Problem 1

(a) Let the input voltage $v_1$ consist of a sinusoidal wave of frequency $\frac{1}{2} f_c$ (i.e., half the desired carrier frequency) and the message signal $m(t)$:

$$v_1 = A_c \cos(\pi f_c t) + m(t)$$

Then, the output current $i_o$ is

$$i_o = a_1 v_1 + a_3 v_1^3$$

$$= a_1 [A_c \cos(\pi f_c t) + m(t)] + a_3 [A_c \cos(\pi f_c t) + m(t)]^3$$

$$= a_1 [A_c \cos(\pi f_c t) + m(t)] + \frac{1}{4} a_3 A_c^3 [\cos(3\pi f_c t) - 3\cos(\pi f_c t)]$$

$$+ \frac{3}{2} A_c^2 m(t)(1 + \cos(2\pi f_c t)) + 3 a_3 A_c \cos(\pi f_c t) m^2(t) + a_3 m^3(t)$$

Assume that $m(t)$ occupies the frequency interval $-W \leq f \leq W$. Then, the amplitude spectrum of the output current $i_o$ is as shown below.

From this diagram we see that in order to extract a MSBSC wave, with carrier frequency $f_c$ from $i_o$, we need a band-pass filter with mid-band frequency $f_c$ and bandwidth $2W$, which satisfy the requirement:

$$f_c - W > \frac{f_c}{2} + 2W$$

that is, $f_c > 4W$
Therefore, to use the given nonlinear device as a product modulator, we may use the following configuration:

\[ \frac{3}{2} a_3 A_c^2 m(t) \cos(2\pi f_c t) \]

(b) To generate an AM wave with carrier frequency \( f_c \), we require a sinusoidal component of frequency \( f_c \) to be added to the DSBSC generated in the manner described above. To achieve this requirement, we may use the following configuration involving a pair of the nonlinear devices and a pair of identical bandpass filters.

The resulting AM wave is therefore \( \frac{3}{2} a_3 A_c^2 (A_0 m(t)) \cos(2\pi f_c t) \). Thus, the choice of the dc level \( A_0 \) at the input of the lower branch controls the percentage modulation of the AM wave.
Problem 2

(a) The envelope detector output is

\[ v(t) = A_m \cos(2\pi f_m t) \]

which is illustrated below for the case when \( \nu=2 \).

We see that \( v(t) \) is periodic with a period equal to \( f_m \), and an even function of \( t \), and so we may express \( v(t) \) in the form:

\[ v(t) = a_0 + 2 \sum_{n=1}^\infty a_n \cos(2\pi nf_m t) \]

where

\[ a_0 = \frac{1}{2f_m} \int_{0}^{1/2f_m} v(t)dt \]

\[ a_n = \frac{1}{2f_m} \int_{0}^{1/2f_m} v(t)\cos(2\pi nf_m t)dt \]

\[ = \frac{A_m}{3} + \frac{4A_m}{\pi} \sin(\frac{2\pi n}{3}) \]  

(1)

\[ = 2A_m \frac{1}{3f_m} \int_{0}^{1/3f_m} [1+2\cos(2\pi f_m t)]dt \]

\[ = 2A_m \frac{1}{3f_m} \int_{0}^{1/3f_m} [-1-2\cos(2\pi f_m t)]dt \]

\[ = \frac{A_m}{3} + \frac{4A_m}{\pi} \sin(\frac{2\pi n}{3}) \]
Problem 3

\[
+ 2A_c f_m \int_{1/3f_m}^{1/2f_m} \left( -1 - 2\cos(2\pi f_m t) \right) \cos(2\pi f_m t) dt \\
= \frac{A_c}{\pi^2} \left( 2 \sin\left( \frac{2\pi m}{3} \right) - \sin(\pi m) \right) + \frac{A_c}{(n+1)\pi} \left( 2 \sin\left( \frac{2\pi (n+1)}{3} \right) - \sin(\pi (n+1)) \right) \\
+ \frac{A_c}{(n-1)\pi} \left( 2 \sin\left( \frac{2\pi (n-1)}{3} \right) - \sin(\pi (n-1)) \right)
\] (2)

For \( n = 0 \), Eq. (2) reduces to that shown in Eq. (1).

(b) For \( n = 1 \), Eq. (2) yields

\[ a_1 = \frac{\sqrt{3}}{2\pi} \left( \sqrt{3} + \frac{1}{3} \right) \]

For \( n = 2 \), it yields

\[ a_2 = \frac{\sqrt{3}}{2\pi} \]

Therefore, the ratio of second-harmonic amplitude to fundamental amplitude in \( v(t) \) is

\[ \frac{a_2}{a_1} = \frac{3\sqrt{3}}{2\pi + 3\sqrt{3}} \approx 0.452 \]

Problem 3

(a) For \( f_c = 1.25 \) kHz, the spectra of the message signal \( m(t) \), the product modulator output \( s(t) \), and the coherent detector output \( v(t) \) are as follows, respectively:
To avoid sideband overlap, the carrier frequency $f_c$ must be greater than or equal to 1 kHz. The lowest carrier frequency is therefore 1 kHz for each sideband of the modulated wave $s(t)$ to be uniquely determined by $m(t)$. 

(b) For the case when $f_c = 0.75$, the respective spectra are as follows:
Problem 4

An error $\Delta f$ in the frequency of the local oscillator in the demodulation of an SSB signal, measured with respect to the carrier frequency $f_c$, gives rise to distortion in the demodulated signal. Let the local oscillator output be denoted by $A_c' \cos(2\pi f_c t + \Delta f)$. The resulting demodulated signal is given by (for the case when the upper sideband only is transmitted)

$$v_0(t) = \frac{1}{4} A_c A_c' \left[ m(t) \cos(2\pi \Delta f t) + m(t) \sin(2\pi \Delta f t) \right]$$

This demodulated signal represents an SSB wave corresponding to a carrier frequency $\Delta f$.

The effect of frequency error $\Delta f$ in the local oscillator may be interpreted as follows:

(a) If the SSB wave $s(t)$ contains the upper sideband and the frequency error $\Delta f$ is positive, or equivalently if $s(t)$ contains the lower sideband and $\Delta f$ is negative, then the frequency components of the demodulated signal $v_0(t)$ are shifted inward by the amount $\Delta f$ compared with the baseband signal $m(t)$, as illustrated in Fig. 1(b).

(b) If the incoming SSB wave $s(t)$ contains the lower sideband and the frequency error $\Delta f$ is positive, or equivalently if $s(t)$ contains the upper sideband and $\Delta f$ is negative, then the frequency components of the demodulated signal $v_0(t)$ are shifted outward by the amount $\Delta f$, compared with the baseband signal $m(t)$. This is illustrated in Fig. 1c for the case of a baseband signal (e.g., voice signal) with an energy gap occupying the interval $-f_s \leq f \leq f_s$, in part (a) of the figure.
Problem 5

(a,b) The spectrum of the message signal is illustrated below:

Correspondingly, the output of the upper first product modulator has the following spectrum:
Correspondingly, the output of the upper first product modulator has the following spectrum:

\[ \frac{1}{2} M(f + f_o) \]
\[ \frac{1}{2} M(f - f_o) \]

The output of the lower first product modulator has the spectrum:

\[ \frac{1}{2} M(f - f_o) \]
\[ \frac{1}{2} M(f + f_o) \]

The output of the upper low pass filter has the spectrum:

\[ \frac{1}{2} M_+ (f + f_o) \]
\[ \frac{1}{2} M_- (f - f_o) \]
The output of the lower low pass filter has the spectrum:

\[ -\frac{f_0}{2} - \frac{f_a}{4} \quad \frac{1}{2j} M_+ \left( f - \frac{f_0}{2} \right) \]

\[ -\frac{1}{2j} M_+ \left( f + \frac{f_0}{2} \right) \]

The output of the upper second product modulator has the spectrum:

\[ -\frac{1}{4} M_+ \left( f + \frac{f_0}{2} + \frac{f_c}{2} \right) \]

\[ \frac{1}{4} M_- \left( f - \frac{f_0}{2} + \frac{f_c}{2} \right) \]

The output of the lower second product modulator has the spectrum:

\[ -\frac{1}{4} M_- \left( f + \frac{f_0}{2} + \frac{f_c}{2} \right) \]

\[ \frac{1}{4} M_+ \left( f - \frac{f_0}{2} - \frac{f_c}{2} \right) \]

Adding the two second product modulator outputs, their upper sidebands add constructively while their lower sidebands cancel each other.

(a) To modify the modulator to transmit only the lower sideband, a single sign change is required in one of the channels. For example, the lower first product modulator could multiply the message signal by \(-\sin(2f_0t)\). Then, the upper sideband would be cancelled and the lower one transmitted.