DETECTING CHANGE USING PSEUDO POWER SIGNATURES

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Abstract: The problem considered here is that of detecting events from the analysis of sensing signals. Our approach is based on the continuous wavelet transform and defines *pseudo power signatures*, as functions of the resolution and characterizing the power distribution of an event. Detection of their presence can be used to identify an event. These pseudo power signatures are, ideally, independent of the duration of the event and can be therefore used to provide fast detection of changes as required, for example, in fault detection problems.
The paper gives an overview of the concept of pseudo power signatures, some of the issues associated with their determination and their application to signal classification problems, focusing in fault detection.

Keywords: Fault detection, change detection, pseudo power signature

1. INTRODUCTION

The initial impetus for this research is a situation in shallow stratigraphy (first 50m)where one estimates underground layers by illuminating the subsurface with electromagnetic pulses and analyzing the echo signal. The formalization of the problem leads to the following classification problem:
There exists a collection, $\mathsf{C} = \{E_1, E_2, \ldots, E_n\}$, *of events. Each event may leave an imprint on a sensing signal,* $x(t), t_0 \leq t \leq t_f$. *Assuming that only one event may affect the signal at any given time, the time interval may be partitioned as* $t_0 < t_1 < t_2 \ldots < t_k, \ldots < t_f$ *such that the segment,* $x_k(t), t_{k-1} < t < t_K$, *is only affected by the event* E_j. *Determination of the time partition and the event sensed in each segment is the* classification *of the signal.*

It is easy to see that the formulation fits many different problems in signal processing. In fact some of the techniques developed have been tested on speech signals. In this case, the events would be phonemes and the classification would become a speech recognition problem.

An application of particular importance, and that is the focus of our current research, is the issue of fault detection. In the simplest form, the signal is a sensor reading and there are only two events; i.e, *normal operation* and *faulty operation.* The classification of the signal would be a fault detection system. As opposed to residue-based fault detection, this approach does not require an explicit mathematical model of the system and, in particular, does not require measurements of the inputs to a system. The fault detection scenario has some additional features that make it specially attractive as a signal processing problem.

Our approach for solving the classification problem is to use the continuous wavelet transform to associate to each event a signature that can searched for in the test signal. This signature should be independent of the time that the event

[1] Partially supported by NASA under award NCC5-573 and contract NASA/LEQSF(2001-04)-1

affects the test signal. Transitions from one signature to another would mark transition from one event to another, e.g., from normal to faulty operation.

Initially, we tried to formulate the problem as a wavelet selection issue; i.e, given a "suitable class of signals," C_s, find a wavelet that would allow the representation of the continuous wavelet transform (CWT), for members of the class, as a product of a function of the scale and a function of time; i.e., *Find a wavelet, $\psi(t)$, such that*

$$c_\psi^x(a,b) = s(a)r(b), x \in C_s.$$

The function $s(a)$ would give the power distribution and would be independent of the duration of the event. It would be a power signature for the event. Early in the research, we determined that the selection issue has a negative answer. For any non-trivial, admissible wavelet, no energy signal can have a separable CWT [1]. Hence, in order to pursue this line one must establish approximations and thus, develop *pseudo power signatures.* In effect, if the wavelet transform, $c_\psi^x(a,b)$, of a signal $x(t) \in L_2$ can be well approximated (in some sense) by a function of the form $s(a)r(b)$ then the function $s(a)$, normalized, could be construed to be a pseudo power signature.

We now establish the notation used, give a more formal statement to the pseudo power signature problem and show some preliminary results using singular value techniques. Next, we show why we decided to state the problem as an inverse projection issue. And we outline the frequency domain approach that we are currently using to solve the problem of determining signatures.

For the application to the detection of change in fault detection, we show results from processing sensor data from mathematical models of airplanes.

1.1 Notation and Mathematical Preliminaries

In the following, $\psi(t) \in L^2(\Re)$ is an admissible wavelet and the family of its translations and dilations is $\psi_{ab}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)$. We need to introduce two additional function spaces

$$\mathcal{H} = \left\{ c(a,b) : C_\psi^{-1} \int_a \int_b |c(a,b)|^2 \frac{dadb}{a^2} < \infty \right\}$$

$$\mathcal{A} = \left\{ s(a) : C_\psi^{-1} \int_a |s(a)|^2 \frac{da}{a^2} < \infty \right\}$$

We know that $\mathcal{H} = \mathcal{A} \otimes L^2(\Re)$ and the continuous wavelet transform is the map $\Gamma : L^2(\Re) \to \mathcal{H}$ defined by $c_\psi^x = \Gamma[x];\ x \in L^2(\Re)$ with

$$c_\psi^x(a,b) = \int x(t)\overline{\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)}dt \qquad (1)$$

The adjoint transformation, $\Gamma^* : \mathcal{H} \to L^2(\Re)$, has the definition $x^c = \Gamma^*[c]; c \in \mathcal{H}$, with

$$x^c = C_\psi^{-1} \int_a \int_b c(a,b) \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)\frac{dadb}{a^2}$$

It is essential to our developments that the space of the continuous wavelet transforms (CWT) is a proper closed subspace, $\mathcal{M} \subset \mathcal{H}$. In particular, regardless of the wavelet chosen, no function in $\mathcal{M}$ can be of the form $s(a)r(b)$. Hence the inverse transformation cannot be applied to this type of functions. Instead, one has (see [1]):

Lemma 1.1. If Γ is the wavelet transform operator defined in Eq(1) then

$$\mathcal{K} = \Gamma\Gamma^* \qquad (2)$$

is an orthogonal projector in $\mathcal{H}$ with range $\mathcal{M}$. Moreover, one has $\Gamma^*\Gamma = I_{L^2(\Re)}$

2. A SINGULAR VALUE DECOMPOSITION APPROACH

Since no element of the form $s(a)r(b)$ can be a wavelet transform, given a wavelet transform, $c_\psi^x(a,b)$, it is reasonable to look for the separable term that is, in some sense, closer to the transform. The usual approach is based on the singular value decomposition. In this case one solves the minimization problem

$$J[s,r] = \left\langle c_\psi^x(a,b) - s(a)r(b), c_\psi^x(a,b) - s(a)r(b) \right\rangle_{\mathcal{H}}$$

(actually one would write $\sigma s(a)r(b)$ with $s(a), r(b)$ on their respective unit balls). For our first numerical implementations [1] we established that, under certain conditions, one can well approximate the problem with a conventional matrix SVD problem. The numerical implementation of the discretized CWT is based on an algorithm developed by Shensa [2] . As supporting example, we created three chirp signals $\{x1, x2, x3\}$ given by

$$x_1(t) = e^{j.5\pi t} sinc(\frac{t}{3})$$
$$x_2(t) = e^{j.55\pi t} sinc(\frac{t}{3})$$
$$x_3(t) = e^{j1.55\pi t} sinc(\frac{t}{3})$$

The signals, their frequency spectra $\{f1, f2, f3\}$ (the axis is expressed as a fraction of π) and their

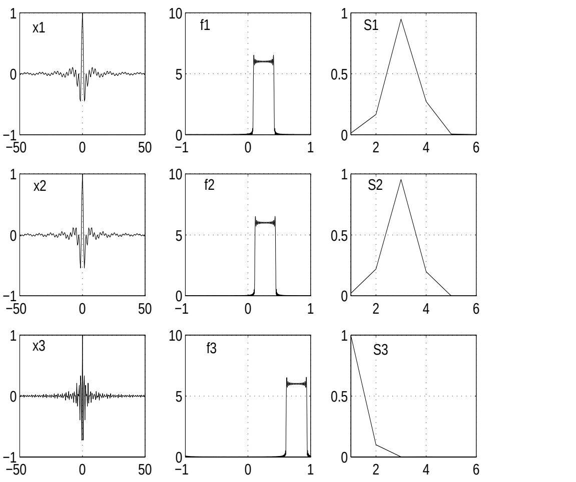

Fig. 1. The 3 signals and their signatures

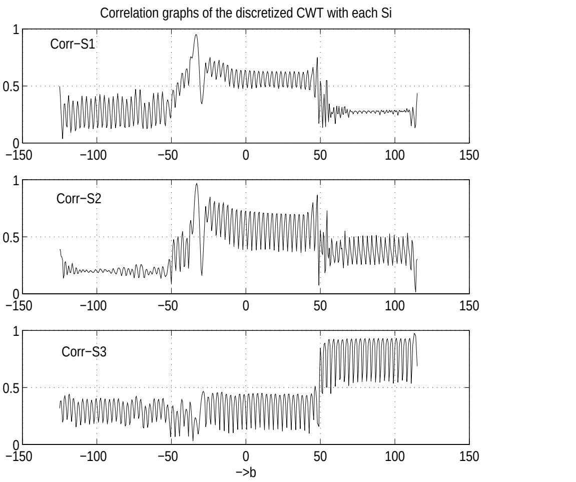

Fig. 2. Correlation graphs of the discretized CWT

pseudo power signatures $\{S1, S2, S3\}$ are shown in figure 1. The signatures were generated using the $Db4$ wavelet. Now consider a signal created by concatenating segments of each signal class: $x1$ over the interval [-125:-50], $x2$ over the interval [-50:50], and $x3$ over the interval [50:115]. In [1] we show that in this case, neither the $STFT$, nor the CWT permit a clear the identification of the component signals or the transition points. Furthermore, direct comparison of the $CWTs$ of each signal class with the CWT of the composite signal is not feasible either because the CWT support is dependent on the signal duration which is, in general, unknown.

One can get an accurate picture of the signal composition, with particular reference to the location of the transition points, if one determines the correlation of each Si with the discretized CWT of the composite signal for each b. The results are presented in Figure 2. The results show quite clearly that there are 2 transition points in the signal, (the first around -50, and the second around 50), a situation which is not very evident upon examination of the signal. Here, one can make the legitimate assumption that the correlation values must remain fairly constant over a range for the signal to be classified as having support in that range. Hence, one can conclude from the graphs that the support of $x1$ is $[-125 : -50]$, that of $x2$ is $[-50 : 50]$, and that of $x3$ is $[50 : 115]$. The high correlation values of $S1$ in the range $[-50 : 50]$ can be disregarded since $S2$ has a higher correlation in that range than $S1$, and is more likely to be present in the range $[-50 : 50]$.

Further experimentation with the SVD approach established the necessity of improving the techniques to compute the continuous wavelet transform, in particular by giving more flexibility in the selection of scales. The next subsection presents a numerical approach for computing the CWT that relies on a mixed *scale-frequency* representation.

2.1 CWT Computations using FFT

If in the definition of the CWT given in Eq. (1) one takes the Fourier transform of $c_{\psi}^{x}(a, b)$ with respect to the time parameter b one obtains the new transformation

$$\begin{aligned}\mathcal{C}(a, \omega) &= \int_{-\infty}^{\infty} c_{\psi}^{x}(a, b) e^{-j\omega b} db \\ &= \sqrt{a}\Psi(a\omega)X(\omega)\end{aligned}$$

In obtaining the previous result one assumes that orders of integration can be interchanged. The representation shows that for any given scale the computation of the transform can be efficiently performed in the frequency domain, for any selected set of scales. Thus, one has a scale-discretized wavelet transform. For numerical implementations, one will also perform a discretization in the frequency domain leading to a *discretized wavelet transform (dWT)*. Moreover, one can establish conditions on the set of selected scales that will insure the inversion of the dWT [3]. The overall computational complexity of this dWT is comparable to that of an ordinary 2-D FFT.

3. SVD SIGNATURES FOR MODEL-FREE FAULT DETECTION. A CASE STUDY

In order to establish the validity of a DSP, model free, approach for fault detection we first verified the capability of regular DSP techniques to enhance ordinary sensor data. For this, we collected data of computer simulated faults using a

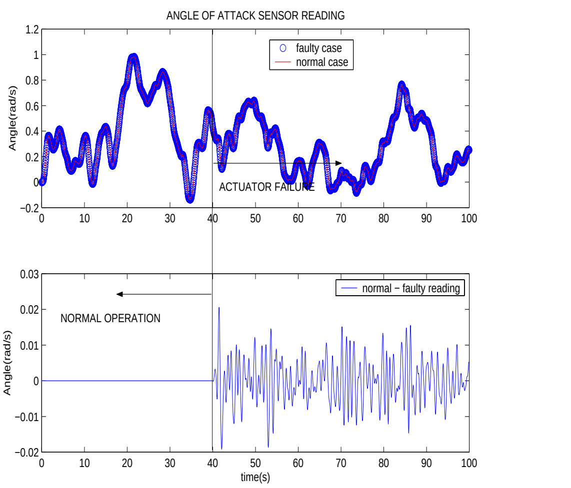

Fig. 3. Angle of attack sensor reading for normal and faulty situation

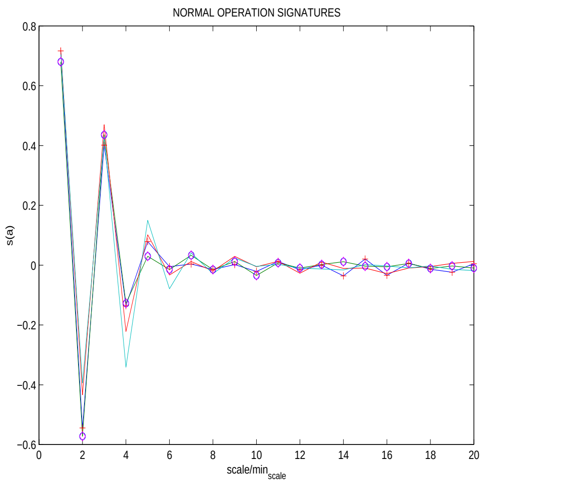

Fig. 5. Signatures obtained using 512 points and window increment of 256 points

public domain F14 model. Results of these experiments are presented in [4]. In a representative experiment we simulated a drastic change in the time constant of the actuator moving one of the ailerons. The value was changed from its nominal value to four times its nominal value in a discontinuous manner. The processing used to enhance the effect of the failure is the decomposition of the signal into 16 orthogonal components using a multi-resolution generated filter bank. The wavelets generating the multiresolution are Daubechis' compact support wavelets [5].

The graphs in figures 3 and 4 display a representative result showing the sixteen orthogonal components of the angle of attack when the stick is driven with a band-limited random signal, emulating combat action. Figure 3 shows the angles of attack in the faulted case and the difference with the angle for the case of no fault. As can be seen, the differences are very small and essentially invisible in the sensor reading. The graph in figure 4, on the other hand, shows that some orthogonal components have very different behavior pre and post fault. Hence, the onset of the fault can be readily established.

We decided to used the data from the F14 simulation and apply to it the pseudo power signature detection technique. The effect of the fault is very small and a pseudo power signature of the sensor signal would not be sensitive enough. In the case of residue-based detection, one obviates the problem by using readings referred to a normal model. Since we assume no model and the orthogonal components appear sensitive to the fault, we created a baseline behavior using the lowest resolution view of the sensor data. The difference between this baseline and the actual sensor reading is the *details signal*. The assumption is that the effect of the signal will be, most likely, more significant at high frequencies. Hence the details signal will show better the effect of the fault. In view of the behavior of the orthogonal components, the assumption appears reasonable.

The first step in our detection approach is to create a pseudo power signature for the normal, pre-fault condition. This is accomplished by computing the dWT of pre-fault details, computing the SVD of the dWT and selecting the principal component of the scale matrix. In order to establish the consistency of the signatures we used a sliding window and determined pre-fault signatures using 512 data points with distance between window centers of 256 data points. Figure 5 shows three such pre-fault signatures. The number of scales used is 20. As can be seen, the consistency of the signatures is very good, supporting the concept of a signature for normal operation.

This normal operation signature was used in an attempt to detect the onset of the fault. For this, we computed the dWT for a record of sensor data containing pre- and post-fault behavior. For each value of the time parameter the correlation between the dWT and the signature was computed. Figure 6 shows a typical result. It is evident that, from the display, no conclusion can be derived with regard to the onset of the fault.

A post-mortem analysis of these results suggests several possibilities. First, a comparison of normal operation signatures with faulty operation signatures showed that, even though there are differences between signatures, these differences are not very significant, especially at the lower scales. Hence, either the details signal is not sensitive enough, or the SVD approach for computing signatures is not suitable.

In this paper we focus on the problem of improving the determination of signatures. We first examine the approximation problem and show

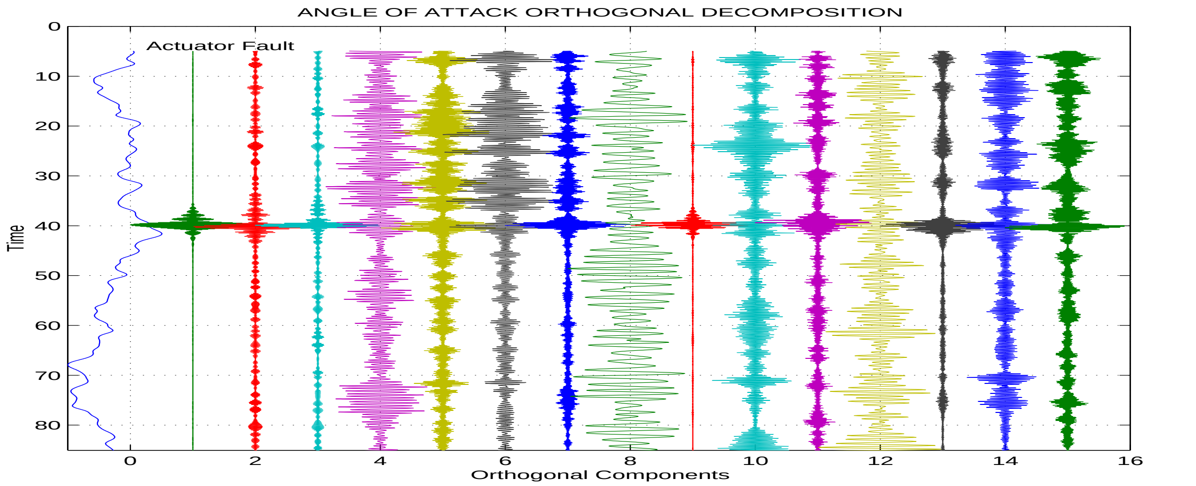

Fig. 4. Sensor reading decomposed into 16 orthogonal components. Fault onset time is 40

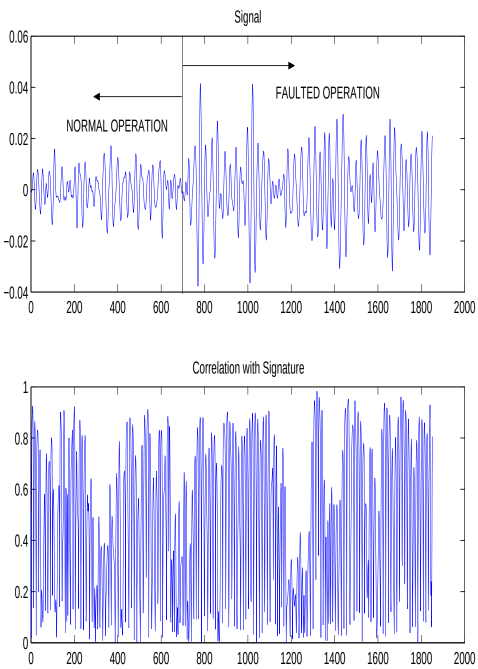

Fig. 6. Enhanced sensor reading and correlation of its dWT with normal signature

that the best approximation to the CWT need not yield the best approximation to the signal. We then proceed to develop a new approach to compute pseudo power signatures.

Using the facts that $\mathcal{K} = \Gamma\Gamma^*$ is an orthogonal projector and that separable terms cannot belong to the range of this projector, for any separable term one can write

$$e_{\mathcal{H}} = c^x_\psi - \mathcal{K}[s \otimes r] + (I - \mathcal{K})[s \otimes r] \quad (3)$$
$$= \Gamma[x - \Gamma^*[s \otimes r]] + m^\perp \quad (4)$$

where $m^\perp \neq 0 \in \mathcal{M}^\perp$. Hence

$$||e_{\mathcal{H}}||^2 = ||x - \Gamma^*[s \otimes r]||^2 + ||m^\perp||^2$$

The SVD approach minimizes the left hand side; that is to say, it minimizes the sum on the right but does not guarantee that the time function obtained from the separable term is a good approximation to the signal. We postulate that the better signatures can be obtained by minimizing the term $||x - \Gamma^*[s \otimes r]||^2$. The next section presents results in solving such minimization problem. The approach offers several intriguing possibilities that we are currently under research.

4. INVERSE PROJECTION SIGNATURES

In [3] we show how the minimization of the approximation error $e_{\mathcal{H}}$, defined in 3, can be approached from frequency domain approach. We give here the essential details of such approach. The map Γ^* when applied to separable terms takes the form

$$\Gamma^*[s \otimes r] =$$

$$= C_\psi^{-1} \int_a \int_b s(a) r(b) \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \frac{dadb}{a^2} \quad (5)$$

Let now $x^{sr} = \Gamma^*[s \otimes r] \in L^2(\Re)$. For its Fourier transform, one can show that

$$X^{sr}(\omega) = C_\psi^{-1} \int_a s(a)\sqrt{a}\Psi(a\omega)\frac{da}{a^2} R(\omega) \quad (6)$$

This last equation permits the definition of a map $\hat{U}$ as follows

$$\hat{U}[s](\omega) = C_\psi^{-1} \int_a s(a)\sqrt{a}\Psi(a\omega)\frac{da}{a^2} \quad (7)$$

The expression for $e_{\mathcal{H}}$, transformed to the frequency domain, becomes

$$E_{\mathcal{H}}(\omega) = X(\omega) - \hat{U}[s](\omega)R(\omega) \quad (8)$$

In particular, if the signal $r(b)$ is robust enough so that $|R(\omega)| > 0$ whenever $X(\omega) \neq 0$ then one can attempt to make the error zero. A special, but significant, case where one can make further progress is for band-limited signals where the support, χ_x, of $X(\omega)$ is a finite interval and one can define an energy signal, $r(b)$, such that

$$R(\omega) = e^{-j\omega\tau}, \tau \neq 0, \omega \in \chi_x$$

In this case the determination of a signature would be equivalent to the determination of scale function $s(a)$, such that $X(\omega) = \hat{U}[s](\omega)$. This problem is approached from a computational point of view where both the scale and the frequency are discretized. Consider the equation

$$X(\omega) = C_\psi^{-1} \int_a s(a)\sqrt{a}\Psi(a\omega)\frac{da}{a^2}$$

Numerically, to solve this equation for $s(a)$, one replaces it by the set of equations

$$X(\omega_n) = C_\psi^{-1} \int_a s(a)\sqrt{a}\Psi(a\omega_n)\frac{da}{a^2};\ n = 1, 2, \ldots N$$

Our first approach to define signatures uses the scale function

$$s(a) = \sum_{k=1}^{q} \sigma_k \sqrt{a}\overline{\Psi(a\omega_k)} \quad (9)$$

In this case one can show that to determine the vector $\sigma = col\{\sigma_1, \ldots, \sigma_q\}$ one must solve the linear equation

$$X_d = U_\psi \sigma \quad (10)$$

where U_ψ is a matrix with entries

$$U(n,k) = \int_a \overline{\Psi(a\omega_n)}\Psi(a\omega_k)\frac{da}{a}$$

The evaluation of the matrix U can be efficiently carried out using a rectangular rule on a basic grid and using linear interpolation for other values. In general, the system of equations in 10 is underdefined and one uses the minimum norm solution. One of the problems still pending is a criterion to select the centers ω_k for the basis functions $\Psi(a\omega_k)$. Our computations show that this approach is sensitive to those values. Once we determine improved signatures, the detection issue follows easily.

5. CONCLUSIONS

We have solid evidence that signal processing can be effectively used to process sensor data and provide early fault detection. We are proposing the use of pseudo power signatures as tools to implement a fault detector. However, signatures based on SVD of the discretized continous wavelet transform appear not to have sufficient discriminatory capability. One current line of research is developing refined signatures using a frequency domain approach. Another important issue that is also under investigation is based on the effect of noise on the sensitivity of the signatures. Finally, it should be pointed out that the concept of pseudo power signatures, as used in our work, is not restricted to the wavelet transform and can be applied to any time-frequency distribution

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