Implied Costs in Wireless Networks

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Abstract — **We calculate implied costs for wireless networks and use them for evaluating trade-offs between calls of different rates. We model user mobility by assigning probabilities for the departure of calls. We use Fixed Channel Assignment (FCA) with priority for handoffs over new call arrivals by reserving a number of channels for handoff calls in all the cells. The performance measures used are new call blocking and handoff drop probabilities. The implied cost is calculated for the network net revenue which considers the revenue generated by accepting a new call arrival into the network as well as the cost of a handoff drop in any cell. Simulation and numerical results are presented as well as confidence intervals showing the accuracy of the model. The implied costs also show that matching capacity distribution to not only exogenous traffic but also to mobility can significantly increase network revenue and quantifies this increase.**

I. Introduction

Recently, demand for wireless services has increased reflecting a need for person-to-person communication, which in turn brings new issues to consider such as mobility management for the service providers, [1]. One of those issues is the treatment of handoffs where, because of a user's mobility, the call has to be handed off from one base station to another. If the new base station does not have a channel available, the handoff call will be blocked. Typically, on account of customer indignation, the rejection of a handoff is considered to be more detrimental than the rejection of new incoming calls. Thus, the handoff blocking probability is an important criterion for the performance of wireless networks.

Essentially, admission of handoffs is done using one of three criteria. The first method is to consider handoffs and new call arrivals equally for occupancy of the channels, the second reserves channels in each cell to give priority to handoffs and the third sends handoffs to a queue if no channel is available. Several performance evaluation algorithms have been introduced for these handoff strategies such as [3] for the reservation strategy and the queueing strategy and in [7] for the reservation and no reservation strategy. In this paper we model a network where users are allowed to move to adjacent cells with certain probability once their call is in progress and we use a model with reservation to give priority to handoff calls versus new calls.

In general, the analysis techniques used for evaluating the performance of wireless networks require fixed point computations to obtain blocking probability and/or handoff drop probability. The use of fixed point computations and the consequent implicitness of the dependence of the blocking probability or the handoff drop performance on the entire network traffic obscure the effects of variables such as exogenous inputs on the performance measures. In this paper we use the concept of implied costs, [4], [5], to evaluate these.

Implied costs have been used in other applications. In [4], the author calculates implied costs, i.e., the derivative of the network rate of return with respect to external traffic and with respect to link capacities, for circuit-switched networks. In [5], this work is extended to include the case of trunk reservation and in [9] and [10] to the case of adaptive routing. Implied costs can be used in a number of applications such as in algorithms to aid capacity expansion decisions [2], [8], in pricing policy [6] and for the apportionment of revenue between various sections of a network, [11], to mention some.

An extension of a fixed point algorithm similar to the one in [7] is used to evaluate performance given by the new call blocking and the handoff drop probabilities for multiple classes of customers with different service rates, different call arrival rates and different bandwidth requirements for each class. We evaluate the accuracy of the algorithm by comparing analytical results to simulations. We present numerical results for several examples. We use implied costs with respect to new call arrival for multi-rate wireless networks.

In Section II, the analytical model is introduced as well as the notation and definitions used with assumptions needed in order to obtain the numerical results. In Section III the calculation of the implied costs is introduced and the definition of the net revenue function. In Section IV, the numerical results and examples obtained are presented together with the network used in the numerical analysis. Finally, the last section includes the conclusions.

II. Model Description

Consider an asymmetric cellular network where $\mathcal{N}$ is the set of cells and N, the total number of cells. Each cell i has C_i channels assigned to it. Let $\mathcal{A}_i$ be the set of cells adjacent to cell i. There are M classes of traffic which share the network resources. Let b_m be the number of channels required by traffic of class m, $m = 1, 2, \ldots, M$. The new call arrival process of class m to cell i is a Poisson Process with mean $\lambda_{i,m}$ independent of other new call arrival processes. Once a call is in progress in a cell, it can attempt one of three events: a handoff to an adjacent cell, stay within the cell or leave the network with the corresponding call termination. The time between these events for a call of class m in cell i, the dwell time, is a random variable with exponential distribution and mean $1/\mu_{i,m}$ and it is independent of earlier arrival times, call durations and elapsed times of other users.

Mobility of the users is achieved by considering distributions which determine the probability of occurrence of any of the three events that a call in progress may undergo. For each cell and for each class of traffic m: let $q_{ij,m}$ be the probability that a call in progress in cell i after completing its dwell time *goes* to cell j, i.e., there is a handoff from cell i to cell j of class m, if cell i and cell j are not adjacent then $q_{ij,m} = 0$. Let $q_{iT,m}$ be the probability that a call of class m in progress in cell i after completing its dwell time terminates and leaves the network and let $q_{ii,m}$ be the probability that a call in progress in cell i of class m after completing its dwell time will remain within the cell. All the cells have a channel reservation parameter T_m for traffic of class m. The reservation parameters are intended to give priority to handoff calls with respect to new calls through a reservation policy, that is, if the occupancy of a cell is such that its state is less than $C-T_m$, i.e., unreserved states, any call (new call or handoff) that requests service will be accepted, whereas if the occupancy is greater than or equal to $C-T_m$ only handoffs will be accepted and new calls will be denied service (blocked)

We consider that occupancy of the cells evolves according to an M-dimensional birth-death process independent of other cells, where the arrival rate or offered traffic to cell i of class m is $\rho_{i,m}$ for the unreserved states and $\alpha_{i,m}$ for reserved states, and the departure rate when cell i is in state $\mathbf{n}$ for calls of class m is $n_m \mu_{i,m}(1-q_{ii,m})$. Let $p_i(\mathbf{n})$ be the stationary probability that cell i is in state $\mathbf{n}$, this distribution can be found by solving the global balance equations of the multidimensional Markov chain.

Let $\nu_{ji,m}$ be the handoff rate of class m out of cell j offered to cell i, for adjacent cells i and j. The handoff traffic that can be offered from cell j to an adjacent cell i of class m depends on the proportion of new calls accepted of class m in cell j that goes into cell i, i.e., $\lambda_{j,m}(1-B_{j,m})q_{ji,m}$, and the proportion of handoff calls accepted of class m from cells adjacent to cell j that goes into cell i, i.e., $(1-B_{hj,m})q_{ji,m}\sum_{x\in\mathcal{A}_j}\nu_{xj,m}$. Thus, the handoff rate out of cell j offered to cell i of traffic class m is given by

$$\nu_{ji,m} = \lambda_{j,m}(1-B_{j,m})q_{ji,m} + (1-B_{hj,m})q_{ji,m}\sum_{x\in\mathcal{A}_j}\nu_{xj,m}\,. \quad (1)$$

This set of linear simultaneous equations in $\nu_{ji,m}$ can be solved to compute the total offered traffic of class m to cell i. This is given by

$$\rho_{i,m} = \lambda_{i,m} + \sum_{j\in\mathcal{A}_i}\nu_{ji,m}\ , \quad \mathbf{n}\in\mathcal{U}_i^{(m)}, \quad (2)$$

$$\alpha_{i,m} = \sum_{j\in\mathcal{A}_i}\nu_{ji,m}\ , \quad \mathbf{n}\in\mathcal{Q}_i^{(m)}\setminus\mathcal{B}_i^{(m)}. \quad (3)$$

For $\mathbf{n}\in\mathcal{B}_i^{(m)}$ the total offered traffic is zero.

The new call blocking probability for class m in cell i, $B_{i,m}$, and the handoff drop probability for class m in cell i, $B_{hi,m}$, are given as follows:

$$B_{i,m} = \sum_{\mathbf{r}\in\mathcal{Q}_i^{(m)}} p_i(\mathbf{r}), \quad \text{and} \quad B_{hi,m} = \sum_{\mathbf{r}\in\mathcal{B}_i^{(m)}} p_i(\mathbf{r}). \quad (4)$$

III. Implied Costs

In order to calculate the implied costs we need to define a function that captures the global effects of traffic demand on performance of the network, and we will obtain the implied costs of this function with respect to the new call arrival rates for any traffic class. Define the net revenue, W, as the revenue generated by the traffic which is carried succesfully. This revenue consists of two components: the first one is the revenue generated by accepting in each cell j a new call of class m, the second takes into account the cost of a forced termination due to handoff failure of those new calls of class m that have arrived and been accepted in cell j, Hence the net revenue is

$$\begin{aligned} W(\mathbf{B},\mathbf{B}_h,\underline{\lambda},\underline{\alpha}) &= \sum_{m=1}^{M}\sum_{i\in\mathcal{N}}\Big\{ w_{i,m}\lambda_{i,m}\left(1-B_{i,m}(\underline{\lambda},\mathbf{p})\right) \\ &- c_{i,m}B_{hi,m}(\underline{\lambda},\mathbf{p})\left\{\mathcal{I}_{\{T_m>0\}}\alpha_{i,m}(\mathbf{v})\right. \\ &+ \left.\mathcal{I}_{\{T_m=0\}}\left[\rho_{i,m}(\underline{\lambda},\mathbf{v})-\lambda_{i,m}\right]\right\}\Big\}, \end{aligned} \quad (5)$$

where $w_{j,m}$ is the revenue generated by accepting a call of class m in cell j, and $c_{j,m}$ is the cost of a forced termination of a call of class m due to a handoff failure. $\mathbf{v}$ denotes the vector whose components are the handoff rates $\nu_{ji,m}$ for all $i,j\in\mathcal{N}$ and for $m=1,2,\ldots,M$, $\mathbf{p}$ denotes the vector whose components are the stationary probabilities for each state of all the cells, $\mathbf{B}$, the vector of the new call blocking probabilities for all the cells and classes, $\mathbf{B}_h$, the vector of the handoff call blocking probabilities for all the cells and classes, $\underline{\lambda}$, the vector of new call arrival rates and $\underline{\alpha}$, the vector of offered traffic to the cells for both, reserved and unreserved states. $B_{i,s}(\underline{\lambda},\mathbf{p})$ and $B_{hi,s}(\underline{\lambda},\mathbf{p})$ are given by equation (4) and are written here to explicitly show their dependence on $\underline{\lambda}$ and $\mathbf{p}$.

The choice of the revenue for cell i and traffic class m, $w_{i,m}$, depends on the number of channels each call of that class occupies, b_m, and the average holding time of the calls of that class. The average holding time depends on the average number of handoffs the calls undergo before departure from the network and since every time a call is accepted in a cell its duration in that cell is the dwell time with mean $1/\mu_{i,m}$, the average holding time of a call will be given by the average number of handoffs times the dwell time. Therefore, in (5), we take the revenues to be $w_{i,m} = b_m\cdot$(average holding time).

Note that since the equilibrium probability vectors and blocking probabilities we use are those given by the fixed point model, rather than the actual equilibrium probabilities and blocking probabilities, the revenue W in (5) is an *approximate* network net revenue. As a result, the implied costs we calculate are also approximations to the actual implied costs. The fixed point model describes the $\mathbf{p}$ as an implicit function of $\underline{\lambda}$. $\mathbf{B}$ and $\mathbf{B}_h$ are, in turn, functions of $\mathbf{p}$ and thereby implicit functions of $\underline{\lambda}$. Consequently, $W(\mathbf{B},\mathbf{B}_h,\underline{\lambda},\underline{\alpha})$ is also an implicit function of $\underline{\lambda}$. We therefore undertake a careful and extensive effort to obtain relations of total and partial derivatives of the new call and handoff blocking probabilities by differentiating the fixed point equations. These relations are manipulated to obtain a system of linear equations in the derivatives of the new call and handoff blocking probabilities with respect to new call traffic.

IV. Numerical Results

This section presents numerical and simulation results for the model presented. We evaluate network revenue for various cases of load and mobility, present examples of how the implied costs can be incorporated into call pricing.

A Blocking and Revenue Results

The calculations and simulations were done for the 10-cell network shown in Figure 1. In the figures, $T=[x,y]$ refers to the reservation of x channels for calls of class 1 and y channels for class 2 in all the cells of the network, and $\mu=[x,y]$ refers to a dwell time of $1/x$ for calls of class 1 and $1/y$ for class 2 in

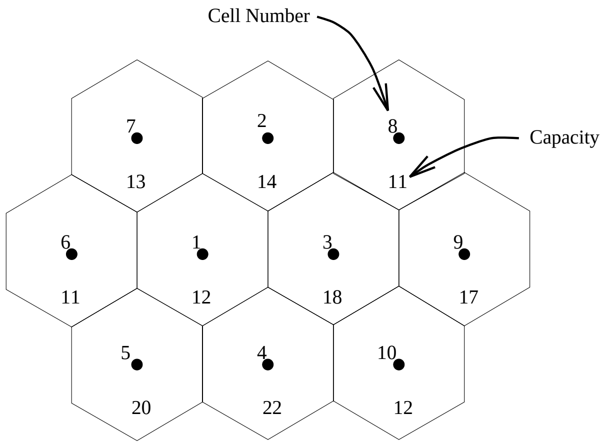

Figure 1: Ten-Cell Network Used in Examples

all the cells. The parameters of the examples are summarized in Table 1.

Table 1: Parameters Ten-Cell Network, 20% Load, ($\lambda_{1,1}$ varied)

Simulation Parameters							
Cell i	$\lambda_{i,1}$	$\lambda_{i,2}$	C_i	$\mu_{i,1}$	$\mu_{i,2}$	$q_{iT,1}$	$q_{iT,2}$
1	*	0.1090	12	1.0	0.5	0.8	0.85
2	2.5454	0.1272	14	1.0	0.5	0.8	0.85
3	3.2727	0.1636	18	1.0	0.5	0.8	0.85
4	4.0000	0.2000	22	1.0	0.5	0.8	0.85
5	3.6363	0.1818	20	1.0	0.5	0.8	0.85
6	2.0000	0.1000	11	1.0	0.5	0.8	0.85
7	2.3636	0.1181	13	1.0	0.5	0.8	0.85
8	2.0000	0.1000	11	1.0	0.5	0.8	0.85
9	3.0909	0.1545	17	1.0	0.5	0.8	0.85
10	2.1818	0.1090	12	1.0	0.5	0.8	0.85

In the examples, the channel reservation parameters of the classes were varied and the performance evaluated in terms of the new call blocking probability, $B_{i,m}$, and the handoff drop probability, $B_{hi,m}$, of both classes. All the results were obtained with class 1 traffic occupying one channel per call and class 2 traffic 2 channels. The figures shown were obtained by keeping the new call arrival for all the cells and both classes constant except for class 1 of cell 1 since this is one of the cells with the most number of adjacent cells. In Figure 2, the new call blocking for both classes is shown as the external arrival rate of class 1 traffic of cell 1 is varied. It can be seen that the new call blocking for class 2 is higher than that of class 1 due to the bandwidth requirement of two channels instead of one, the figure also shows the close agreement between simulation and numerical analysis in the performance evaluation model obtained. The handoff blocking is the same as that in Figure 2 since there is no reservation for any of the classes of traffic.

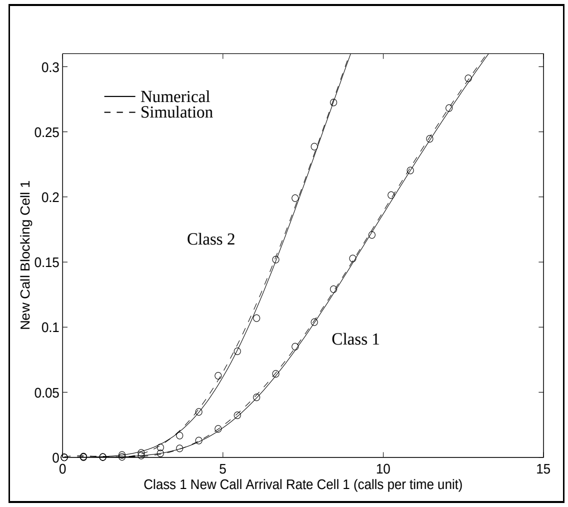

Figure 2: New Call Blocking, $T = [0, 0]$, $\mu = [1, \frac{1}{2}]$, 20% load

B Implied Costs Results

Figure 3 compares the network net revenue as a function of the class 1 new call arrival of cell 1 for the 10-cell network obtained from simulation and from the fixed point model. It can be seen that the net revenue decreases due to the increase in new call blocking. It is also seen that simulation and numerical results are within agreement as expected since the blocking probabilities are in close agreement. From the figure it can also be seen that as the new call arrival increases and stays below a level of 8 calls per time unit, the increase in revenue is constant, whereas revenue is decreased once the external arrival increases above this value.

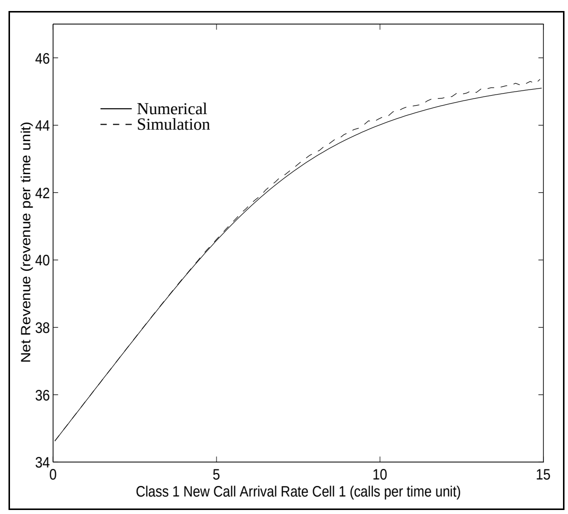

Figure 3: Net Revenue for 10-Cell Network, 20% load, $T = [0, 0]$, $\mu = [1, \frac{1}{2}]$

To verify the accuracy of our numerical calculation of the implicit costs, in Figure 4 we compared these obtained from numerical calculation with those obtained from simulation for the 10-cell network as a function of the new call arrival rate of cell 1 of class 1.

In Figure 4, the implicit cost of the net revenue is shown with respect to the new call arrival of class 1 of cell 1, together with the implied cost obtained by simulation. The case analyzed was the ten cell network for 20% of load and no reservation for any of the classes of traffic. The 99% confidence intervals are also included to show the close agreement of the model with the simulation. The implicit cost with respect to class 1 decreases rapidly after five units of new call arrival rate because of the degradation on performance by accepting more calls, increasing blocking.

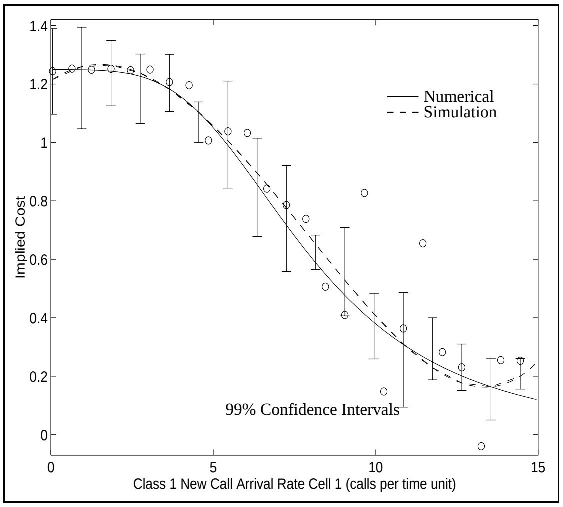

Figure 4: Implied cost for 10-Cell Network, 20% load, $T = [0, 0]$, $\mu = [1, \frac{1}{2}]$

Figures 5 and 6 show the net revenue and implied cost, respectively, for the same ten cell network for 50% of load varying mobility, meaning low and high, where low mobility is fixed with $q_{iT,1} = 0.8$ and $q_{iT,2} = 0.85$ as in all the previous examples, and high mobility is fixed with $q_{iT,1} = 0.5$ and $q_{iT,2} = 0.55$. It can be seen that due to the increase in traffic in the network, because users tend to stay more time connected, the blocking of new and handoff calls will increase and the net revenue will decrease. It can also be seen that for different levels of mobility reservation has to be chosen wisely since performance will be degraded differently for each class of traffic. For the high mobility case, we have less revenue, than that with low mobility and if we chose some reservation levels, we will improve revenue. This is presented as load is increased.

In Figure 6, we can see that mobility is captured by the implied costs by decreasing their value. We can also see that as the implied cost is decreased, the revenue is also decreased, hence, the larger the implied cost the larger the revenue and the better the blocking level the network is experiencing. The figure also shows that reservation has to be chosen wisely depending on the implied cost obtained, that is, if we have a small value of the implied cost, we will have that a connection establishment will be more expensive and it is not a good idea to accept more traffic of that class in that cell, therefore, we could determine some admission control criteria based on the implied costs.

V. Conclusions

We described the calculation of implied costs with respect to the new call arrival rates for wireless networks with multiple classes of customers, and show their use for evaluating trade-offs between calls of different rates and for different calls based on mobilities and bandwidth. Reservation parameters need to be chosen wisely depending on the mobility, the bandwidth, the arrival rate and the dwell time of the calls. This provides evidence that matching capacity distribution to exogenous traffic and mobility is important.

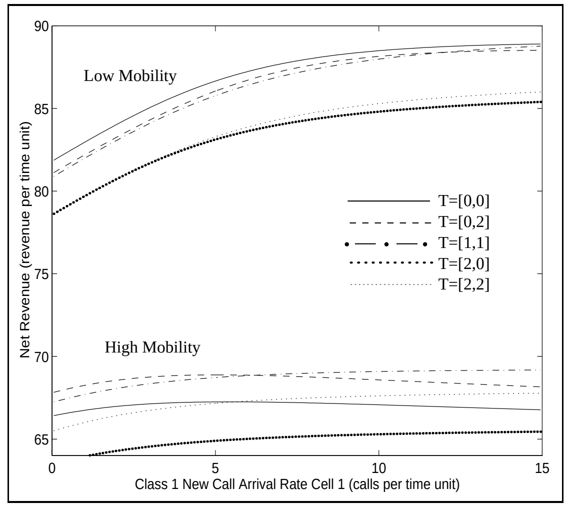

Figure 5: Net revenue for 10-Cell Network, 50% load, $\mu = [1, \frac{1}{2}]$

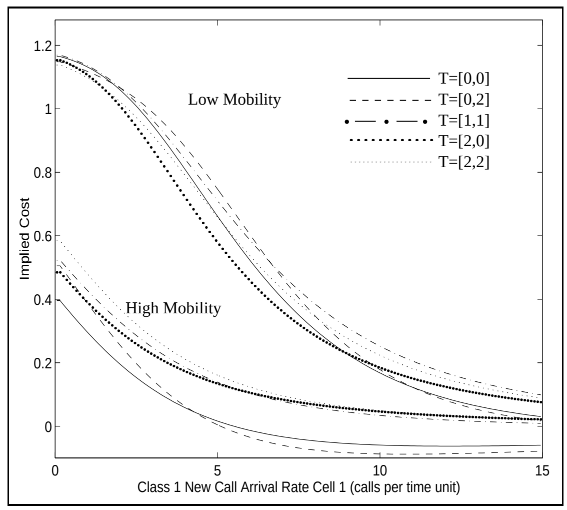

Figure 6: Implied cost for 10-Cell Network, 50% load, $T = [0, 0]$, $\mu = [1, \frac{1}{2}]$

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