1. Potential customers arrive at a full-service, one-pump gas station at a Poisson rate of 20 cars per hour. However, customers will only enter the station for gas if there are no more than two cars (including the one currently being attended to) at the pump. Suppose the amount of time required to service a car is exponentially distributed with a mean of five minutes.

(a) What fraction of the attendant’s time will be spent servicing cars?
(b) What fraction of potential customers are lost?

2. A service center consists of two servers, each working at an exponential rate of two services per hour. If customers arrive at a Poisson rate of three per hour, then, assuming a system capacity of at most three customers,

(a) What fraction of potential customers enter the system?
(b) What would the value of part (a) be if there was only a single server, and his rate was twice as fast (that is, $\mu = 4$)?

3. Each time a machine is repaired it remains up for an exponentially distributed time with rate $\lambda$. It then fails, and its failure is either of two types. If it is a type 1 failure, then the time to repair the machine is exponential with rate $\mu_1$; if it is a type 2 failure, then the repair time is exponential with rate $\mu_2$. Each failure is, independently of the time it took the machine to fail, a type 1 failure with probability $p$ and a type 2 failure with probability $1 - p$. What proportion of time is the machine down due to a type 1 failure? What proportion of time is it down due to a type 2 failure? What proportion of time is it up?

4. After being repaired, a machine functions for an exponential time with rate $\lambda$ and then fails. Upon failure, a repair process begins. The repair process proceeds sequentially through $k$ distinct phases. First a phase 1 repair must be performed, then a phase 2, and so on. The times to complete these phases are independent, with phase $i$ taking an exponential time with rate $\mu_i$, $i = 1, \ldots, k$.

(a) What proportion of time is the machine undergoing a phase $i$ repair?
(b) What proportion of time is the machine working?

5. Customers arrive at a single-server queue in accordance with a Poisson process having rate $\lambda$. However, an arrival that finds $n$ customers already in the system will only join the system with probability $1/(n + 1)$. That is, with probability $n/(n + 1)$ such an arrival will not join the system. Show that the limiting distribution of the number of customers in the system is Poisson with mean $\frac{\lambda}{\mu}$. 