Progress on High Performance Robust and Fault Tolerant Control

Kemin Zhou
Electrical and Computer Engineering
Louisiana State University

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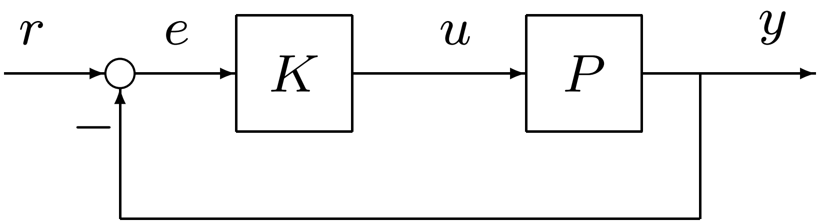
Example: Robust Stabilization



$P(\Delta)$ stable for all Δ

- $P(\Delta) = P_0 + W_1 \Delta W_2$
- $P(\Delta) = P_0(I + W_1 \Delta W_2)$
- $P(\Delta) = P_{11} + P_{12}\Delta(I - P_{22}\Delta)^{-1}P_{21}$

$$\|P_{22}\Delta\|_\infty < 1$$

$$\max_{K\ Stabilizing} \{\gamma : \quad \|\Delta\|_\infty < \gamma\}$$

$$K_{opt} = 0$$

Performance: None

Controller Architecture

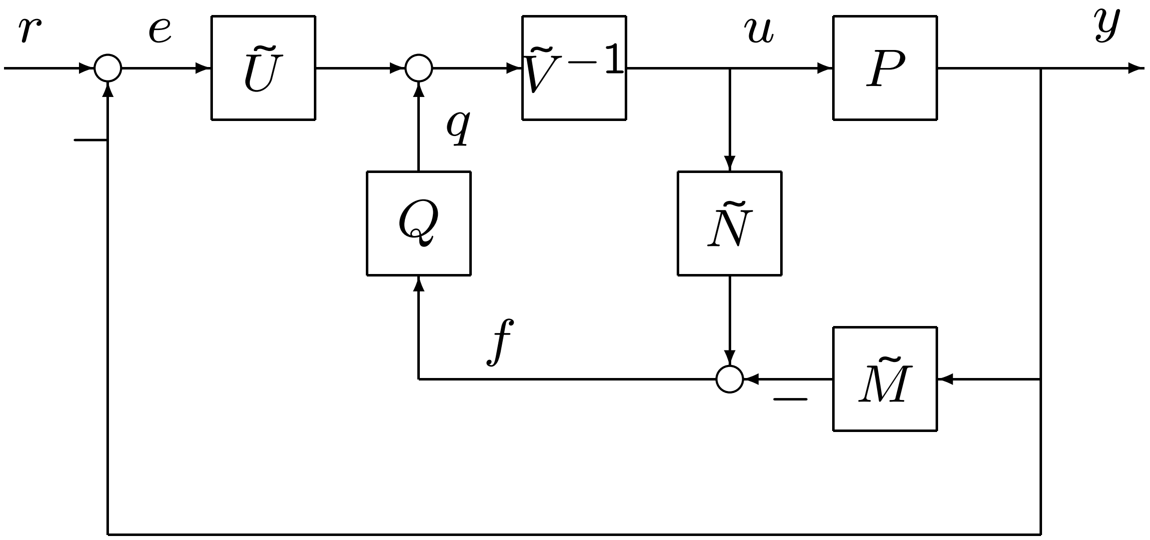
Suppose K_0 is a stabilizing controller and let

$$K_0 = \tilde{V}^{-1}\tilde{U}, \quad P_0 = \tilde{M}^{-1}\tilde{N}.$$

Then every stabilizing controller can be written as:

$$K = (\tilde{V} - Q\tilde{N})^{-1}(\tilde{U} + Q\tilde{M})$$

for some $Q \in \mathcal{H}_\infty$



Then $u = -Ky$

$f = 0$ if the plant model is perfect, i.e., if $P = P_0$.

Controller Design

===Separation of Performance and Robustness===

A high performance robust system can be designed in two steps:

(a) Design $K_0 = \tilde{V}^{-1}\tilde{U}$ to satisfy the system performance specifications with a nominal plant model P_0;

(b) Design Q to satisfy the system robustness requirements.

Note that the controller Q will not affect the system nominal performance.

K_0 can be any stabilizing controller: PI, lead-lag, LQG, H_∞, ...

$\boxed{\text{Estimation Error } f}$

$$P_0 = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = \tilde{M}^{-1}\tilde{N}$$

$$\left[\begin{array}{cc} \tilde{N} & \tilde{M} \end{array}\right] = \left[\begin{array}{c|cc} A+LC & B+LD & L \\ \hline C & D & I \end{array}\right].$$

Denote the state vector of $\left[\begin{array}{cc} \tilde{N} & \tilde{M} \end{array}\right]$ by $\hat{x}$ and note that

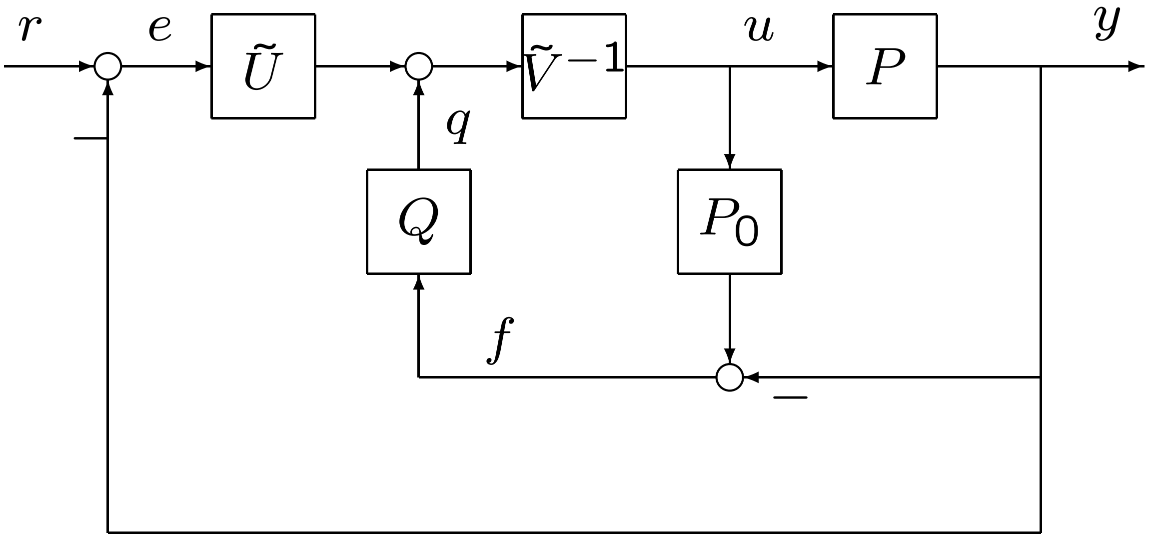
$$f = \tilde{N}u - \tilde{M}y.$$

Then we have

$$\begin{aligned} \dot{\hat{x}} &= (A+LC)\hat{x} + (B+LD)u - Ly \\ f &= (C\hat{x} + Du) - y \end{aligned}$$

i.e., f is the estimated output error.

Stable Plant



When the plant itself is stable, we can take

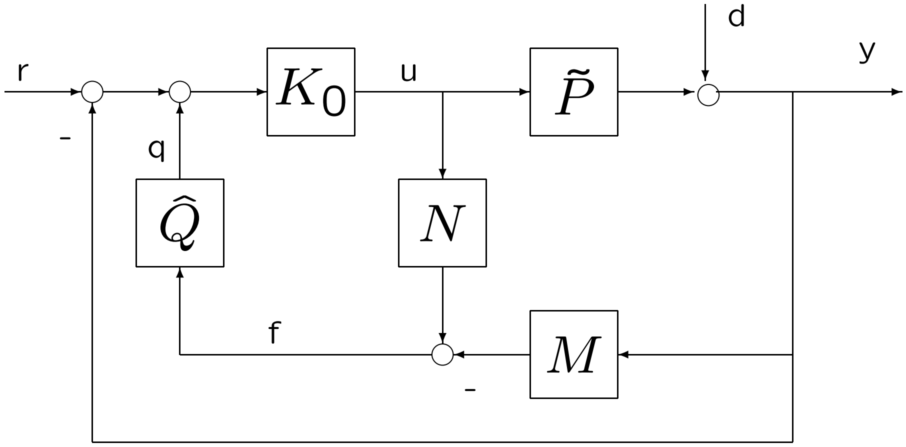
$$\tilde{N} = P_0, \quad \tilde{M} = I.$$

$f = (P_0 - P)u$ is the error between the output of the nominal model and the output of the true system.

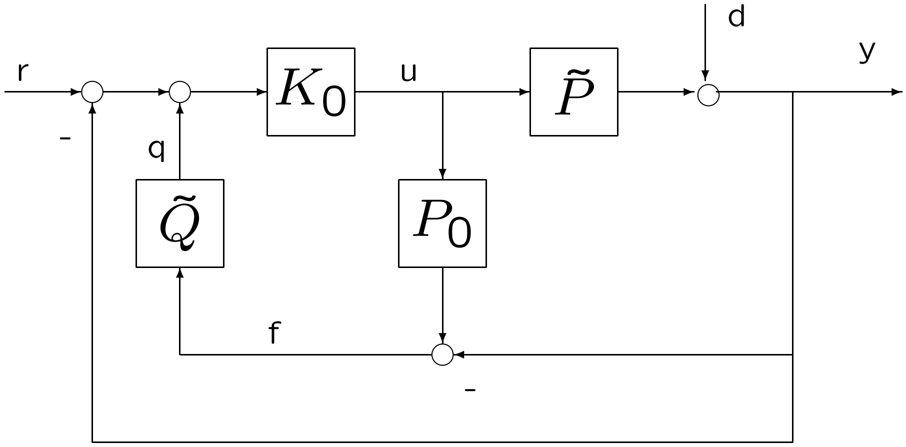
Minimum Phase Controller

When $\tilde{U}$ is minimum phase, take $Q = \tilde{U}\hat{Q}$ for some stable $\hat{Q}$. Then the controller can be written as

$$K = (I - K_0\hat{Q}\tilde{N})^{-1}(K_0 + K_0\hat{Q}\tilde{M})$$

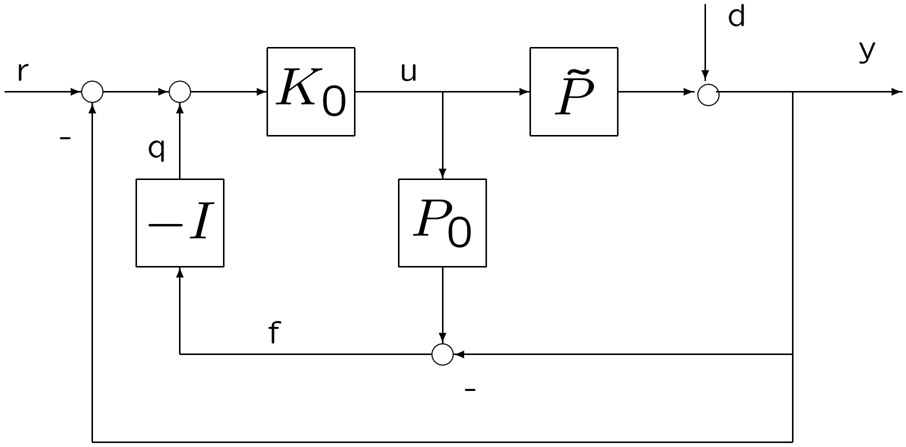


If P_0 is also stable, then



Example: Robust Stabilization Again

Optimal $\tilde{Q} = -I$.



Open Loop control with

$$u = 0\ y$$

$$u = Kr$$

with

$$K = K_0(I + P_0K_0)^{-1}$$

Robustness Example from μ-Toolbox

Nominal plant

$$P = \frac{1}{s-1}.$$

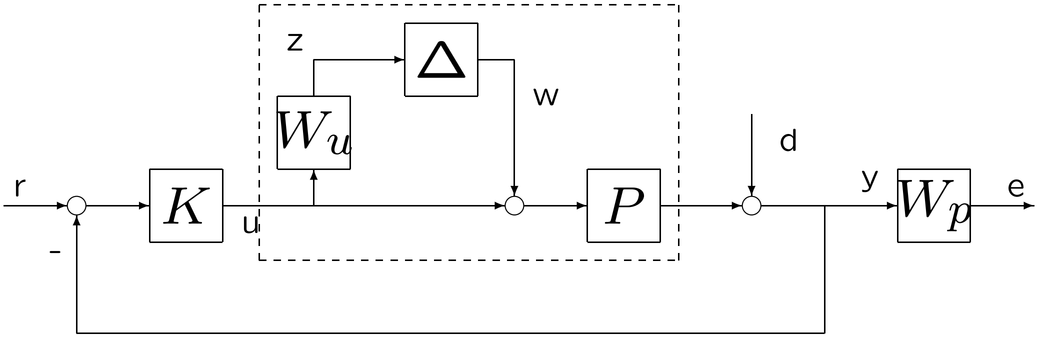
True plant in a multiplicative set

$$\mathcal{M}(P, W_u) := \left\{ P(1 + \Delta W_u) : \ \max_{\omega} |\Delta(j\omega)| \leq 1 \right\}$$

with

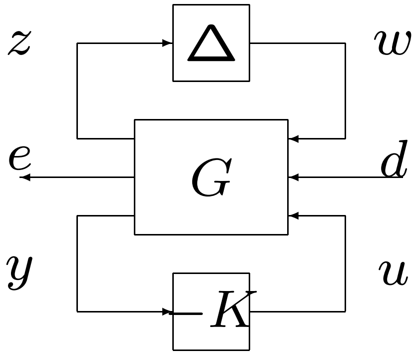
$$W_u = \frac{\frac{1}{4}\left(\frac{1}{2}s + 1\right)}{\frac{1}{32}s + 1}$$

such that $P(1 + \Delta W_u)$ and P have the same number of unstable poles.



LFT Form and Performance

$$G = \begin{bmatrix} 0 & 0 & W_u \\ W_p P & W_p & W_p P \\ P & I & P \end{bmatrix}.$$



closed-loop stability and disturbance rejection up to 0.6 rad/sec, with at least 100 : 1 at DC for all possible models.

approximately:

$$\|T_{ed}\|_\infty = \left\| \frac{W_p}{1 + \tilde{P}K} \right\|_\infty \leq 1$$

for all $\tilde{P} \in \mathcal{M}(P, W_u)$ with the weighting

$$W_p = \frac{\frac{1}{4}s + 0.6}{s + 0.006}.$$

Let M be

$$\begin{bmatrix} z \\ e \end{bmatrix} = M(s) \begin{bmatrix} w \\ d \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} w \\ d \end{bmatrix}$$

Nominal performance (i.e., when $\Delta = 0$):

$$\left\| T_{ed}^{0} \right\|_\infty := \| T_{ed} |_{\Delta=0} \|_\infty = \| M_{22} \|_\infty = \left\| \frac{W_p}{1 + PK} \right\|_\infty$$

Robust stability margin:

$$\| T_{zw} \|_\infty = \| M_{11} \| = \left\| \frac{W_u PK}{1 + PK} \right\|_\infty .$$

Robust performance:

$$\| T_{ed} \|_\infty = \left\| \frac{W_p}{1 + \tilde{P} K} \right\|_\infty \leq 1$$

is satisfied if and only if

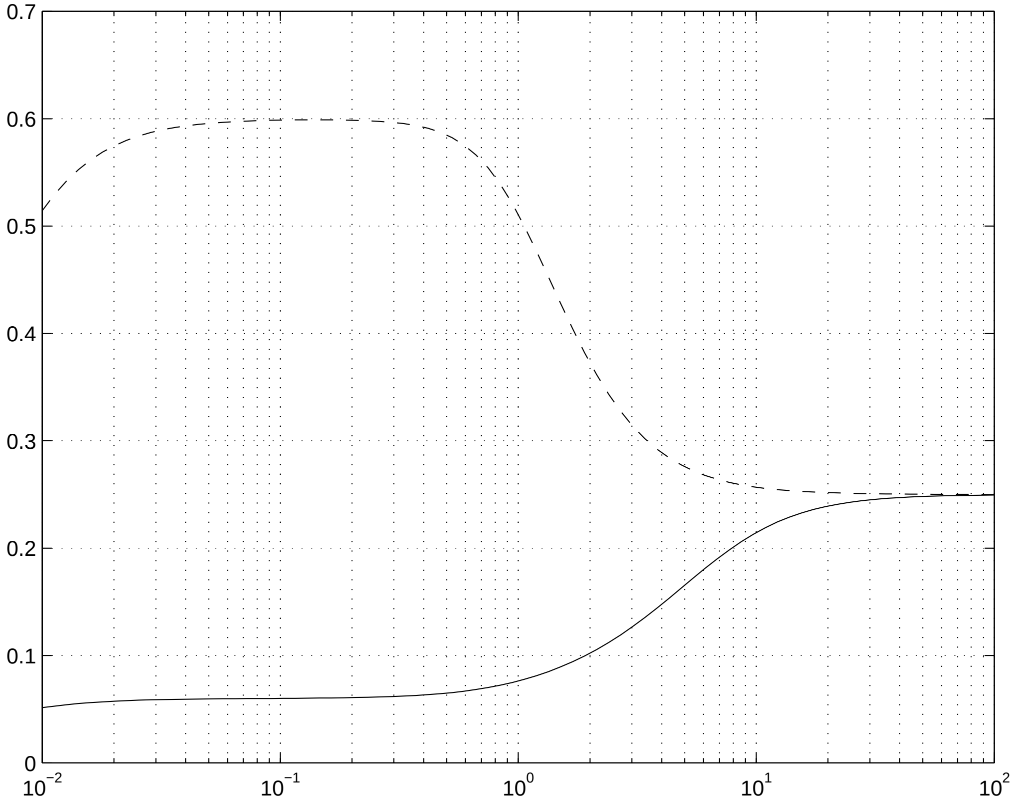
$$\mu_{\Delta_P}(M(j\omega)) \leq 1, \quad \forall \omega$$

where $\Delta_P = \text{diag}(\Delta, \Delta_f)$.

NP of Two PI Controllers

from μ-toolbox:

$$K_1 = \frac{10(0.9s+1)}{s}, \qquad K_2 = \frac{2.8s+1}{s}.$$

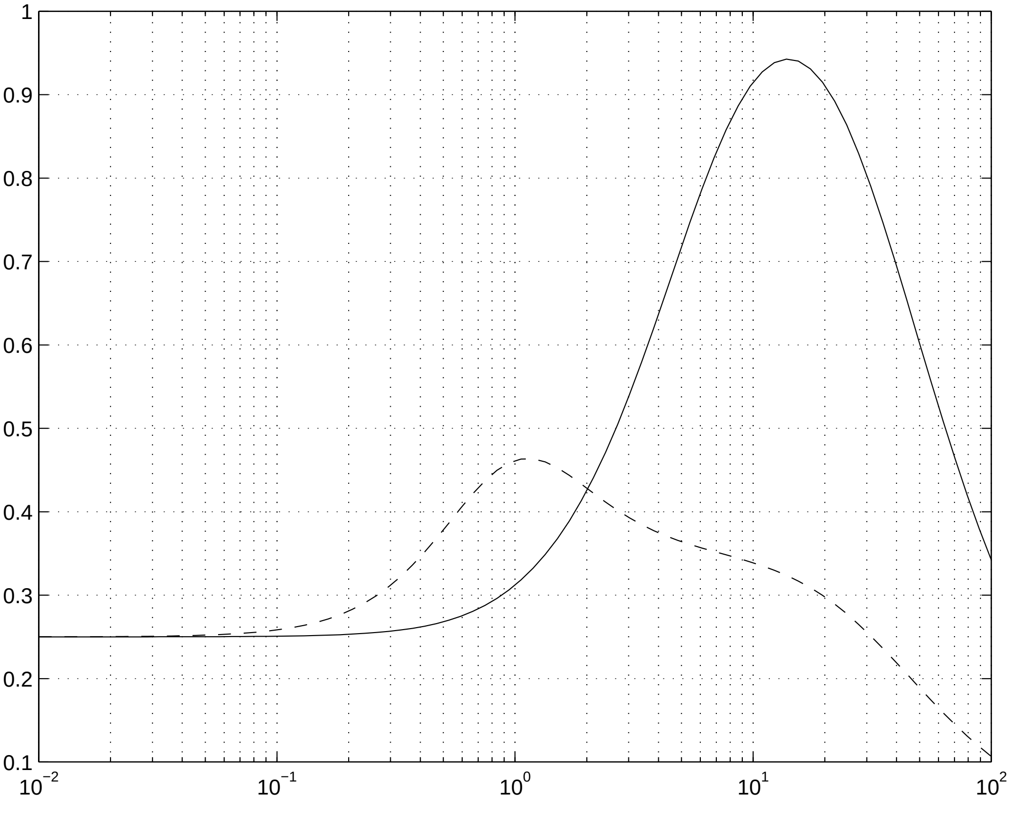


Frequency Responses of T_{ed}^0 for Nominal Performance: K_1 (solid) and K_2 (dashed)

RS of Two PI Controllers

from μ-toolbox:

$$K_1 = \frac{10(0.9s+1)}{s}, \qquad K_2 = \frac{2.8s+1}{s}.$$

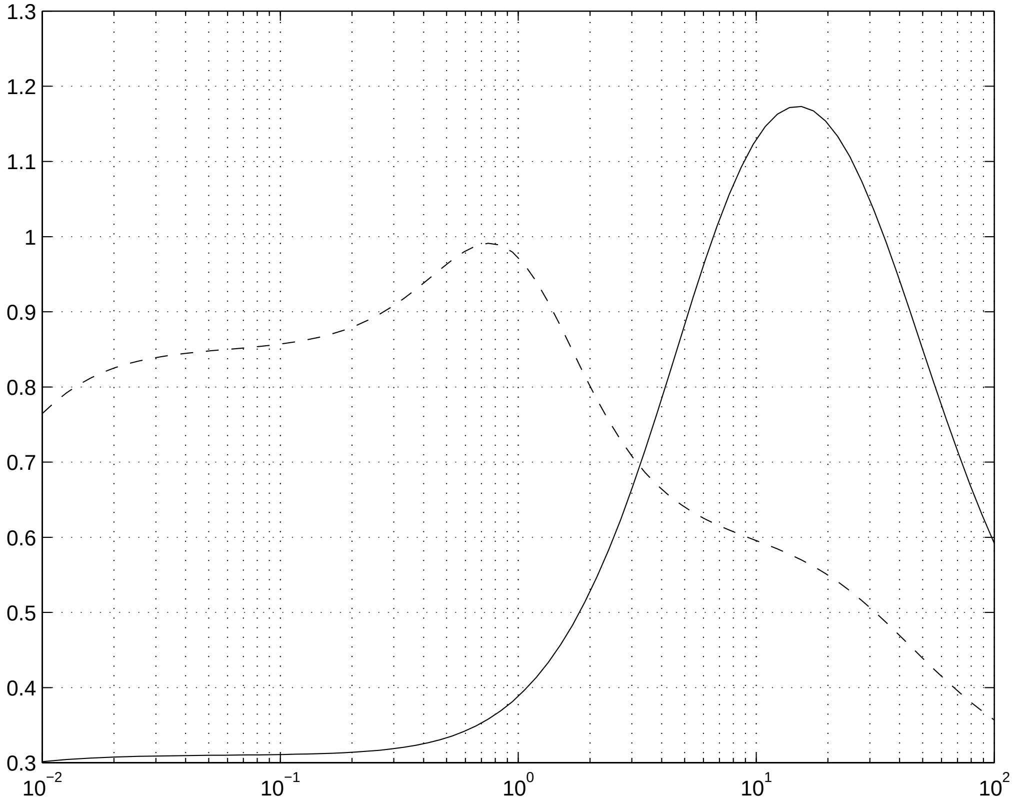


Frequency Responses of T_{zw} for Robust Stability: K_1 (solid) and K_2 (dashed)

RP of Two PI Controllers

from μ-toolbox:

$$K_1 = \frac{10(0.9s+1)}{s}, \qquad K_2 = \frac{2.8s+1}{s}.$$



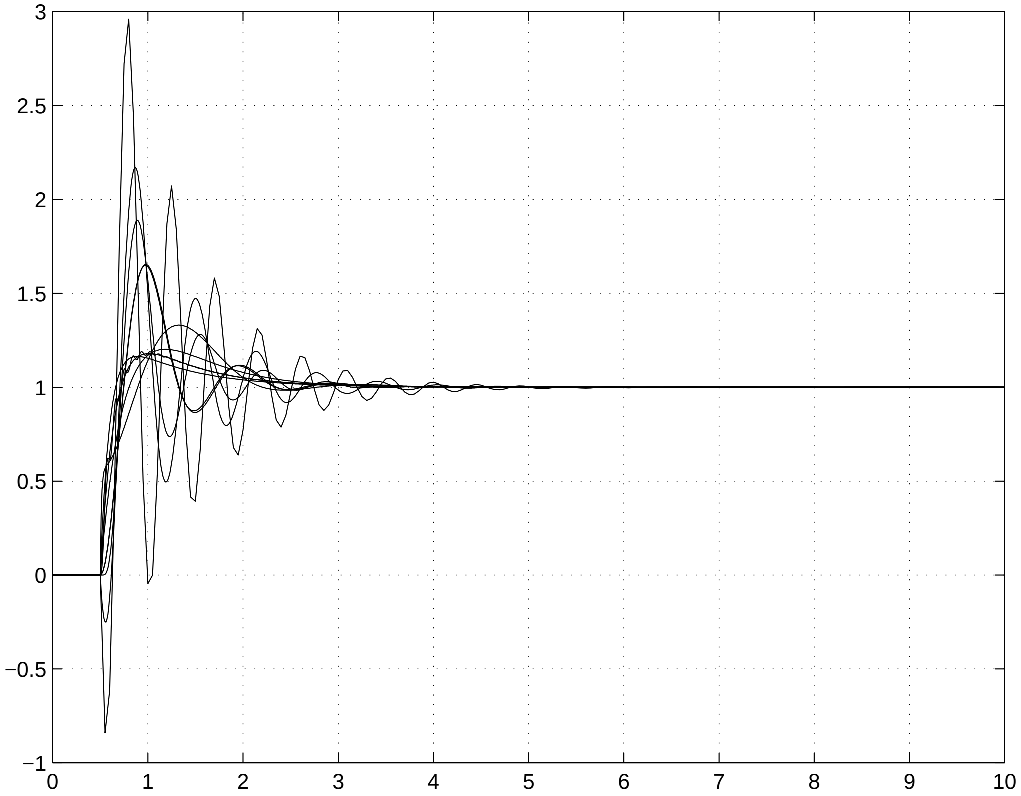
Frequency Responses of $\mu_{\Delta_P}(M(j\omega))$ for Robust Performance: K_1 (solid) and K_2 (dashed)

Time Domain Evaluation (1)

10 plants including the nominal and two "worst-case" plants in the set $\mathcal{M}(P, W_u)$

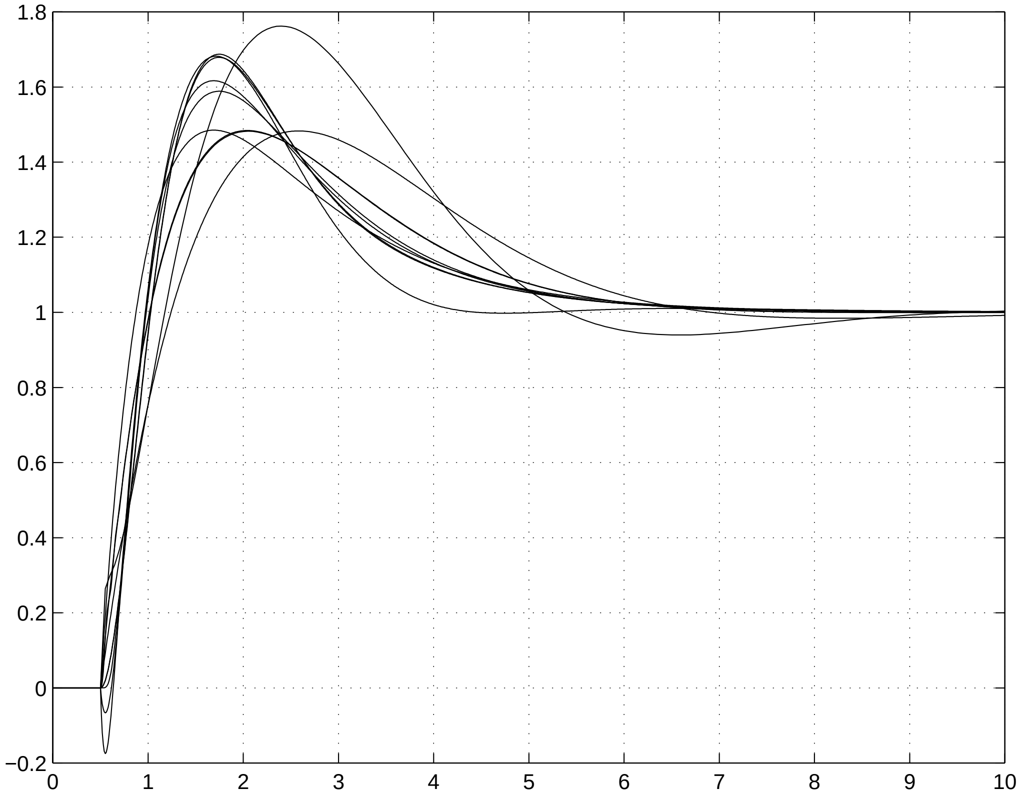
$$\begin{aligned}
P &= \frac{1}{s-1} \\
P_1 &= \frac{1}{s-1}\frac{6.1}{s+6.1} \\
P_2 &= \frac{1.425}{s-1.425} \\
P_3 &= \frac{0.67}{s-0.67} \\
P_4 &= \frac{1}{s-1}\frac{-0.07s+1}{0.07s+1} \\
P_5 &= \frac{1}{s-1}\frac{70^2}{s^2+2\cdot 0.15\cdot 70s+70^2} \\
P_6 &= \frac{1}{s-1}\frac{70^2}{s^2+2\cdot 5.6\cdot 70s+70^2} \\
P_7 &= \frac{1}{s-1}\left(\frac{50}{s+50}\right)^6 \\
P_{wc1} &= \frac{1}{s-1}\frac{-2.9621(s-9.837)(s+0.76892)}{(s+32)(s+0.56119)} \\
P_{wc2} &= \frac{1}{s-1}\frac{s^2+3.6722s+34.848}{(s+7.2408)(s+32)}
\end{aligned}$$

Time Domain Evaluation (2)



Step Response with K_1 and Various Plants for Standard Feedback Implementation

Time Domain Evaluation (3)



Step Response with K_2 and Various Plants
for Standard Feedback Implementation

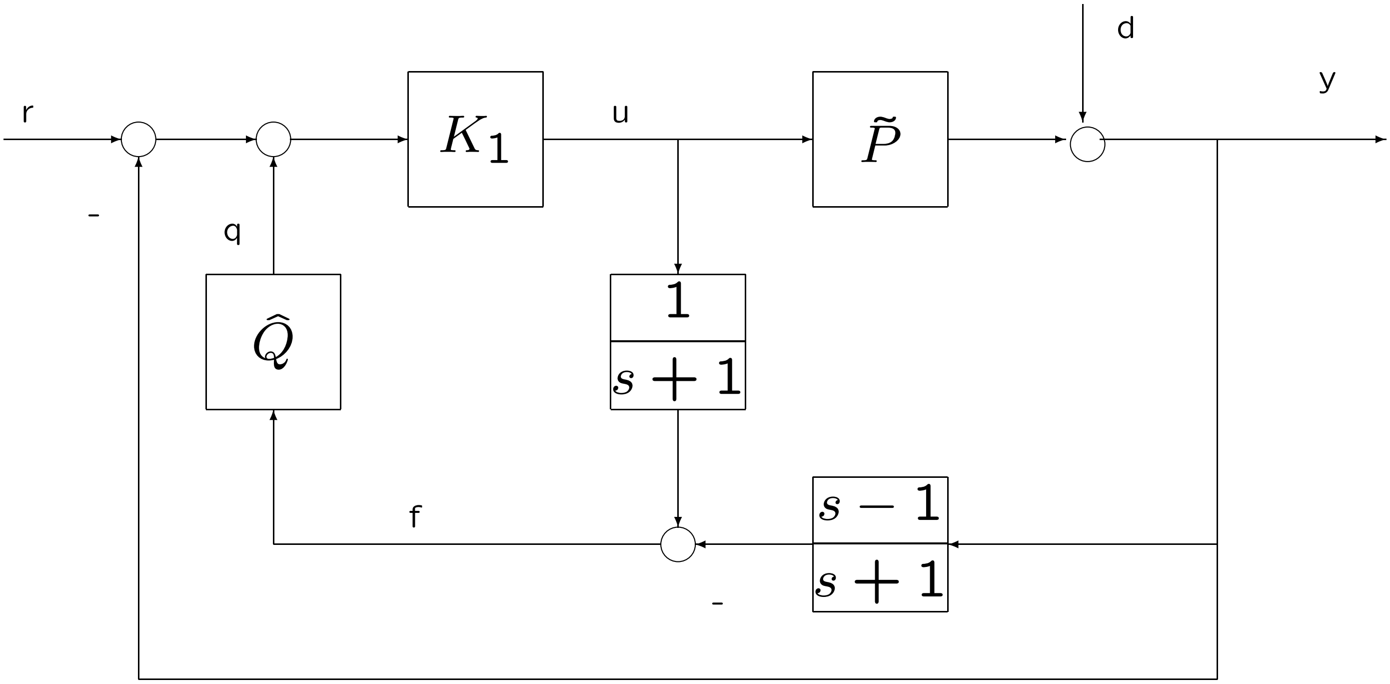
New Implementation

$$P = N/M, \quad N = \frac{1}{s+1}, \quad M = \frac{s-1}{s+1}.$$

Then

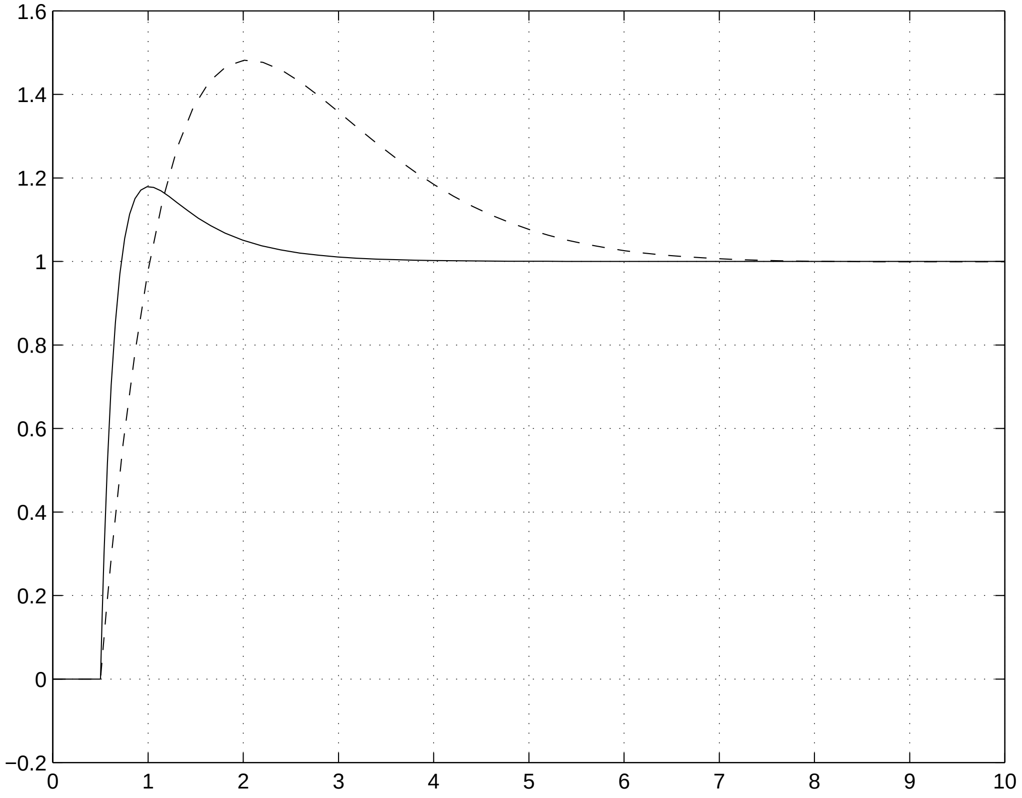
$$K_2 = \frac{K_1(1+\hat{Q}M)}{1-K_1\hat{Q}N}$$

$$\hat{Q}(s) = -\frac{0.1s(6.2s+1)(s+1)}{(0.9s+1)(s^2+1.8s+1)}.$$



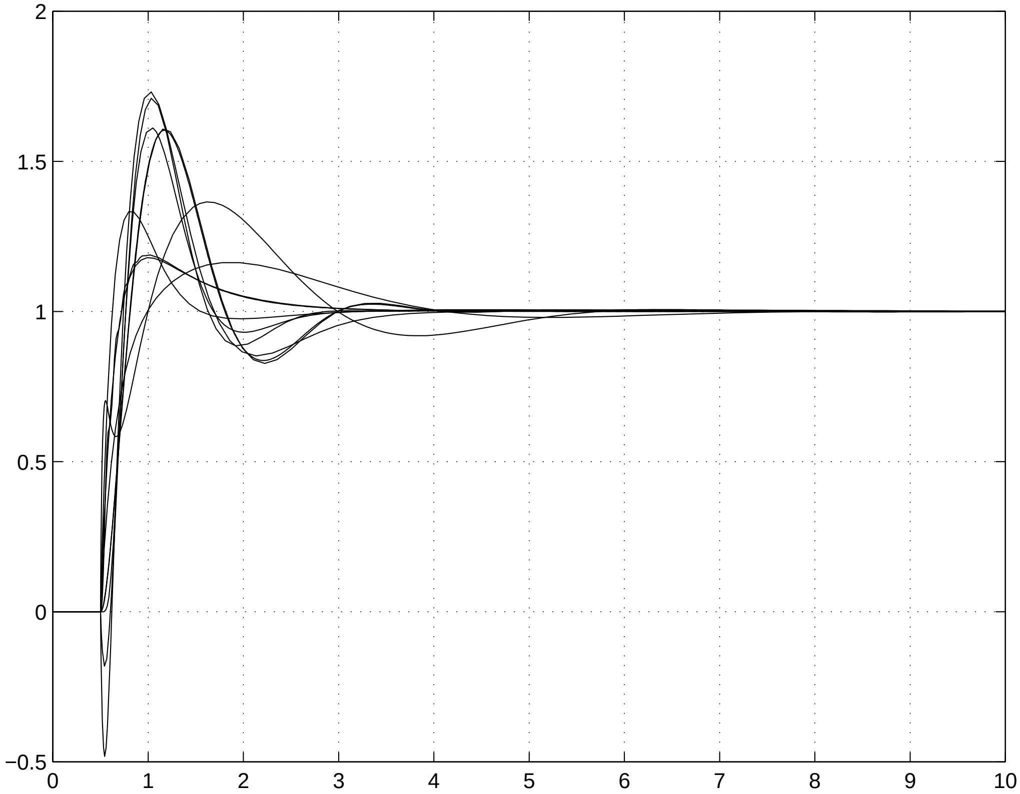
GIMC Implementation of K_2

Nominal Performance



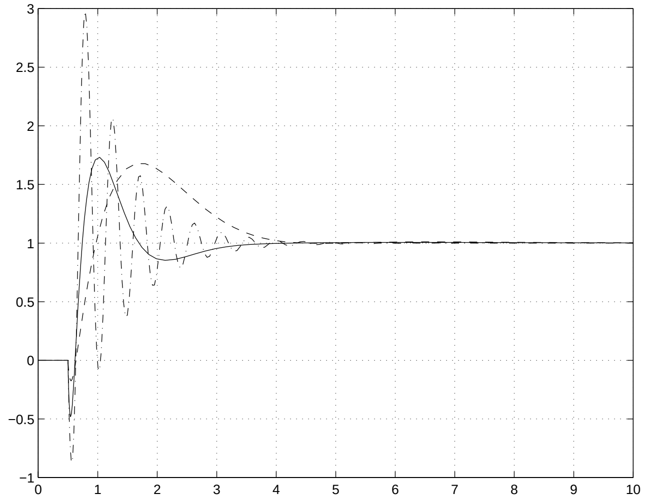
Step Responses of the Nominal P: K_1 (solid), K_2 (dashed), and GIMC (solid)

Robust Performance

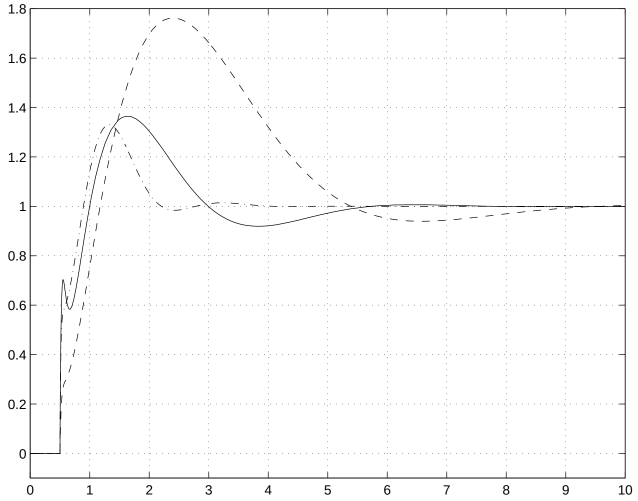


Step Responses of the Closed-loop System with K_2 Implemented Using the GIMC Structure Under Various Perturbations

Comparsion of Worst Cases



Step Responses of P_{wc1}: K_1 (dash-dot), K_2 (dashed), and GIMC (solid)



Step Responses of P_{wc2}: K_1 (dash-dot), K_2 (dashed), and GIMC (solid)

Fault Tolerant Control

= Conventional robust control approach =

f is the *residual signal* used in fault diagnosis

a possible actuator fault in the first channel of an m actuator system with $B = [B_1, B_2, \ldots, B_m]$ can be represented by introducing an uncertainty in the corresponding input matrix

$$\dot{x} = Ax + B_1(1+\delta)u_1 + B_2u_2 + \ldots + B_mu_m, \;\; \delta \in [-1, 0]$$

where $\delta = -1$ implies a total failure of the actuator and $\delta = 0$ implies no actuator failure.

Worst Case Design: a robust controller is designed for this uncertain system

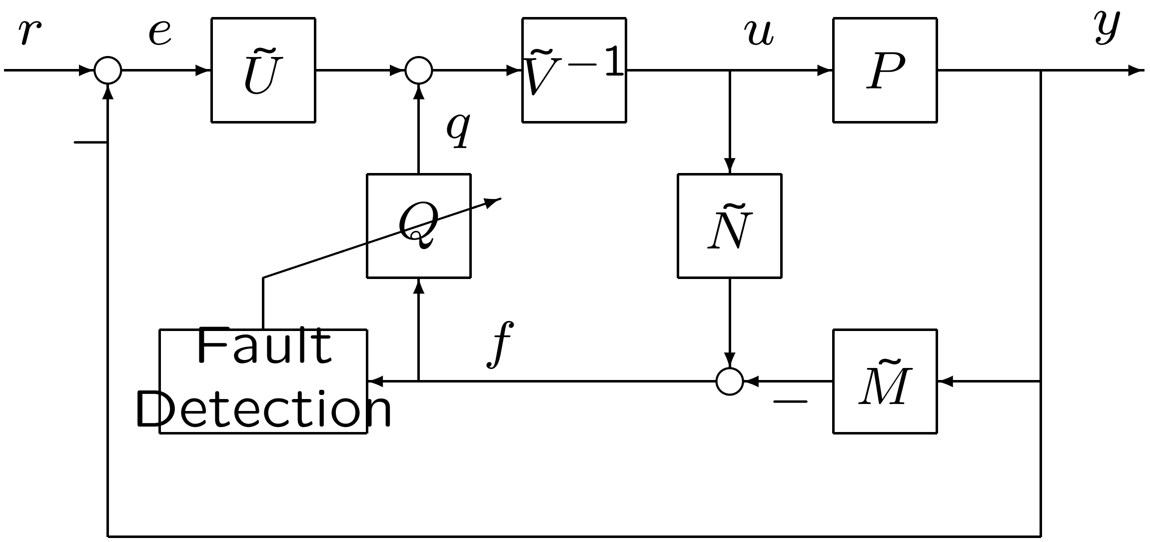
Performance: Too conservative—-worst case is rare.

Fault Tolerant Control

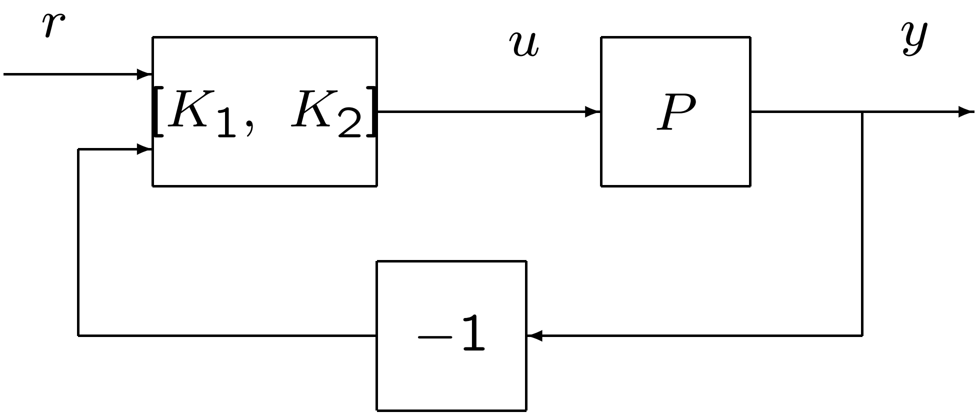
======== Our approach ========

(a) Design $K_0 = \tilde{V}^{-1}\tilde{U}$ to satisfy the system performance by assuming no faults (and model uncertainties).

(b) Design Q to tolerate possible actuators and/or sensors failures (and model uncertainties). This Q can be designed using standard robust control techniques, fuzzy control methods, adaptive control techniques, etc.



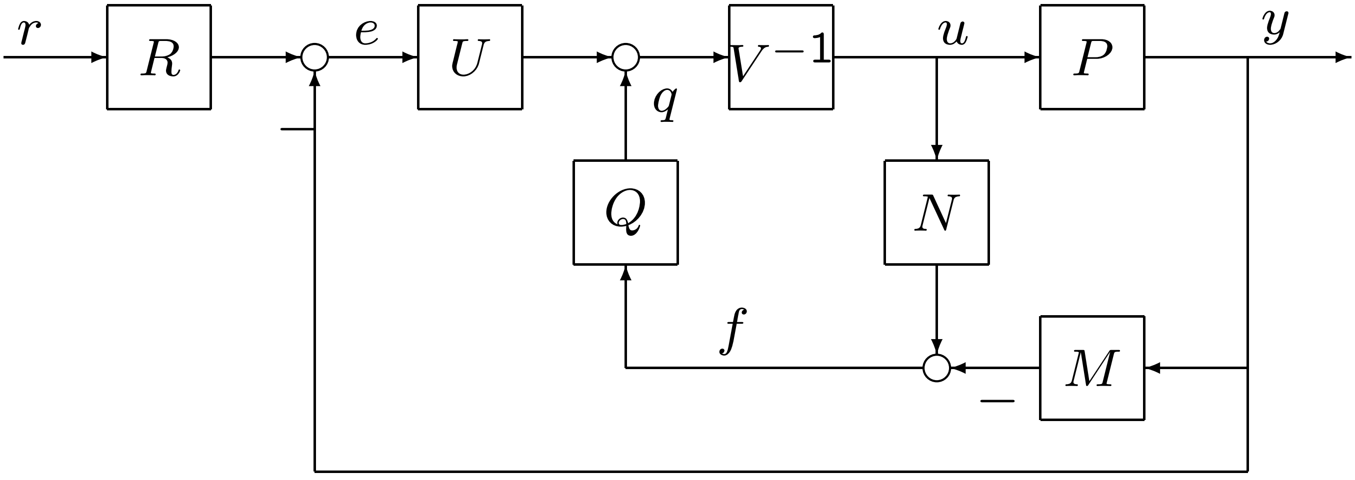
Connections with 2DOF



All 2DOF controllers

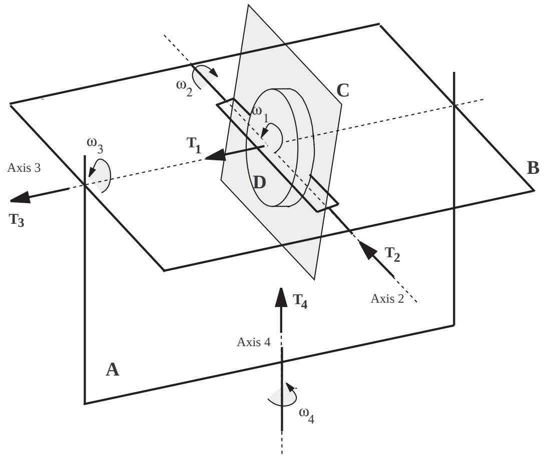
$$\begin{bmatrix} K_1 & K_2 \end{bmatrix} = (V - QN)^{-1} \begin{bmatrix} R & U + QM \end{bmatrix}$$

Now take $R = UR$ for any $R \in \mathcal{H}_\infty$

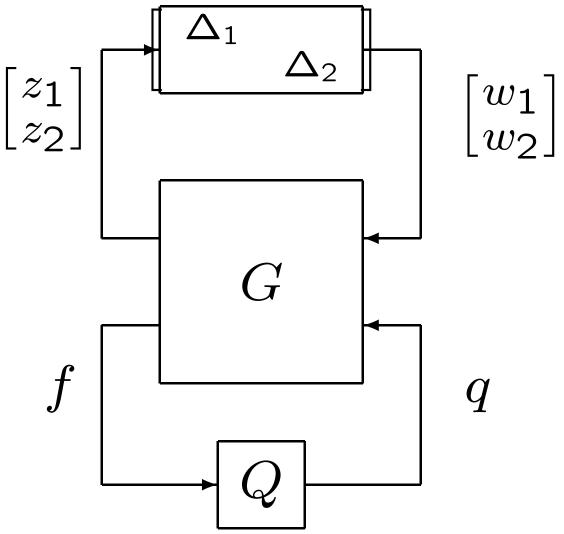


Experimental Study

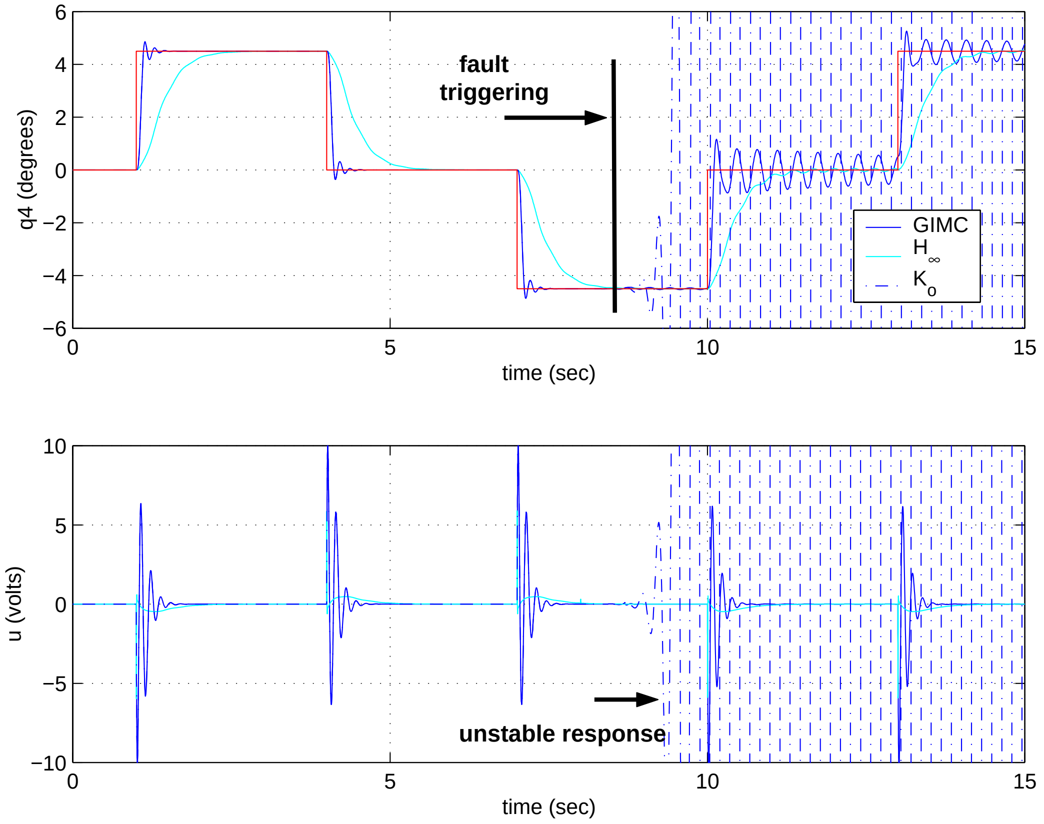
===A Gyroscope System===



Sensor faults: $\hat{y} = (I + \Delta)y$, $\Delta = \begin{bmatrix} \Delta_1 & \\ & \Delta_2 \end{bmatrix}$



Simulation with Fault

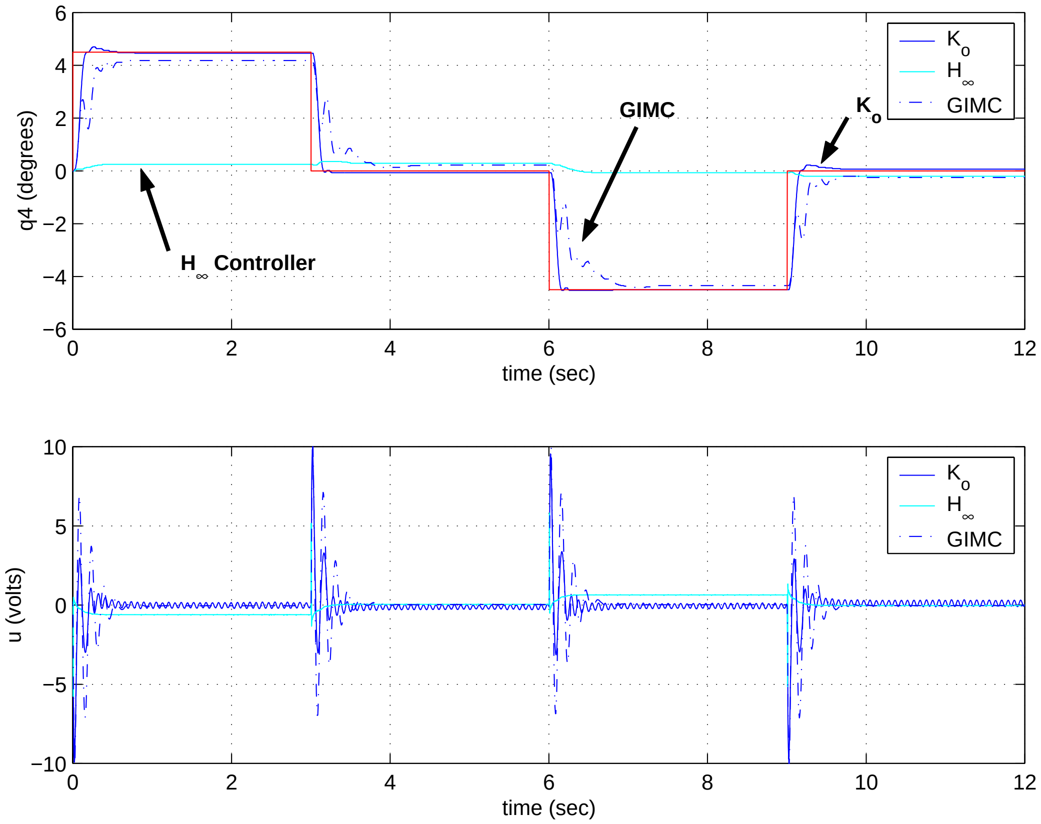


Sensor fault at 8 sec.

Our new controller performs exactly the same as the nominal LQG contorller when there is no fault and maintains stability when there is a fault.

H_∞ controller is robust but slow.

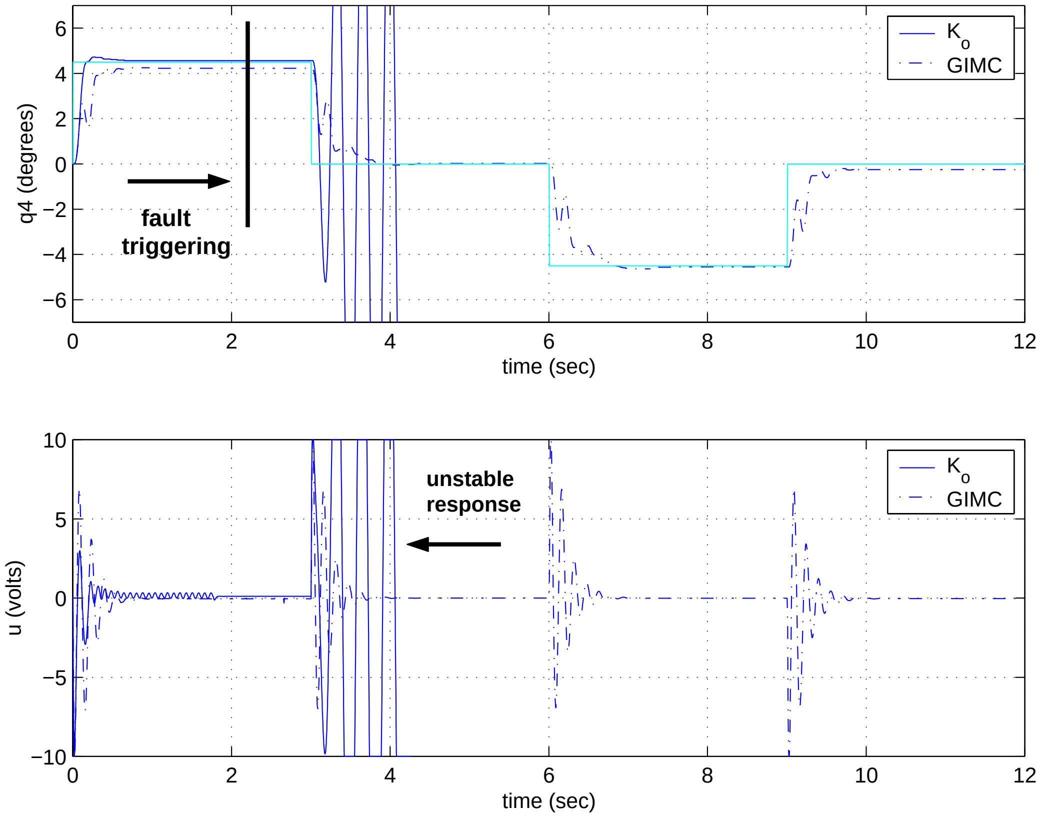
Experiments without Sensor Fault



Our new controller performs closely to the LQG controller. The difference is caused by the model uncertainties.

H_∞ controller fails to perform because of dead zone.

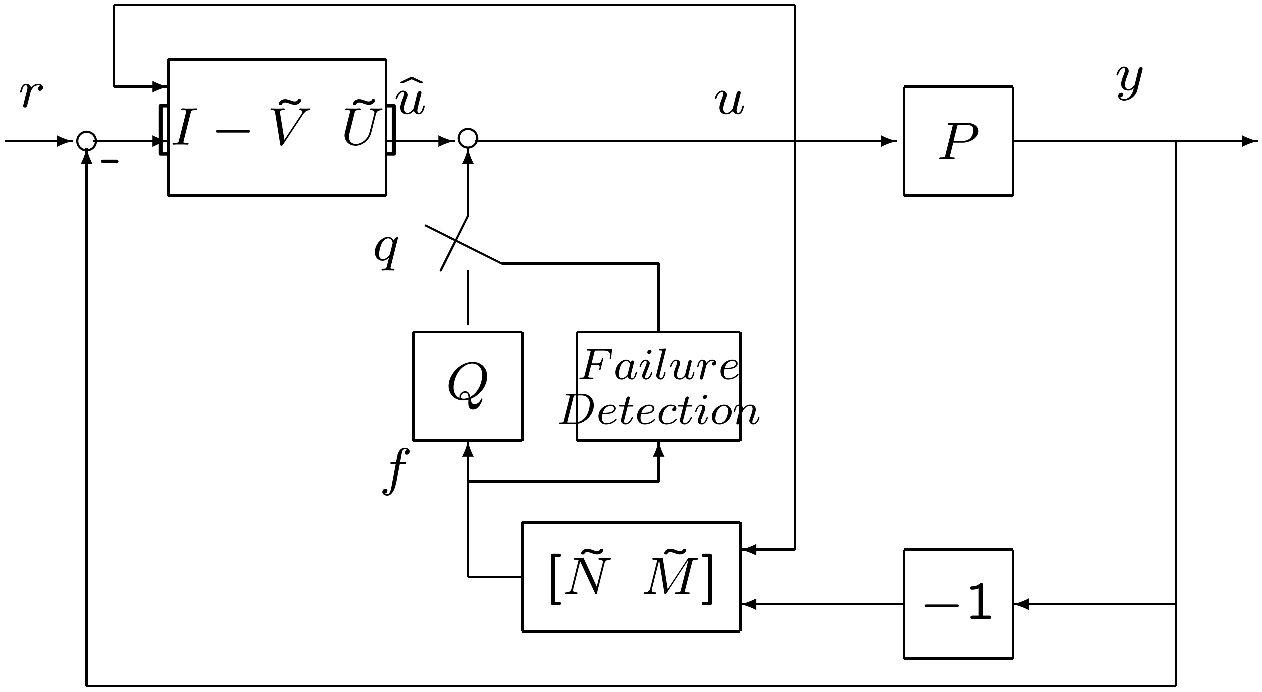
Experiments with Fault



Experimental response with sensor fault at 2.2 sec.

Experimental data is consistent with the simulation data.

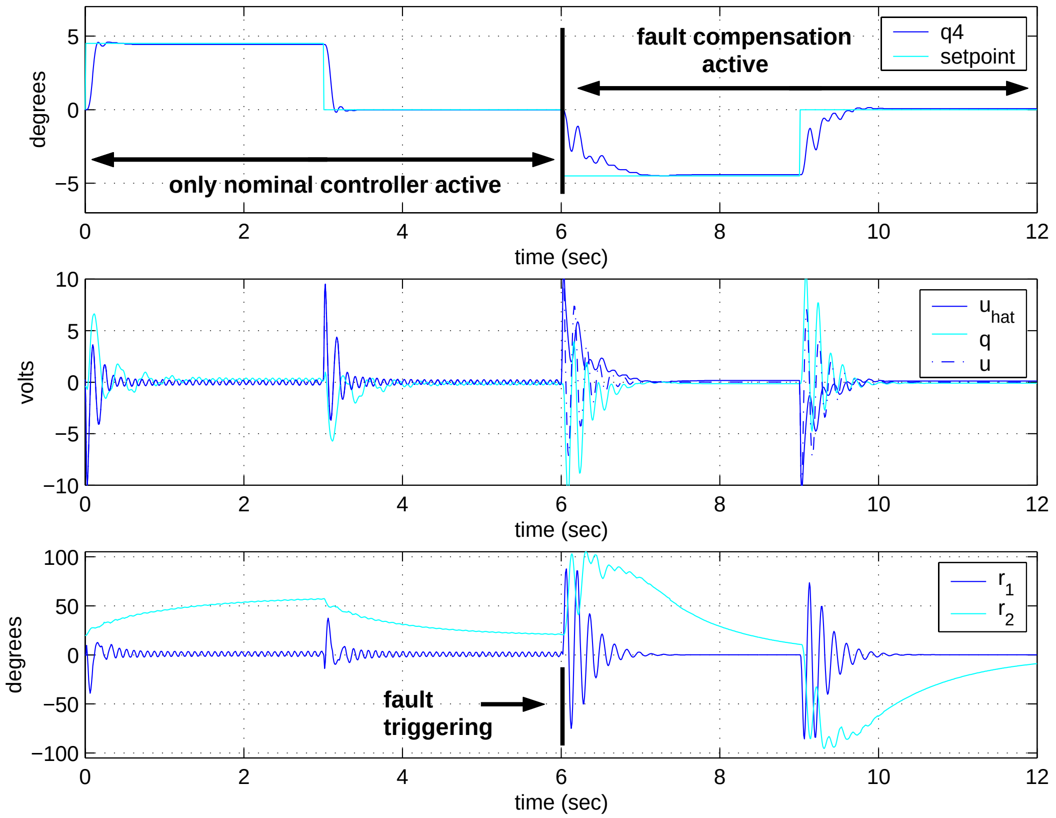
Detection and Switching



Robust controller Q is off when there is no fault.

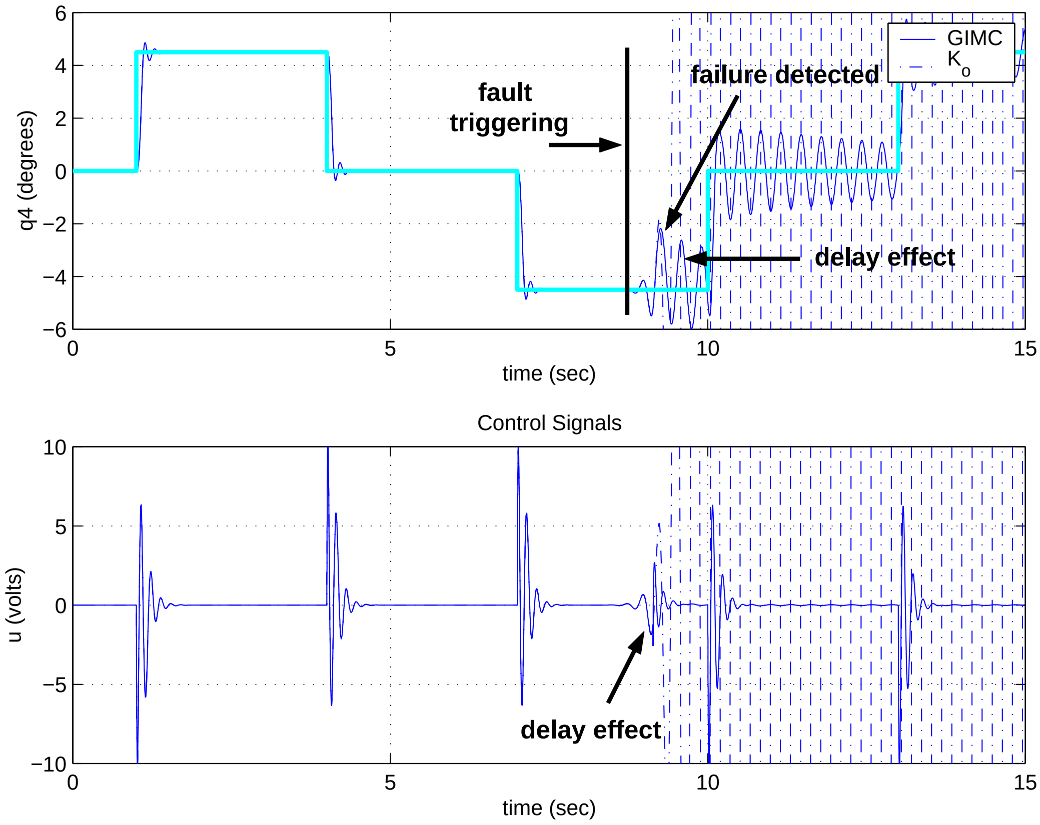
Q is switched on when the error exceeds a threshold

Experiment



Experimental response: switching on the robustness signal q after detecting sensor failure (6 sec.)

Time Delay



Experimental response: switching on the robustness signal q after detecting sensor failure (6 sec.) with delay 0.35 sec