An Approximation Approach to Decentralized H_∞ Control[1]

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Abstract

In this paper, we propose a decentralized H_∞ control design technique through approximation. We show that a decentralized H_∞ control problem as well as any fixed structured H_∞ control problem can be (conservatively) converted into a model approximation problem. We then propose some explicit parameterizations of the decentralized controllers and the final decentralized controllers are obtained through some convex optimization.

1 Introduction

In many aerospace and other industrial applications, the control systems are highly complex and consist of many subsystems where local controllers are used. It is highly desirable for a high performance control system that those local decentralized controllers are designed such that the global performance of the system is taken into consideration. Thus it is of great importance to develop high performance and robust decentralized control techniques for highly complex systems. Many decentralized control design techniques have been proposed in the literature over the years. For example, a sequential design method can be used [6]. A decentralized control stabilization using time varying controllers is described in [7]. An LMI approach to decentralized H_∞ control is proposed in [8]. An H_2 decentralized control design using the local controller parameterization is considered in [5]. The parameterization of all decentralized stabilizing controller is considered in [9]. In general, the generalization of the centralized control design technique to the decentralized setting is fairly complicated and conservative. Indeed, the conservativeness in decentralized control design seems to be the way of life. The work described here is no exception.

In this paper, we shall propose a decentralized H_∞ control design technique based on model reduction and optimization technique.

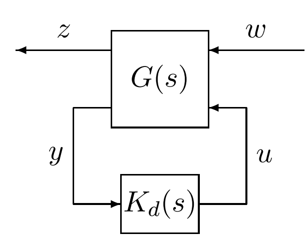

Figure 1: Closed-loop system diagram

To start with, let us assume that we can formulate the control design problem as an H_∞ optimization problem with a generalized plant G and our purpose is to design a decentralized

[1] This research was supported in part by grants from AFOSR (F49620-99-1-0179), NASA (NCC5-573) and LEQSF (NASA/LEQSF(2001-04)-01).
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stabilizing controller

$$K_d(s) = \begin{bmatrix} K_1(s) & & & \\ & K_2(s) & & \\ & & \ddots & \\ & & & K_m(s) \end{bmatrix}$$

such that $\|T_{zw}\|_\infty < \gamma$ for some $\gamma > 0$ where T_{zw} is the transfer function from w to z:

$$T_{zw} = G_{11} + G_{12}K_d(I - G_{22}K_d)^{-1}G_{21}$$

and

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

2 Centralized H_∞ Control

In this section, we shall first summarize the standard H_∞ control result where centralized controllers are used. Consider a feedback system shown in Figure 2 with a generalized plant realization given by

$$G(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right].$$

Assumed that the following standard assumptions are satisfied:

(A1) (A, B_2) is stabilizable and (C_2, A) is detectable;

(A2) D_{12} has full column rank and D_{21} has full row rank;

(A3) $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank for all ω;

(A4) $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank for all ω.

Then it is well known that all stabilizing controllers satisfying $\|T_{zw}\|_\infty < \gamma$ can be parameterized as

$$K = \mathcal{F}_\ell(M_\infty, Q), \;\; Q \in RH_\infty, \;\; \|Q\|_\infty < \gamma \quad (1)$$

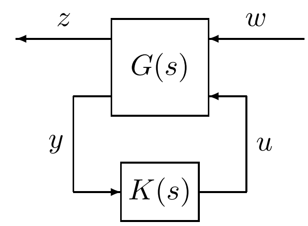

Figure 2: Closed-loop system diagram

where M_∞ is of the form

$$M_\infty = \begin{bmatrix} M_{11}(s) & M_{12}(s) \\ M_{21}(s) & M_{22}(s) \end{bmatrix} = \left[\begin{array}{c|cc} \hat{A} & \hat{B}_1 & \hat{B}_2 \\ \hline \hat{C}_1 & \hat{D}_{11} & \hat{D}_{12} \\ \hat{C}_2 & \hat{D}_{21} & \hat{D}_{22} \end{array}\right]$$

such that $\hat{D}_{12}$ and $\hat{D}_{21}$ are invertible and $\hat{A} - \hat{B}_2\hat{D}_{12}^{-1}\hat{C}_1$ and $\hat{A} - \hat{B}_1\hat{D}_{21}^{-1}\hat{C}_2$ are both stable (i.e., M_{12}^{-1} and M_{21}^{-1} are both stable). See [2, 11, 12] for details.

Thus it is clear that our desired decentralized controllers must be in the family of the parameterization. Unfortunately, it is very hard to find such decentralized controllers directly.

3 Decentralized H_∞ Controller Design by Approximation

The problem to be considered here is to find a controller K_d with a certain structure and/or order such that the H_∞ performance requirement $\|\mathcal{F}_\ell(G, K_d)\|_\infty < \gamma$ is satisfied for a given $\gamma > 0$. This is clearly equivalent to finding a Q so that it satisfies the preceding constraint so that K_d has certain structure and/or order. Instead of choosing Q directly, we shall approach this problem from approximation point of view.

3.1 Additive Controller Approximation

Consider a class of controllers that can be represented in the form

$$K_d = K_0 + W_2\Delta W_1,$$

where K_0 may be interpreted as a nominal and centralized controller and Δ is a stable perturbation with stable, minimum phase and invertible weighting functions W_1 and W_2. Suppose that $\|\mathcal{F}_\ell(G, K_0)\|_\infty < \gamma$. A natural question is whether it is possible to obtain a controller K_d with some specific structure in this class such that the H_∞ performance bound remains valid when K_d is in place of K_0. Note that this is somewhat a special case of the preceding general problem: The specific form of K_d means that K_d and K_0 must possess the same right-half plane poles, thus to a certain degree limiting the set of attainable structured controllers.

Define

$$\bar{K}_a^{-1} := \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} M_\infty^{-1} \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}.$$

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} = \mathcal{S}(\bar{K}_a^{-1}, \begin{bmatrix} K_o & I \\ I & 0 \end{bmatrix}).$$

and

$$\tilde{R} = \begin{bmatrix} \gamma^{-1/2} I & 0 \\ 0 & W_1 \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} \gamma^{-1/2} I & 0 \\ 0 & W_2 \end{bmatrix}$$

Then the following result follows immediately from results in [4].

Theorem 1 *Let K_0 be a stabilizing controller such that $\|\mathcal{F}_\ell(G, K_0)\|_\infty < \gamma$ and suppose W_1 and W_2 are stable, minimum phase and invertible transfer matrices such that $\tilde{R}$ is a contraction. Then K_d is also a stabilizing controller such that $\|\mathcal{F}_\ell(G, K_d)\|_\infty < \gamma$ if K_d has the same number of unstable poles as K_0 and*

$$\|\Delta\|_\infty = \left\| W_2^{-1}(K_d - K_0) W_1^{-1} \right\|_\infty < 1.$$

Since $\tilde{R}$ can always be made contractive for sufficiently small W_1 and W_2, there are infinite many W_1 and W_2 that satisfy the conditions in the theorem. It is obvious that it would be easier to make $\left\| W_2^{-1}(K_d - K_0) W_1^{-1} \right\|_\infty < 1$ for some K_d if the "largest" W_1 and W_2 are selected such that $\tilde{R}$ is a contraction.

Lemma 2 *Assume that $\|R_{22}\|_\infty < \gamma$ and define*

$$L = \begin{bmatrix} L_1 & L_2 \\ L_2^\sim & L_3 \end{bmatrix}$$

$$= \mathcal{F}_\ell(\left[\begin{array}{cc:cc} 0 & -R_{11} & 0 & R_{12} \\ -R_{11}^\sim & 0 & R_{21}^\sim & 0 \\ \hdashline 0 & R_{21} & 0 & -R_{22} \\ R_{12}^\sim & 0 & -R_{22}^\sim & 0 \end{array}\right], \gamma^{-1} I).$$

Then $\tilde{R}$ is a contraction if W_1 and W_2 satisfy

$$\begin{bmatrix} (W_1^\sim W_1)^{-1} & 0 \\ 0 & (W_2 W_2^\sim)^{-1} \end{bmatrix} \geq \begin{bmatrix} L_1 & L_2 \\ L_2^\sim & L_3 \end{bmatrix}.$$

A numerical algorithm that maximizes $\det(W_1^\sim W_1)\det(W_2 W_2^\sim)$ has been developed by Goddard and Glover in [4]. Thus we have reduced (conservatively) the original decentralized H_∞ controller design problem into an decentralized approximation problem.

A reasonable assumption is that the controller K_o has all the correct poles! Let

$$K_o(s) = \left[\begin{array}{c|c} A_0 & B_0 \\ \hline C_0 & D_0 \end{array}\right]$$

Then the poles of $K_o(s)$ are given by the roots of the polynomial $d(s) = \det(sI - A_0)$. Suppose A_0 is stable. Then we shall parameterize the decentralized controller K_d as follows

$$K_d = \frac{\begin{bmatrix} N_1(s) & & & \\ & N_2(s) & & \\ & & \ddots & \\ & & & N_m(s) \end{bmatrix}}{d(s)}$$

where $N_i(s)$ are polynomial matrices to be optimized.

The procedure below, devised directly from the preceding theorem, can be used to generate a required decentralized controller that will preserve the closed-loop H_∞ performance bound $\|\mathcal{F}_\ell(G, K_d)\|_\infty < \gamma$.

1. Let K_0 be a full-order stable controller such that $\|\mathcal{F}_\ell(G, K_0)\|_\infty < \gamma$;
2. Compute W_1 and W_2 so that $\tilde{R}$ is a contraction;
3. The decentralized controller can be found from the following minimization

$$\min_{N_i} \left\| W_1^{-1} (K_0 - K_d) W_2^{-1} \right\|_\infty < 1$$

We note that the reduced order decentralized controller design can be easily dealt with in this framework.

3.2 Coprime Factor Approximation

The H_∞ controller reduction problem can also be considered in the coprime factor framework. One of the advantages in using coprime factor approximation is that K_0 and K_d need not be stable or have the same number of unstable poles [11, 12]. For that purpose, we need the following alternative representation of all admissible H_∞ controllers:

Lemma 3 *The family of all admissible controllers such that $\|T_{zw}\|_\infty < \gamma$ can also be written as*

$$\begin{aligned} K(s) &= \mathcal{F}_\ell(M_\infty, Q) \\ &= (\Theta_{11}Q + \Theta_{12})(\Theta_{21}Q + \Theta_{22})^{-1} := UV^{-1} \\ &= (Q\tilde{\Theta}_{12} + \tilde{\Theta}_{22})^{-1}(Q\tilde{\Theta}_{11} + \tilde{\Theta}_{21}) := \tilde{V}^{-1}\tilde{U} \end{aligned}$$

where $Q \in RH_\infty$, $\|Q\|_\infty < \gamma$, and UV^{-1} and $\tilde{V}^{-1}\tilde{U}$ are, respectively, right and left coprime factorizations over RH_∞, and

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{bmatrix}$$

$$= \left[\begin{array}{c|cc} \hat{A} - \hat{B}_1\hat{D}_{21}^{-1}\hat{C}_2 & \hat{B}_2 - \hat{B}_1\hat{D}_{21}^{-1}\hat{D}_{22} & \hat{B}_1\hat{D}_{21}^{-1} \\ \hline \hat{C}_1 - \hat{D}_{11}\hat{D}_{21}^{-1}\hat{C}_2 & \hat{D}_{12} - \hat{D}_{11}\hat{D}_{21}^{-1}\hat{D}_{22} & \hat{D}_{11}\hat{D}_{21}^{-1} \\ -\hat{D}_{21}^{-1}\hat{C}_2 & -\hat{D}_{21}^{-1}\hat{D}_{22} & \hat{D}_{21}^{-1} \end{array}\right]$$

$$\tilde{\Theta} = \begin{bmatrix} \tilde{\Theta}_{11} & \tilde{\Theta}_{12} \\ \tilde{\Theta}_{21} & \tilde{\Theta}_{22} \end{bmatrix}$$

$$= \left[\begin{array}{c|cc} \hat{A} - \hat{B}_2\hat{D}_{12}^{-1}\hat{C}_1 & \hat{B}_1 - \hat{B}_2\hat{D}_{12}^{-1}\hat{D}_{11} & -\hat{B}_2\hat{D}_{12}^{-1} \\ \hline \hat{C}_2 - \hat{D}_{22}\hat{D}_{12}^{-1}\hat{C}_1 & \hat{D}_{21} - \hat{D}_{22}\hat{D}_{12}^{-1}\hat{D}_{11} & -\hat{D}_{22}\hat{D}_{12}^{-1} \\ \hat{D}_{12}^{-1}\hat{C}_1 & \hat{D}_{12}^{-1}\hat{D}_{11} & \hat{D}_{12}^{-1} \end{array}\right].$$

Let

$$d(s) = \det\left(sI - (\hat{A} - \hat{B}_1\hat{D}_{21}^{-1}\hat{C}_2)\right)$$

and

$$\tilde{d}(s) = \det\left(sI - (\hat{A} - \hat{B}_2\hat{D}_{12}^{-1}\hat{C}_1)\right)$$

Let

$$K_d = U_d V_d^{-1} = \tilde{V}_d^{-1}\tilde{U}_d$$

where

$$U_d = \frac{\begin{bmatrix} U_1(s) & & & \\ & U_2(s) & & \\ & & \ddots & \\ & & & U_m(s) \end{bmatrix}}{d(s)},$$

$$V_d = \frac{\begin{bmatrix} V_1(s) & & & \\ & V_2(s) & & \\ & & \ddots & \\ & & & V_m(s) \end{bmatrix}}{d(s)}$$

$$\tilde{U}_d = \frac{\begin{bmatrix} \tilde{U}_1(s) & & & \\ & \tilde{U}_2(s) & & \\ & & \ddots & \\ & & & \tilde{U}_m(s) \end{bmatrix}}{\hat{d}(s)},$$

$$\tilde{V}_d = \frac{\begin{bmatrix} \tilde{V}_1(s) & & & \\ & \tilde{V}_2(s) & & \\ & & \ddots & \\ & & & \tilde{V}_m(s) \end{bmatrix}}{\hat{d}(s)}$$

Theorem 4 *Let $K_0 = \Theta_{12}\Theta_{22}^{-1}$ be the central H_∞ controller such that $\|\mathcal{F}_\ell(G, K_0)\|_\infty < \gamma$ and let $U_d, V_d \in RH_\infty$ with $\det V_d(\infty) \neq 0$ be such that*

$$\left\| \begin{bmatrix} \gamma^{-1}I & 0 \\ 0 & I \end{bmatrix} \Theta^{-1} \left(\begin{bmatrix} \Theta_{12} \\ \Theta_{22} \end{bmatrix} - \begin{bmatrix} U \\ V \end{bmatrix} \right) \right\|_\infty < 1/\sqrt{2}.$$

Then $K_d = U_d V_d^{-1}$ is also a stabilizing controller such that $\|\mathcal{F}_\ell(G, K_d)\|_\infty < \gamma$.

Similarly, we have the following theorem:

Theorem 5 *Let $K_0 = \tilde{\Theta}_{22}^{-1}\tilde{\Theta}_{21}$ be the central H_∞ controller such that $\|\mathcal{F}_\ell(G, K_0)\|_\infty < \gamma$ and let $\tilde{U}_d, \tilde{V}_d \in RH_\infty$ with $\det \tilde{V}_d(\infty) \neq 0$ be such that*

$$\left\| \left(\begin{bmatrix} \tilde{\Theta}_{21} & \tilde{\Theta}_{22} \end{bmatrix} - \begin{bmatrix} \tilde{U}_d & \tilde{V}_d \end{bmatrix} \right) \tilde{\Theta}^{-1} \begin{bmatrix} \gamma^{-1} I & 0 \\ 0 & I \end{bmatrix} \right\|_\infty < 1/\sqrt{2}.$$

Then $K_d = \tilde{V}_d^{-1}\tilde{U}_d$ is also a stabilizing controller such that $\|\mathcal{F}_\ell(G, K_d)\|_\infty < \gamma$.

The preceding two theorems show that the sufficient conditions for the structured H_∞ controller design problems are equivalent to frequency-weighted H_∞ model reduction problems.

Decentralized H_∞ Controller Design

(i) Let $K_0 = \Theta_{12}\Theta_{22}^{-1} (= \tilde{\Theta}_{22}^{-1}\tilde{\Theta}_{21})$ be a suboptimal H_∞ central controller ($Q = 0$) such that $\|T_{zw}\|_\infty < \gamma$.

(ii) Find a reduced-order controller $K = U_d V_d^{-1}$ (or $\tilde{V}_d^{-1}\tilde{U}_d$) such that

$$\left\| \begin{bmatrix} \gamma^{-1} I & 0 \\ 0 & I \end{bmatrix} \Theta^{-1} \left(\begin{bmatrix} \Theta_{12} \\ \Theta_{22} \end{bmatrix} - \begin{bmatrix} U_d \\ V_d \end{bmatrix} \right) \right\|_\infty < 1/\sqrt{2}$$

or

$$\left\| \left(\begin{bmatrix} \tilde{\Theta}_{21} & \tilde{\Theta}_{22} \end{bmatrix} - \begin{bmatrix} \tilde{U}_d & \tilde{V}_d \end{bmatrix} \right) \tilde{\Theta}^{-1} \begin{bmatrix} \gamma^{-1} I & 0 \\ 0 & I \end{bmatrix} \right\|_\infty < 1/\sqrt{2}.$$

Then the closed-loop system with the reduced-order controller K_d is stable and the performance is maintained with the reduced-order controller; that is,

$$\|T_{zw}\|_\infty = \|\mathcal{F}_\ell(G, K_d)\|_\infty < \gamma.$$

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