}V_o}{2} = \frac{\frac{6}{11}V_o - V_o}{4}$$

$$2V_{IN} - \frac{12}{11}V_o = \frac{-5}{11}V_o$$

$$2V_{IN} = \frac{7}{11}V_o$$

$$\boxed{\frac{V_o}{V_{IN}} = \frac{22}{7}}$$