EXPECTED-MODE AUGMENTATION ALGORITHMS FOR VARIABLE-STRUCTURE MULTIPLE-MODEL ESTIMATION[1]

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Abstract: This paper presents a new class of variable-structure algorithms, referred to as expected-mode augmentation (EMA), for multiple-model estimation. In this approach, the original model set is augmented by a variable set of models intended to match the expected value of the *unknown* true mode. These models are generated adaptively in real time as (globally or locally) probabilistically weighted sums of modal states over the model set. This makes it possible to cover a large continuous mode space by a relatively small number of models at a given accuracy level. Performance of the proposed EMA algorithms is evaluated via a simulated example of a maneuvering target tracking problem.

Keywords: Adaptive Estimation, Multiple Model, Variable Structure, IMM, Target Tracking

1. INTRODUCTION

In many applications of multiple-model (MM) estimation, the set of possible values of uncertain system parameters, known as mode space, is continuous. As shown in (Li and Jilkov, 2001), optimal use of more models in such cases does improve the performance of MM estimation. In practice, however, only a limited number of models can be used. The common practice in MM estimation is to design a finite set of models to approximate this mode space. Loosely speaking, the major objective here is to achieve best modelling accuracy at a minimum number of models. Here the variable-structure approach (see e.g., (Li and Bar-Shalom, 1996; Li, 2000*b*; Li, 2000*a*)) to MM estimation has certain advantages. In a general setting, the problem of efficient model set design for MM estimation is still open, although significant progress has been reported in (Li, 2002; Li *et al.*, 2002).

It is known that the performance of a model set depends highly on how close the models in the set is to the true mode. The closer the better. Since the true mode may be time varying over a large space, if a fixed set of models is used, the required number of models to achieve a satisfactory accuracy may easily be prohibitively large. However, nothing really prevents us from using a variable set of models. To capture various possible mode jumps and in the meantime to have at least one model close to the true mode, a natural idea is to augment the original model set by one or more adaptive models that follow closely the true mode. A good candidate for the augmenting models is the expected value of the true mode since it is statistically closest to the true mode. This expected mode can be approximated by a sum of modal states weighted by the corresponding model probabilities, readily available from the underlying MM algorithm. This expected-mode augmentation (EMA) approach, proposed in (Li and Jilkov, 2001), is systematic and general for all problems with a continuous mode space.

Several researchers have considered similar problems and proposed their solution techniques. The use of an initial coarse grid and a subsequent fine grid was proposed in (Gauvrit, 1984) for a static MM algorithm. Also for a static MM algorithm, (Maybeck and Hentz, 1987) presented a filter bank that moves over a predefined fixed grid according to a decision logic. It was proposed in (Munir and Atherton, 1995) to use a moving set of acceleration models centered around a model whose acceleration is determined by an additional Kalman filter. In (Li and Bar-Shalom, 1996), it was suggested to employ the expected mode as the center of an adaptive grid for an example of non-stationary noise identification. In (Layne, 1998), an adaptive IMM algorithm for radar tracking of a maneuvering target was proposed that uses an acceleration model determined by a separate Kalman filter on top of a fixed set of models. Compared with these existing techniques, not only is the EMA approach much more general and systematic, but it is also highly cost-effective and easy to implement.

In this paper, we continue our investigation of the EMA approach started in (Li and Jilkov, 2001). More specifically, in (Li and Jilkov, 2001) we proposed the EMA approach, presented its theoretical foundation, and provided simulation results of a simplest possible practical implementation. In this paper, we outline the EMA approach in a general setting, propose three practical EMA algorithms, and evaluate their performance for a generic maneuvering target tracking problem.

2. EXPECTED-MODE AUGMENTATION

2.1 *Benefit of Model-Set Augmentation*

Denote by s the true mode and by S the mode space (i.e., the set of possible values of s). Consider the problem of adding a model set C to the original model set M (hence $C \cap M = \varnothing$). Assume $(M \cup C) \subset S$. Let

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$$\hat{x}_M = E[x|s \in M, z], \quad \hat{x}_C = E[x|s \in C, z]$$
$$\mu_M = P\{s \in M | s \in (M \cup C), z\}$$
$$\mu_C = P\{s \in C | s \in (M \cup C), z\}$$

where z stands for measurement. Then the estimator of x based on the union of model sets M and C is

$$\hat{x} = E[x|s \in (M \cup C), z] = \mu_M \hat{x}_M + \mu_C \hat{x}_C$$

which is a convex combination of $\hat{x}_M$ and $\hat{x}_C$. As shown in (Li and Jilkov, 2001), if $\hat{x}_M$ and $\hat{x}_C$ are unbiased (and have uncorrelated estimation errors), which can be assumed in most cases, use of the union of M and C is better than use of M alone if and only if

$$\mu_C < \frac{2\text{mse}(\hat{x}_M)}{\text{mse}(\hat{x}_M) + \text{mse}(\hat{x}_C)}$$

where $\text{mse}(\hat{x}_M)$ stands for the mean-square error of $\hat{x}_M$. This inequality is always satisfied if $\hat{x}_C$ is better than $\hat{x}_M$. Even if $\hat{x}_C$ is worse than $\hat{x}_M$, $\hat{x}$ is still better than $\hat{x}_M$ provided μ_C satisfies the above inequality, and in the case where $E[\tilde{x}'_M \tilde{x}_C] \neq 0$, $\hat{x}$ is still better than $\hat{x}_M$ if and only if $E[\tilde{x}'_M \tilde{x}_C] < E[\tilde{x}'_M \tilde{x}_M] = \text{mse}(\hat{x}_M)$, where $\tilde{x} = x - \hat{x}$ is the estimation error.

Note that this result, which holds when $M \cup C \subset S$, does not contradicts the finding presented in (Li and Bar-Shalom, 1996) that the optimal use of more models is not necessarily better because the above result would not necessarily be correct if $C \subset S$ were not true. Since in this paper we focus on problems with a continuous mode space, $M \cup C \subset S$ holds in general and thus $\hat{x}_\alpha = (1-\alpha)\hat{x}_M + \alpha \hat{x}_C$ with some α will be better than $\hat{x}_M$. As a consequence, optimal use of more models for such problems does improve performance because its estimate $\hat{x}$ cannot be worse than $\hat{x}_\alpha$. Of course, this holds true only under the simplifying assumption $s \in (M \cup C)$, which is not necessarily true in general.

2.2 *Expected-Mode Augmentation*

The above results have many potential applications in model-set design for fixed-structure MM estimation as well as model-set adaptation for variable-structure MM (VSMM) estimation. In general, given a model set M, it is up to us to find another model set C such that use of the augmented model set $M \cup C$ enhances the performance. The key here for model-set adaptation is to augment M by a set C of *good* models for the current time in real time. If we use a model set C based on which $\hat{x}_C$ is better than $\hat{x}_M$, then such augmentation is guaranteed to improve the performance. With this in mind, a natural idea is to choose C to be a set of expected modes, which should be close to the true mode and thus lead to good $\hat{x}_C$. In this approach, called **expected-mode augmentation** (EMA), at every time we augment model set M by a set C of adaptive models representing the expected mode. Clearly this approach has a variable structure since the models in the set C are adaptive (and hence variable) although the number of models in C may or may not be fixed.

We formally define the expected-mode augmentation approach in a general setting.

Definition 1 (EMA set). The union of a model set M and the set of models that match the true mode in some statistical average (expectation) sense is called the expected-mode augmented (EMA) set of M.

Definition 2 (EMA algorithm). An MM algorithm using an EMA set is called an EMA algorithm.

Let $M^+(M_1, \ldots, M_q) = M \cup E$ denote the model set M augmented by a set of models E that match the expected modes:

$$E = E(M; M_1, \ldots, M_q) = \{\bar{m}_1, \bar{m}_2, \ldots, \bar{m}_q\} \quad (1)$$

Here $\bar{m}_i$ is the expected mode as calculated based on the model set M_i, given by (say, for time k)

$$\bar{m}_i = \bar{m}_{k|k}^{M_i} = E[s_k | s_k \in M_i, M^{k-1}, z^k] = \sum_{m_j \in M_i} m_j \mu_j(k)$$

or

$$\bar{m}_i = \bar{m}_{k|k-1}^{M_i} = E[s_k | s_k \in M_i, M^{k-1}, z^{k-1}] = \sum_{m_j \in M_i} m_j \mu_j(k|k-1)$$

where M^k and z^k stand for the sequences of model sets and measurements, respectively, $\mu_j(k|k-1) = P\{s_k = m_j | s_k \in M_k, M^{k-1}, z^{k-1}\}$ and $\mu_j(k) = P\{s_k = m_j | s_k \in M_k, M^{k-1}, z^k\}$ denote the predicted and updated probabilities of model j being the correct one, and m_j is the parameter value that characterizes model j.

This approach is general—it is valid for all problems where the above $\bar{m}_i$ is meaningful[2]—and is simple to implement because $\bar{m}_i$ is readily available from an MM estimator with little extra computation.

In general, the expected-mode set E is time-varying and a generic cycle is $E_{k-1} \to E_k$. In a purely EMA algorithm (i.e., a VSMM estimator where the model set is time-varying only due to expected-mode augmentation), the evolution of the model set is

$$M_k = E_k \cup (M_{k-1} - E_{k-1}) \quad (2)$$

Clearly, an EMA algorithm with a generic cycle $E_{k-1} \to E_k$ can be integrated with other VSMM estimators with a generic cycle $M'_{k-1} \to M'_k$ to yield a generic cycle

$$M_{k-1} = M'_{k-1} + E_{k-1} \longrightarrow M_k = M'_k + E_k \quad (3)$$

It is thus clear that the key step in an EMA algorithm is the determination of M, and $M_1, \ldots, M_q$.

3. PRACTICAL EMA ALGORITHMS

We now describe practical EMA algorithms proposed in this paper. Assume for simplicity of presentation that the IMM mechanism is used for model-conditioned reinitialization (Li, 1996).

The proposed EMA algorithms involve the following main functional modules

[2] This is the case whenever the mode space is continuous, although there are problems in which different m_j represent different physical quantities and thus their weighted sum is not necessarily meaningful.

- EMA $M_k := M^+(M_1, \ldots, M_q)$: *expected mode augmentation* procedure;
- VSIMM$[M_k, M_{k-1}]$: recursion for *variable structure IMM* estimation that uses model sets M_{k-1} and M_k at time $k-1$ and k, respectively;
- EF$[M'_k, M''_k; M_{k-1}]$: procedure for *estimation fusion* of two estimates resulting from VSIMM$[M'_k, M_{k-1}]$ and VSIMM$[M''_k, M_{k-1}]$ recursions, respectively.

The VSIMM and EF functions have been developed, utilized, and documented in several publications on VSMM estimation (Li *et al.*, 1999*a*; Li *et al.*, 1999*b*; Li and Zhang, 2000; Li, 2000*a*). For the EMA procedure, a general description has been given above; a more detailed discussion is given next.

Table 1. One cycle of EMA — Algorithm A

S1.	Obtain $E_k = E(M_{k-1}; M_1, \ldots, M_q)$ using the *predicted* model probabilities $\{\mu_i(k\|k-1)\}_{m_i \in M_{k-1}}$
S2.	For $M_k = E_k \cup (M_{k-1} - E_{k-1})$, run VSIMM$[M_k, M_{k-1}]$ to obtain the overall estimates, error covariances, and model probabilities $\{\hat{x}_i(k\|k), P_i(k\|k), \mu_i(k)\}_{m_i \in M_k}$

Table 2. One cycle of EMA — Algorithm B

S1.	For $M_f = M_{k-1} - E_{k-1}$, run VSIMM$[M_f, M_{k-1}]$ to obtain $\{\hat{x}_i(k\|k), P_i(k\|k), \mu_i(k)\}_{m_i \in M_f}$
S2.	Obtain $E_k = E(M_f; M_1, \ldots, M_q)$ using the *current* updated model probabilities $\{\mu_i(k)\}_{m_i \in M_f}$
S3.	Run VSIMM$[E_k, M_{k-1}]$ to obtain $\{\hat{x}_i(k\|k), P_i(k\|k), \mu_i(k)\}_{m_i \in E_k}$
S4.	Run EF$[M_f, E_k; M_{k-1}]$ to obtain overall estimates, error covariances, and model probabilities in the set $M_k = M_f \cup E_k$: $\{\hat{x}_i(k\|k), P_i(k\|k), \mu_i(k)\}_{m_i \in M_k}$

Table 3. One cycle of EMA — Algorithm C

S1.	Obtain $E'_k = E(M_{k-1}; M_1, \ldots, M_q)$ using the *predicted* model probabilities $\{\mu_i(k\|k-1)\}_{m_i \in M_{k-1}}$
S2.	For $M'_k = E'_k \cup (M_{k-1} - E_{k-1})$, run VSIMM$[M'_k, M_{k-1}]$
S3.	Obtain $E_k = E(M'_k; M_1, \ldots, M_q)$ using the *current* updated model probabilities $\{\mu_i(k)\}_{m_i \in M'_k}$
S4.	Run VSIMM$[E_k, M_{k-1}]$
S5.	For $M_f = M_{k-1} - E_{k-1}$, run EF$[M_f, E_k; M_{k-1}]$ to obtain overall estimates, error covariances, and model probabilities in the set $M_k = M_f \cup E_k$: $\{\hat{x}_i(k\|k), P_i(k\|k), \mu_i(k)\}_{m_i \in M_k}$

3.1 *EMA Algorithms*

We now outline three EMA algorithms.

Consider a generic cycle from time $k-1$ to k. Suppose that the model set M_{k-1} used at $k-1$ is given. Three basic EMA algorithms are given in Tables 1, 2, and 3, respectively, using different schemes for determination of the model set M needed to obtain the expected-mode set $E_k = E(M; M_1, \ldots, M_q)$ at time k. Choices of $M_1, \ldots, M_q$ are discussed later.

The main difference among the three algorithms lies in how the expected-mode set E_k is determined. Algorithm A (Step 1) uses M_{k-1} (including E_{k-1}) but not the current measurement z_k to determine E_k. On the contrary, Algorithm B (Step 2) uses z_k but not E_{k-1} to determine E_k. Algorithm C (Step 3) uses both z_k and E_{k-1} to determine E_k. In general, Algorithm B should outperform Algorithm A at the time instant of a system mode jump (e.g., with a faster response and hence a smaller peak error) because of the timely information included in z_k, while Algorithm A should have a better steady-state performance due to the more direct utilization of the old expected modes. Algorithm C provides a trade off between the steady-state performance and the fast response.

Algorithm A is the simplest, while Algorithm C is the most sophisticated. Thanks to the optimal estimation fusion formulas described in (Li, 2000*a*), the computational complexities of Algorithms B and C increased by the use of the current measurement z_k to determine E_k is quite limited.

The above algorithms can be integrated to yield more sophisticated algorithms with improved performance. For example, we can use $E_k = E_k^A \cup E_k^B$ as the set of expected modes, where E_k^A and E_k^B are the sets of (predicted and updated) expected modes obtained by Algorithms A (Step 1) and B (Step 2), respectively; or more preferably, we may use $E_k = E_k^A \cup E_k^C$ as the set of expected modes, where E_k^A and E_k^C are the sets of (predicted and updated) expected modes obtained by Algorithm A (or C) in Step 1 and Algorithm C in Step 3, respectively, which is equivalent to replacing Step 5 of Algorithm C by running EF$[M'_k, E_k; M_{k-1}]$.

In Step 1 of Algorithm A, use of the *predicted* model probabilities at the *current* time step $\{\mu_i(k|k-1)\}_{m_i \in M_{k-1}}$ amounts to $\bar{m}_k = \bar{m}_{k|k-1}$ and should be superior to use of the *updated* model probabilities at the *previous* time step $\{\mu_i(k-1)\}_{m_i \in M_{k-1}}$, which amounts to assuming $\bar{m}_k = \bar{m}_{k-1|k-1}$. The same is true for Algorithm C. Both sets of model probabilities are readily available from an MM estimator.

3.2 *Choices of* $M_1, \ldots, M_q$

Assume that M for $E_k = E(M; M_1, \ldots, M_q)$ at time k is determined as above. We now discuss how to choose $M_1, \ldots, M_q$. Usually, a small number q of expected modes are used; that is, E_k is a small set. (We used only $q = 1$ and 2 in our simulations.) It is clear from the above algorithms that $M_1, \ldots, M_q$ should be subsets of M; otherwise their model probabilities needed to determine E_k are hard to come by.

Naturally, we should choose $M_1 := M$ in general unless there is a strong reason not to do so. For example, for Algorithm C (Step 3) with $q = 1$, we have then

$$E_k = \bar{m}_1 = \sum_{m_j \in M'_k} m_j \mu_j(k) \qquad (4)$$

that is, $\bar{m}_1$ is indeed the expected mode as computed based on all models in the set M'_k, including the expected mode at $k-1$.

There are many ways of choosing $M_2, \dots, M_q$. An idea is the following. Note first that the expected mode $\bar{m}_1$ obtained above is actually a global average because M_1 is equal to the total set M. In this sense, $\bar{m}_i$ will be a local average if M_i is a proper subset of M. By the analysis in Sec. 2, it is reasonable to expect that augmenting a model set M by $\bar{m}_i$ is beneficial if M_i is a set of models in M that are quite likely to be true (i.e., $s \in M_i$) at the time. As such, M_i can be chosen to be the set of those models in M with the highest probabilities, or those models that are close to $\bar{m}_1$ since $\bar{m}_1$ is probably the best single model. As reported in the next section, we implemented the above three EMA algorithms with $q = 1$ and 2 where $\bar{m}_1$ is as obtained above and M_2 is the set of the three most probable models in M.

4. EXAMPLE: MANEUVERING TARGET TRACKING BY EMA

4.1 *The Problem*

The target-measurement model is

$$x_{k+1} = Fx_k + G\left[a\left(k\right) + w_k\right]$$
$$z_{k+1} = Hx_{k+1} + v_{k+1}, \quad k = 0, 1, 2, \dots$$

where $x \triangleq (\mathsf{x}, v_x, \mathsf{y}, v_y)'$ denotes the target state, $a \triangleq (a_x, a_y)'$ is the acceleration, $w_k \sim \mathcal{N}\left[0, Q\right]$ is the acceleration process noise, $z = (z_x, z_y)'$ is the measurement, $v_k \sim \mathcal{N}\left[0, R\right]$ is the random measurement error, and $F = \text{diag}[F_2, F_2]$ and $G = \text{diag}[G_2, G_2]$ with

$$F_2 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad G_2 = \begin{bmatrix} T^2/2 \\ T \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The unknown true acceleration $a\left(k\right)$ is assumed piecewise constant, varying over a given continuous planar region A^c. In the MM framework, we consider a generic finite set (grid) of acceleration values:

$$A_r \triangleq \{a_i \in A^c : i = 1, 2, \dots, r\} \qquad (5)$$

which defines the total model set. We approximate the evolution of the true acceleration over the quantized set A_r via a Markov chain model, that is, $a(k) \in A_r$ with given $P\left\{a(0) = a_i\right\} = P_i$ and $P\left\{a(k) = a_j | a(k-1) = a_i\right\} = \pi_{ij}$ for $i, j = 1, 2, \dots, r$.

4.2 *Designs of EMA Algorithms*

In the simulated well-known example of (Averbuch *et al.*, 1991; Li and Bar-Shalom, 1992; Munir and Atherton, 1995; Li and Bar-Shalom, 1996; Li *et al.*, 1999*b*; Li and Zhang, 2000), the maximum acceleration in any coordinate direction is assumed to be about $4g$. The mode space is thus defined as (Li and Jilkov, 2001)

$$A^c \triangleq \left\{(a_x, a_y) : \sqrt{a_x^2 + a_y^2} \leq 40\right\}$$

The basic set of *fixed* models A_7, designed by quantization of A^c is (Fig. 1)

$$\left\{ \begin{array}{lll} a_1 = \rho\left[0, 0\right]' & a_2 = \rho\left[2, 0\right]' & a_3 = \rho\left[1, \sqrt{3}\right]' \\ a_4 = \rho\left[-1, \sqrt{3}\right]' & a_5 = \rho\left[-2, 0\right]' & a_6 = \rho\left[-1, -\sqrt{3}\right]' \\ a_7 = \rho\left[1, -\sqrt{3}\right]' & & \end{array} \right\}$$

with $\rho = 20 \approx 2g$. This quantization is more efficient than other rectangular designs with 9 and 13 nodes (Li and Jilkov, 2001; Li, 2002).

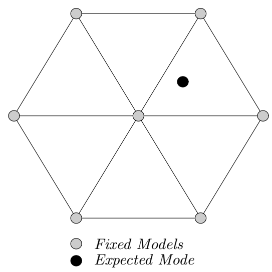

Fig. 1. 7-Model Set Design

Two types of EMA design based on A_7 were considered—single-model augmentation, denoted by $A_{7+1}(k) \triangleq \{A_7, \hat{a}_8(k)\}$ (Fig. 1) and two-model augmentation, denoted by $A_{7+2}(k) \triangleq \{A_7, \hat{a}_8(k), \hat{a}_9(k)\}$. All Algorithms A, B, and C presented above were implemented for both A_{7+1} and A_{7+2}. The corresponding algorithms are denoted for short as $A\{7+i\}$, $B\{7+i\}$, $C\{7+i\}$, $i = 1, 2$, respectively. The EMA$\{7+1\}$ algorithms use always $\hat{a}_8(k) = \bar{m}_1$ as computed by (4). The EMA$\{7+2\}$ algorithms use $\hat{a}_8(k)$ as in EMA$\{7+1\}$ and compute $\hat{a}_9(k)$ as the probabilistically weighted sum of the accelerations of the three most probable models in the respective model set.

The following transition probability matrices for the Markov chains over A_{7+1} and A_{7+2} respectively were used in the simulation

$$\begin{bmatrix} .894 & .001 & .001 & .001 & .001 & .001 & .001 & .1 \\ .05 & .65 & .05 & .0 & .0 & .0 & .05 & .2 \\ .05 & .05 & .65 & .05 & .0 & .0 & .0 & .2 \\ .05 & .0 & .05 & .65 & .05 & .0 & .0 & .2 \\ .05 & .0 & .0 & .05 & .65 & .05 & .0 & .2 \\ .05 & .0 & .0 & .0 & .05 & .65 & .05 & .2 \\ .05 & .05 & .0 & .0 & .0 & .05 & .65 & .2 \\ .001 & .001 & .001 & .001 & .001 & .001 & .001 & .993 \end{bmatrix}$$

and

$$\begin{bmatrix} .964 & .001 & .001 & .001 & .001 & .001 & .001 & .015 & .015 \\ .05 & .65 & .05 & .0 & .0 & .0 & .05 & .1 & .1 \\ .05 & .05 & .65 & .05 & .0 & .0 & .0 & .1 & .1 \\ .05 & .0 & .05 & .65 & .05 & .0 & .0 & .1 & .1 \\ .05 & .0 & .0 & .05 & .65 & .05 & .0 & .1 & .1 \\ .05 & .0 & .0 & .0 & .05 & .65 & .05 & .1 & .1 \\ .05 & .05 & .0 & .0 & .0 & .05 & .65 & .1 & .1 \\ .01 & .01 & .01 & .01 & .01 & .01 & .01 & .9 & .03 \\ .01 & .01 & .01 & .01 & .01 & .01 & .01 & .03 & .9 \end{bmatrix}$$

All other parameters of the IMM algorithms implemented in the simulation are the same as given in (Li *et al.*, 1999*b*), e.g. $T = 1s$, $Q^1 = (0.003)^2 I$, $Q^j = (0.008)^2 I$, $j \neq 1$, $R = 1250I$.

5. PERFORMANCE EVALUATION

5.1 *Test Scenarios*

The performances of the six MM tracking algorithms were investigated first over a large number of deterministic maneuver scenarios with fixed acceleration sequences. Deterministic scenarios serve to evaluate algorithms' peak errors, steady-state errors and response times. We present below results for two of them, referred to as DS1 and DS2, the same as used in

(Li and Jilkov, 2001). The other parameters for both scenarios are $T = 1s,\ Q = O,\ R = 1250I, x_0 = [8000,\ 25,\ 8000,\ 200]'$.

To provide an as fair as possible performance comparison over an ensemble of maneuver trajectories the algorithms were tested on the random scenario, developed in (Li *et al.*, 1999*b*; Li and Zhang, 2000). With such a scenario, it is difficult, if not virtually impossible, to design an MM estimator with subtle tricks that are effective only for certain scenarios. In the random scenario the acceleration vector is a 2-dimensional semi-Markov process. All details and discussions are given in (Li *et al.*, 1999*b*). In the simulation we used the same parameter values as given therein.

5.2 *Simulation Results*

Accuracy comparison results over 100 Monte Carlo runs for DS1 are plotted in Fig. 2 for the EMA$\{7+1\}$ algorithms versus the EMA$\{7+2\}$ algorithms. It is seen that for all three algorithms $A,\ B,$ and C, two-expected-mode augmentation in the EMA$\{7+2\}$ algorithms have substantially reduced peak errors compared with their $\{7+1\}$ counterparts. During the steady-state regimes (no-jumps present) the errors of the respective algorithms $A,\ B,$ and C are virtually indistinguishable. The results for the other deterministic scenario DS2 (not presented here) are very similar. As shown in Fig. 3 over 500 runs of the random scenario, the $\{7+2\}$ algorithms have a small overall error reduction relative to the EMA$\{7+1\}$ algorithms. Also plotted in this figure are the results of the standard fixed-structure IMM algorithm with 13 models (denoted as IMM13) (Li *et al.*, 1999*b*). It is seen that the EMA algorithms, which are of a variable structure, have a substantially better accuracy than the fixed-structure IMM algorithm. As evidenced by Table 4, this performance superiority is achieved by the EMA algorithms at a computational complexity only about half of that of the IMM13. The reason for this performance improvement is clear from Fig. 4—the augmenting model (i.e., the expected mode $\hat{a}_8$) is on average much better than every other model in the set.

Fig. 5 depicts comparative results between the three algorithms A, B, and C over DS2. Although the accuracy differences are not significant, a visible tendency is that Algorithm B provides a faster response (and hence smaller peak errors) than the other two algorithms. As explained before, this is due to the fact that its set of the expected modes relies more on the current measurement information than Algorithms A and C. However, this results in larger steady-state errors.

The computational complexities of the algorithms evaluated in terms of *relative floating point operations* (FLOP) ratios with respect to the standard IMM13 are summarized in Table 4.

Table 4. Computational Load (FLOP Ratio)

$\text{IMM13} = 1$	A	B	C
EMA$\{7+1\}$	0.5069	0.5004	0.5428
EMA$\{7+2\}$	0.5977	0.5919	0.6685

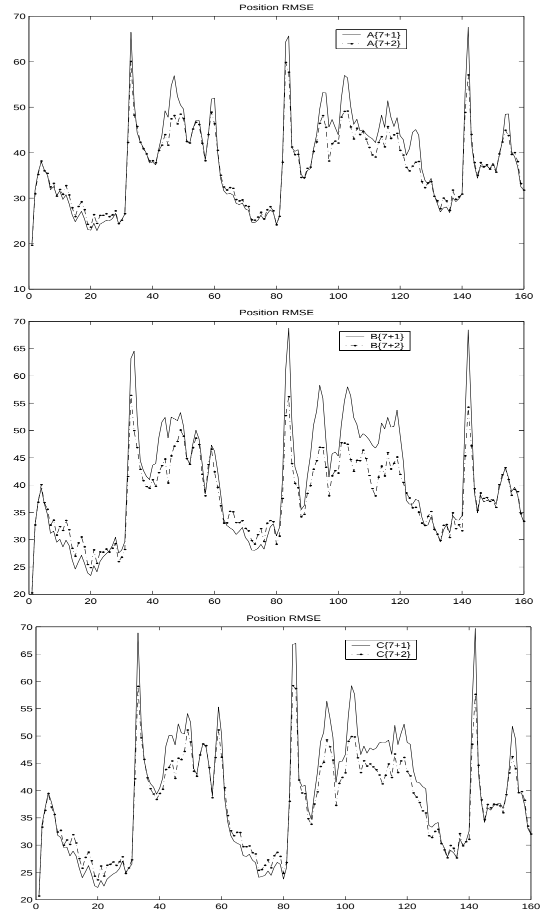

Fig. 2. RMS Position Errors (DS1): 7 + 1 vs. 7 + 2

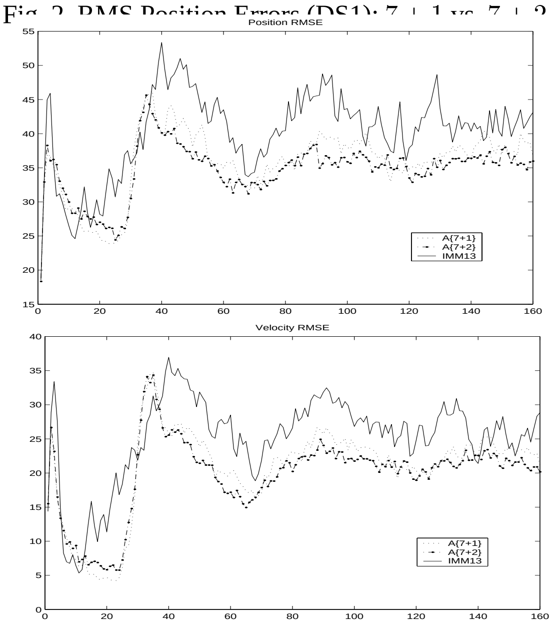

Fig. 3. RMS Errors (RS): $7+1$, $7+2$, IMM

6. CONCLUSIONS

Practical algorithms of expected-mode augmentation, which have a variable structure, for MM state estimation over a continuous mode space have been developed and investigated. The simulations conducted have demonstrated the capabilities of the proposed

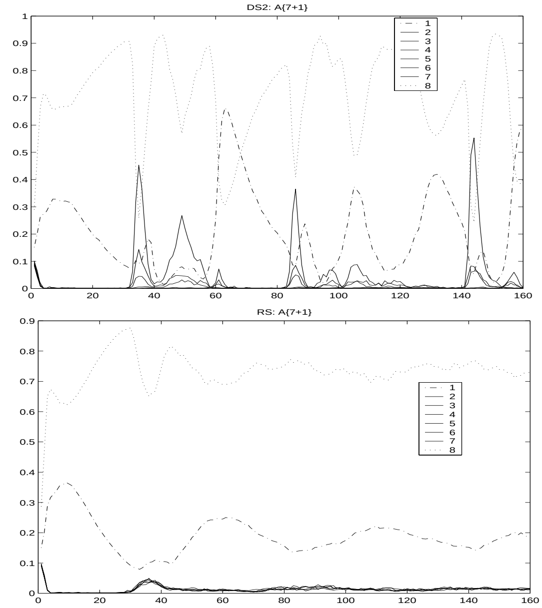

Fig. 4. Average Model Probabilities

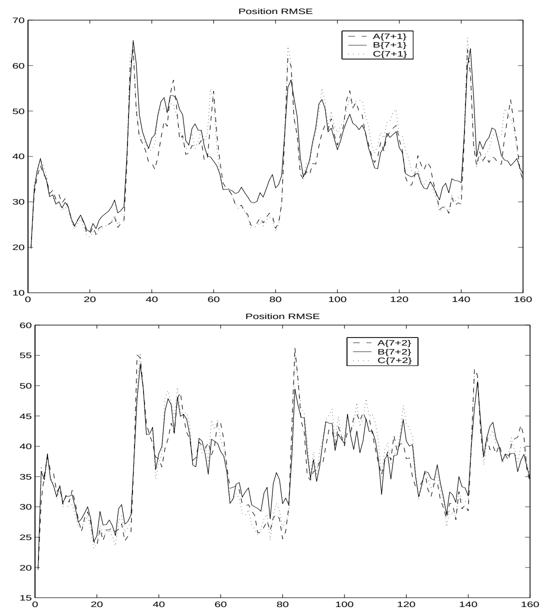

Fig. 5. RMS Position Errors (DS2): Algs. A, B, and C

algorithms for performance enhancement and computation reduction.

The approach is generally applicable, wherever the mode space is continuous. It can be applied to fixed- and variable-structure MM algorithms and supplements the existing methods of variable-structure MM estimation and facilitates the design of more efficient practical MM estimators.

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