Fault-tolerant Control System Design and Analysis

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Outline of the presentation

- Overview of two approaches to fault-tolerant control system design and analysis
- Redundancy in fault-tolerant control systems
- Trade-offs among redundancy, performance and integrity
- An example of passive fault-tolerant control design
- An example of active fault-tolerant control design
- Some open problems

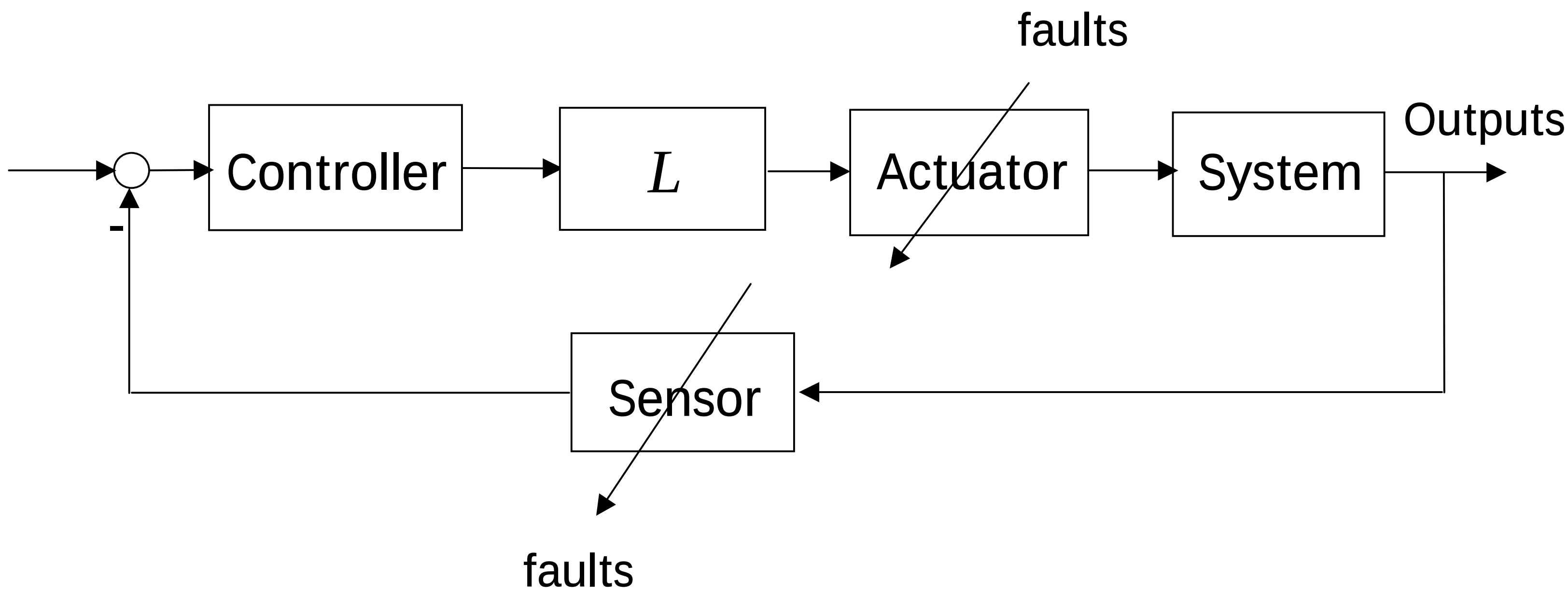
Fault-tolerant control: An overview

- Passive fault-tolerant control systems
  - Robust fixed structure controller
  - Faults have been considered at the controller design stage

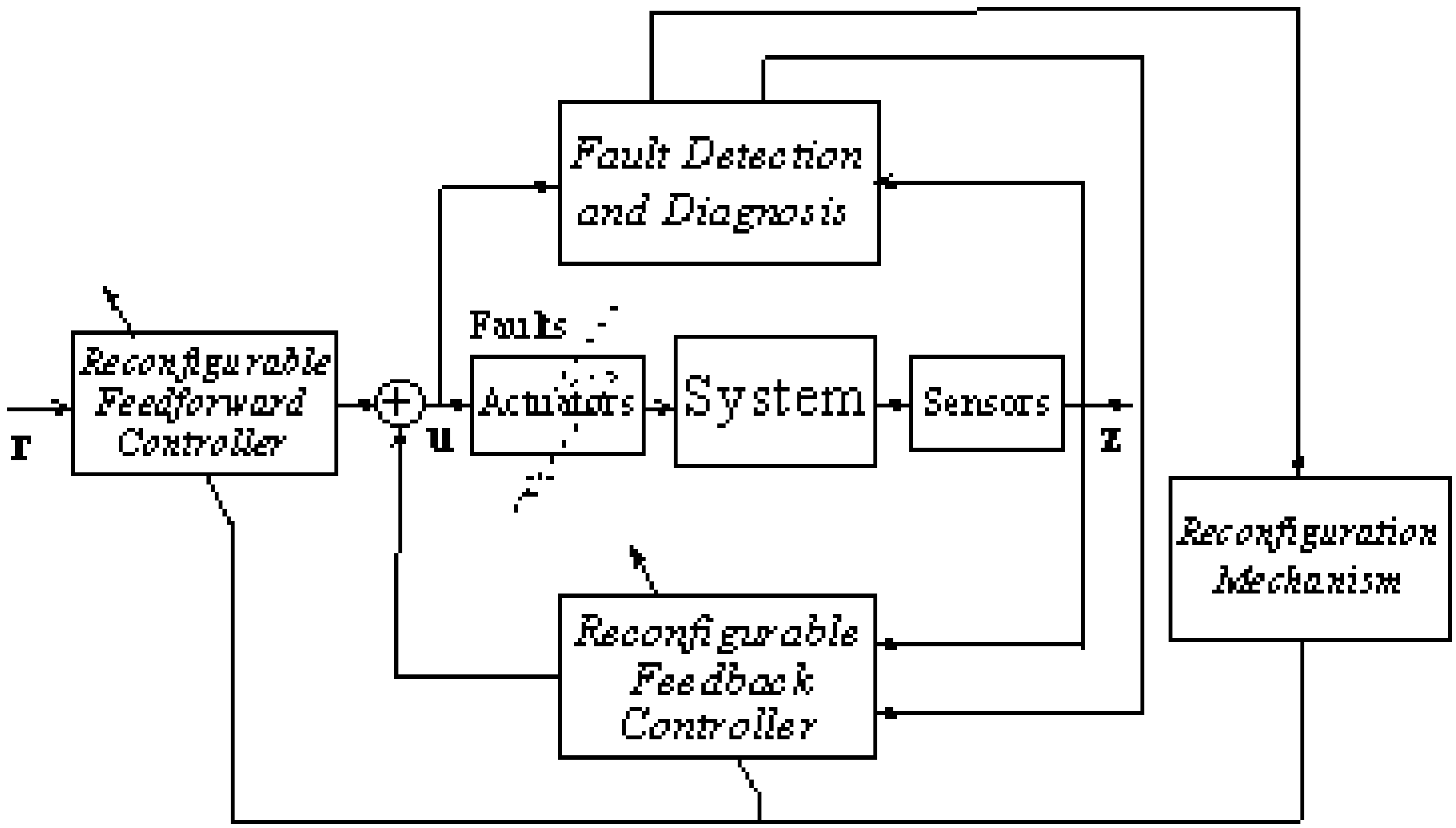
- Active fault-tolerant control systems
  - Explicit fault detection/diagnosis schemes
  - Real-time decision-making and controller reconfiguration

- The key to any fault-tolerant control system
  - Redundancy

Passive fault-tolerant control systems



Active fault-tolerant control systems



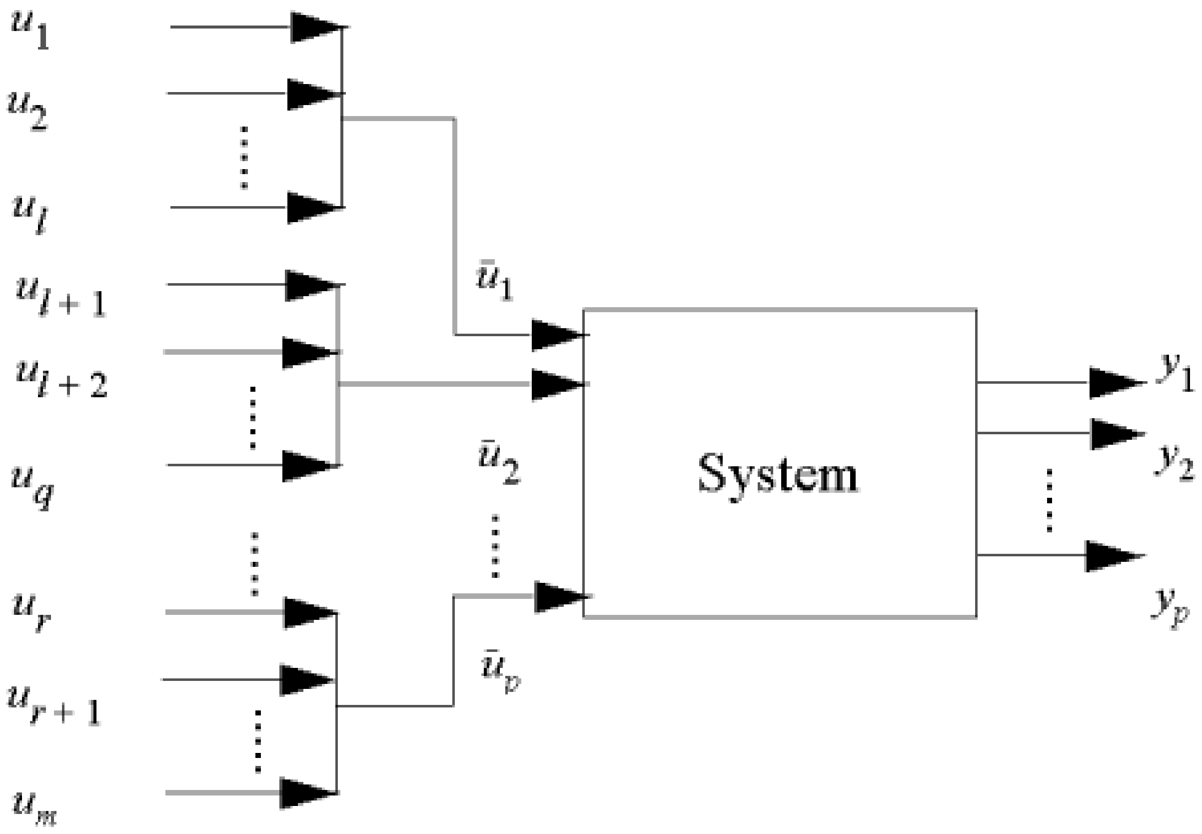
Features and limitations

- Passive fault-tolerant control systems
  - Simple to implement
  - Difficult to account for large number of fault scenarios
  - Unable to deal with unforeseen faults
- Active fault-tolerant control systems
  - Potentially be able to deal with a large number of fault scenarios
  - Can deal with certain number of unforeseen faults
  - More complex to implement
  - Real challenge is real-time decision-making

Redundancies

- Actuator redundancies
  - Multiple physical actuators
  - They usually act on the system at different locations
- Sensor redundancies
  - Multiple physical sensors
  - They usually measure the same physical quality
- Analytical redundancies
  - Rely on mathematical models (FDI)

Actuator redundancies



Actuator Redundancies

For a multi-input linear system with the following state space representation:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

where $x \in \Re^{n \times 1}$, $y \in \Re^{1}$ are the system state and the output, respectively. The system and the output matrices, A and C, are assumed to have appropriate dimensions. The input matrix $B \in \Re^{n \times p}$ can be represented by $B = \begin{bmatrix} b_1 & b_2 & \ldots\ldots & b_p \end{bmatrix}$ with each column being $b_i \in \Re^{n \times 1}$ $\quad 1 \le i \le p$. The system input vector associated with the multiple actuators is given by $u = \begin{bmatrix} u_1 & u_2 & \ldots\ldots & u_p \end{bmatrix}^T$. Three types of redundancy can be defined.

Definitions of actuator redundancies

Definition 2.1:

The system of (EQ 1) is said to have ***(p-1)*** *degree of actuator redundancy,* if the pair (A, b_i) is completely controllable $\forall i \quad (1 \le i \le p)$.

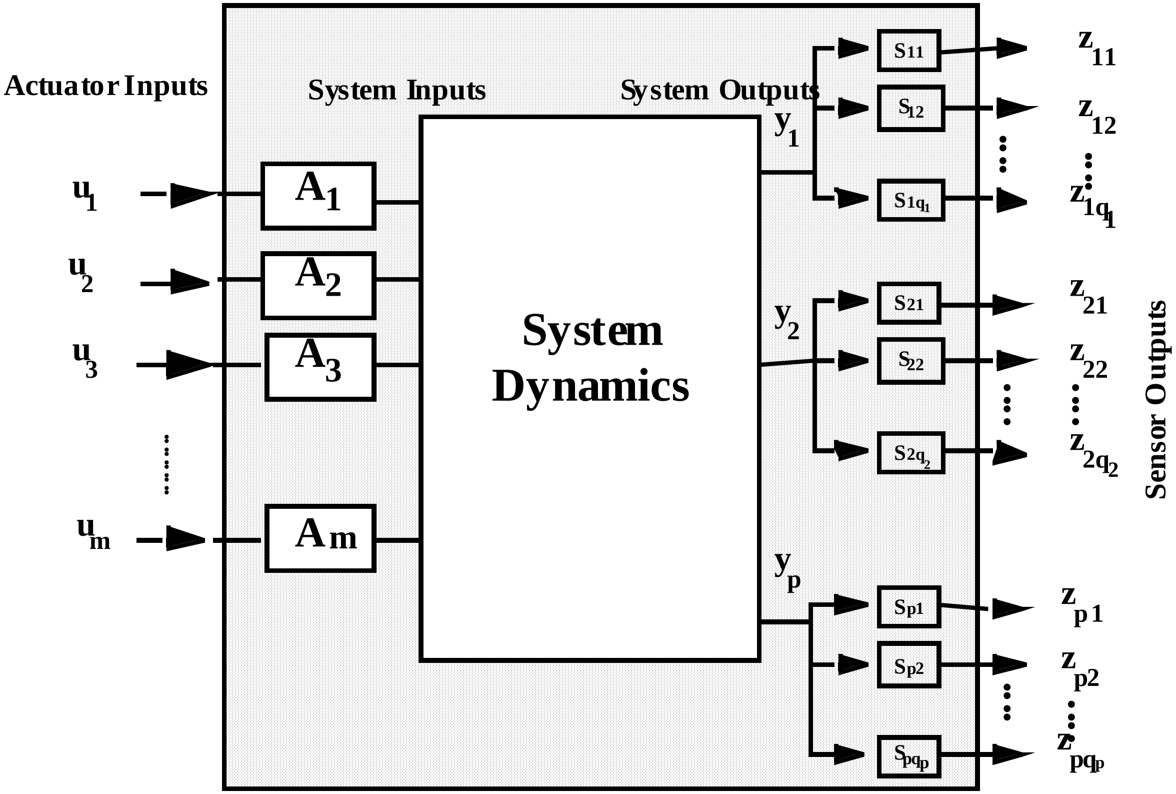
Definition 2.2:

The system of (EQ 1) is said to have ***(p-1)*** *degree of* ***non-uniform*** *actuator redundancy, if* (A, b_i) *is completely controllable* $\forall i \;\; 1 \le i \le p$ *and the* $Rank[B] = p$.

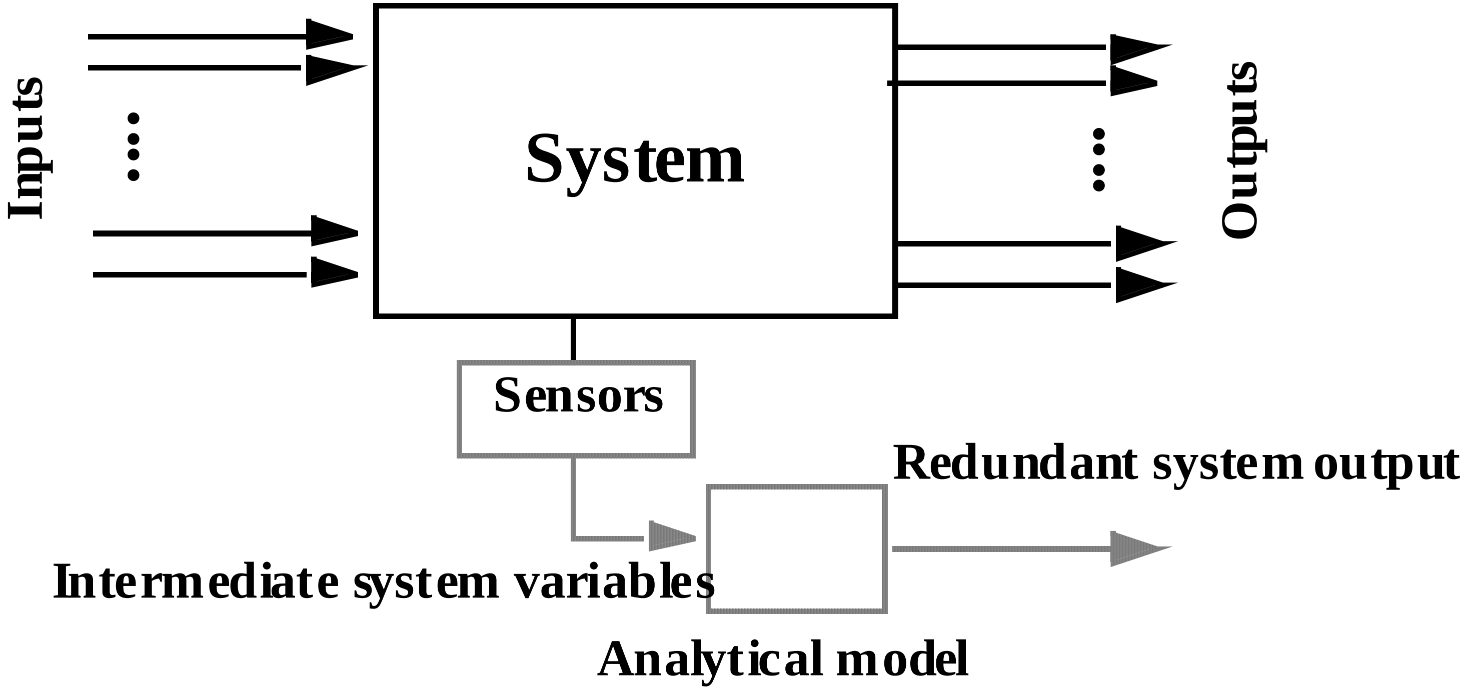
Definition 2.3:

The system of (EQ 1) is said to have ***(p-1)*** *degree of* ***uniform*** *actuator redundancy if* (A, b_i) *is completely controllable* $\forall i \;\; 1 \le i \le p$ *and the* $Rank[B] = 1$.

Sensor redundancies



Analytical Redundancies



Performance trade-offs

Three main factors to consider in any fault-tolerant control system design:

- System Integrity (safety requirements)
- Performance (design specifications)
- Redundancy (physical and financial constraints)

Problem: How to design a control system, under a given degree of redundancy such that the integrity of the system is guaranteed and the performance is satisfactory.

Issue 1: System integrity should always be maintained

Issue 2: Faults should result in reduction of the degree of redundancy first

Issue 3: One should consider performance degradation with available redundancies.

Example of passive fault-tolerant control system

The system used in this example represents a bank-angle control system for a jet transport aircraft flying at the speed of 0.8 Mach, and the attitude of 40,000 ft. There are two manipulated variables: the aileron, and the rudder. The variable being controlled is the bank-angle of the aircraft.

The transfer function matrix for this system is given as follows:

$$G(s) = \left[\frac{1.1476s^2 - 2.0036s - 13.7264}{s^4 + 0.6358s^3 + 0.9389s^2 + 0.5116s + 0.0037} \quad \frac{10.7290s^2 + 2.3169s + 10.237}{s^4 + 0.6358s^3 + 0.9389s^2 + 0.5116s + 0.0037} \right]$$

To convert the non-uniform actuator redundancy to a uniform one, the following dynamic pre-compensator is used:

$$D(s) = diag\left[\frac{(s + 0.108)^2 + 0.9708^2}{s^2} \quad \frac{(s - 4.4399)(s + 2.6940)}{s^2} \right]$$

Description of system

With such a pre-compensator, the augmented system can be represented in the following state-space form:

$$\dot{x}_a(t) = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & -0.0037 & -0.5116 & -0.9389 & -0.6358 \end{bmatrix} x_a(t) + \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \\ 1.1476 & 10.7290 \end{bmatrix} \begin{bmatrix} l_1 & 0 \\ 0 & l_2 \end{bmatrix} u_a(t)$$

and the output equation becomes:

$$y(t) = \begin{bmatrix} -11.4125 & -4.2488 & -11.3838 & -1.53 & 1 & 0 \end{bmatrix} x_a(t)$$

The following state feedback gain matrix is obtained:

$$K = \begin{bmatrix} 1.525\times10^{-2} & 0.14012 & 0.5257 & 1.05 & 1.1154 & 0.9106 \\ 2.256\times10^{-3} & 2.051\times10^{-2} & 7.736\times10^{-2} & 0.1211 & 0.1585 & 0.1241 \end{bmatrix}$$

Control system performance

TABLE 1. The eigenvalues of the closed-loop system under three modes of operation

Normal Operation	Aileron Failure	Rudder Failure
-0.2937+j0.6001	-0.2606+j1.0475	-0.5052+j0.9191
-0.2937-j0.6001	-0.2606-j1.0475	-0.5052-j0.9191
-0.3468	-0.1068+j0.3094	-0.2040+j0.4738
-1.0782	-0.1068-j0.3094	-0.2040-j0.4738
-0.5	-0.7621	-0.2745+j0.0858
-0.5	-0.1840	-0.2745-j0.0858

Control system performance

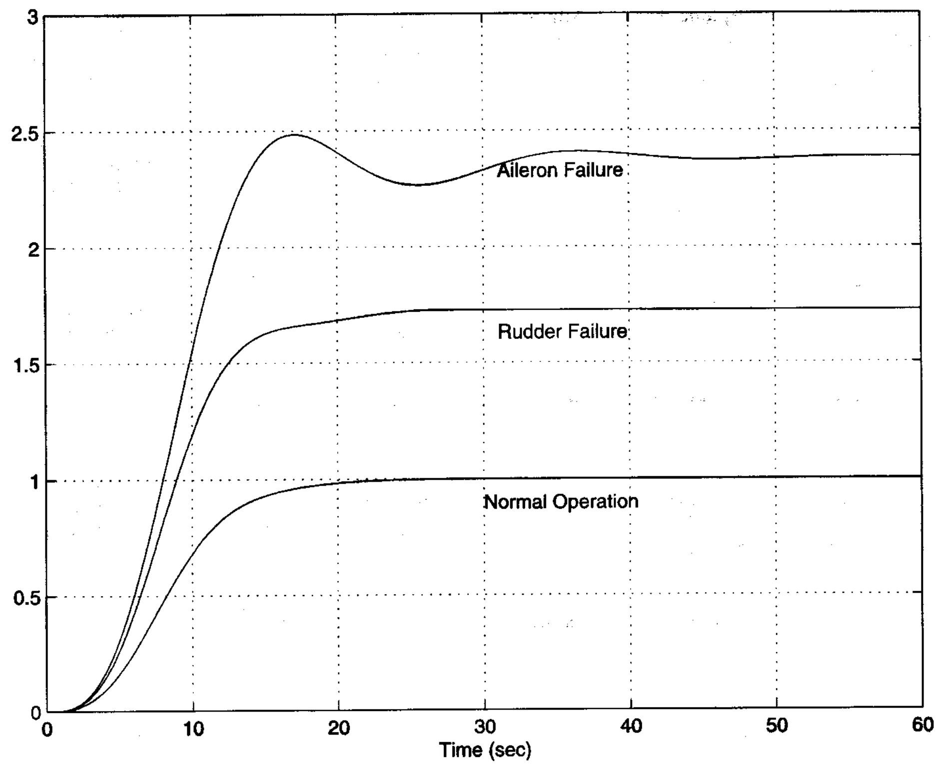
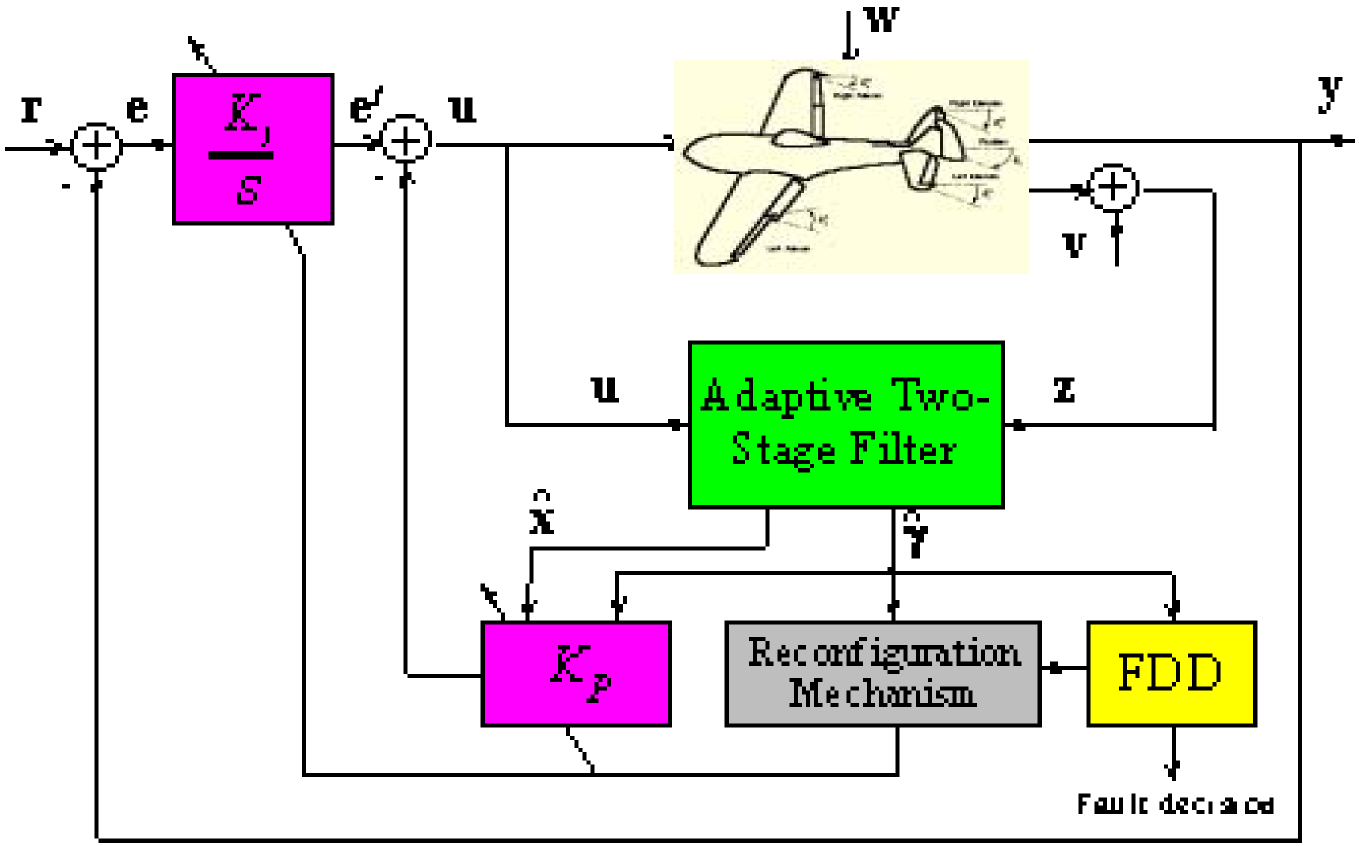
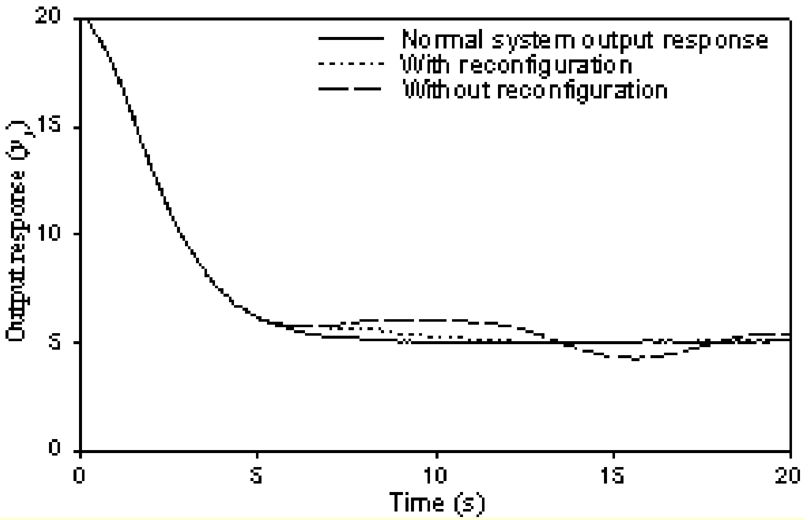


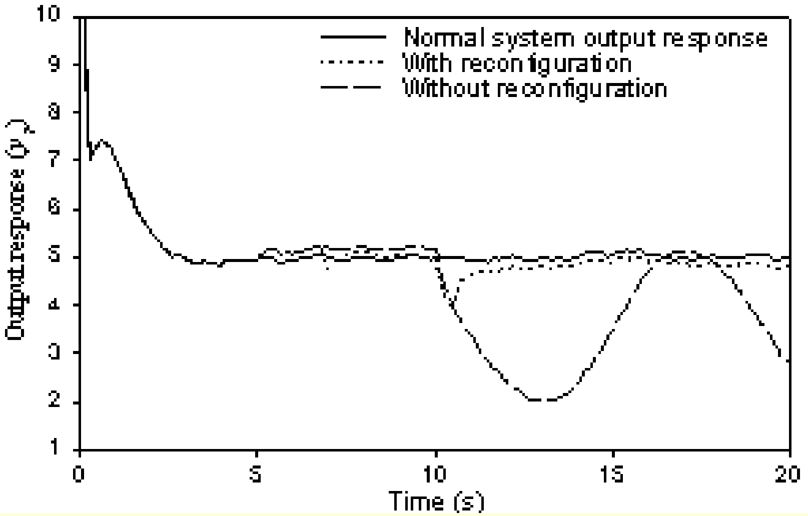
Fig. 3. Step responses of the system for different actuator operating modes.

Example of active fault-tolerant control system

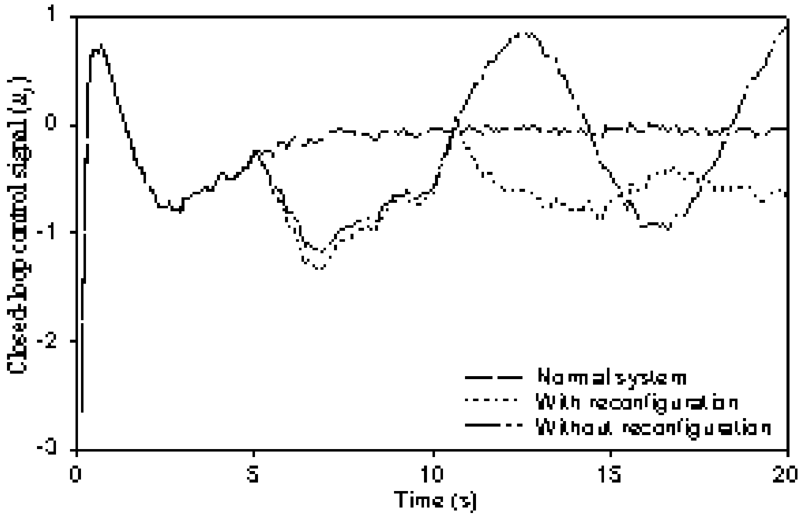


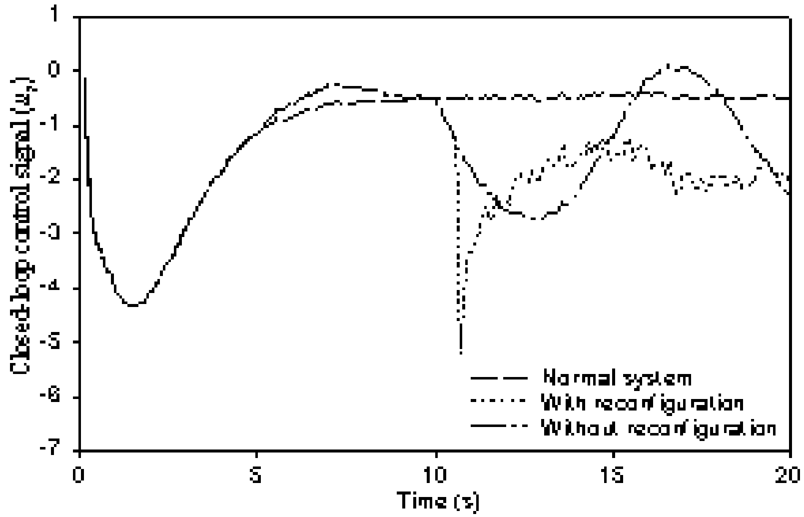
Simulation results





Simulation results





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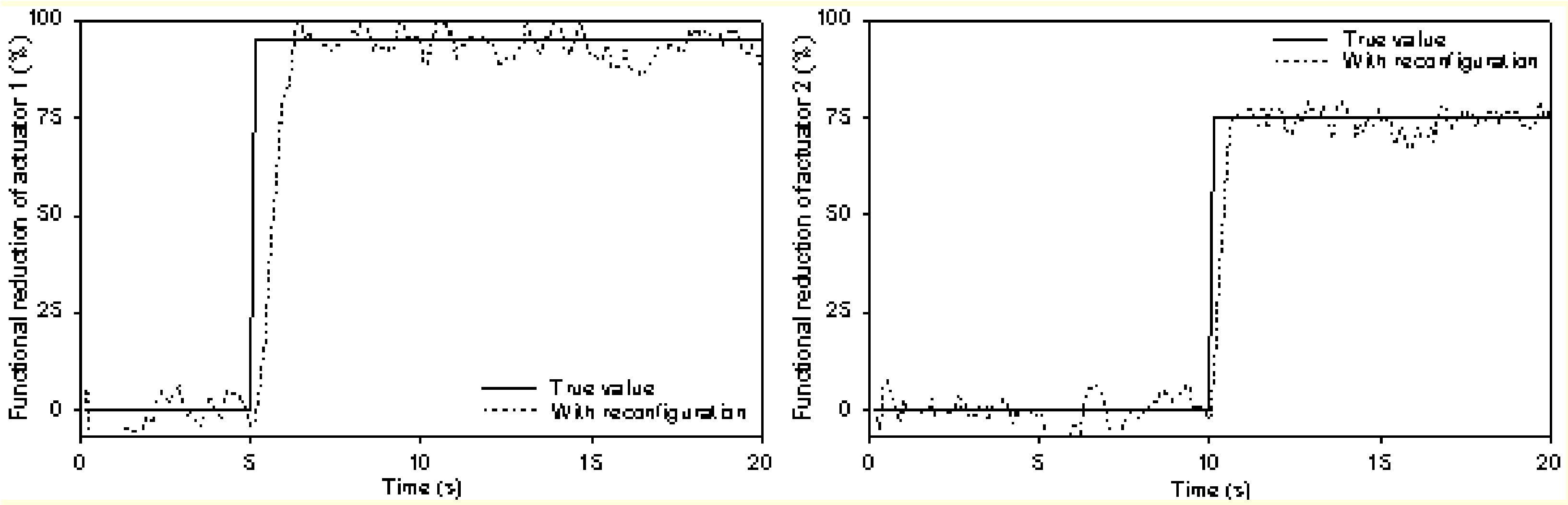
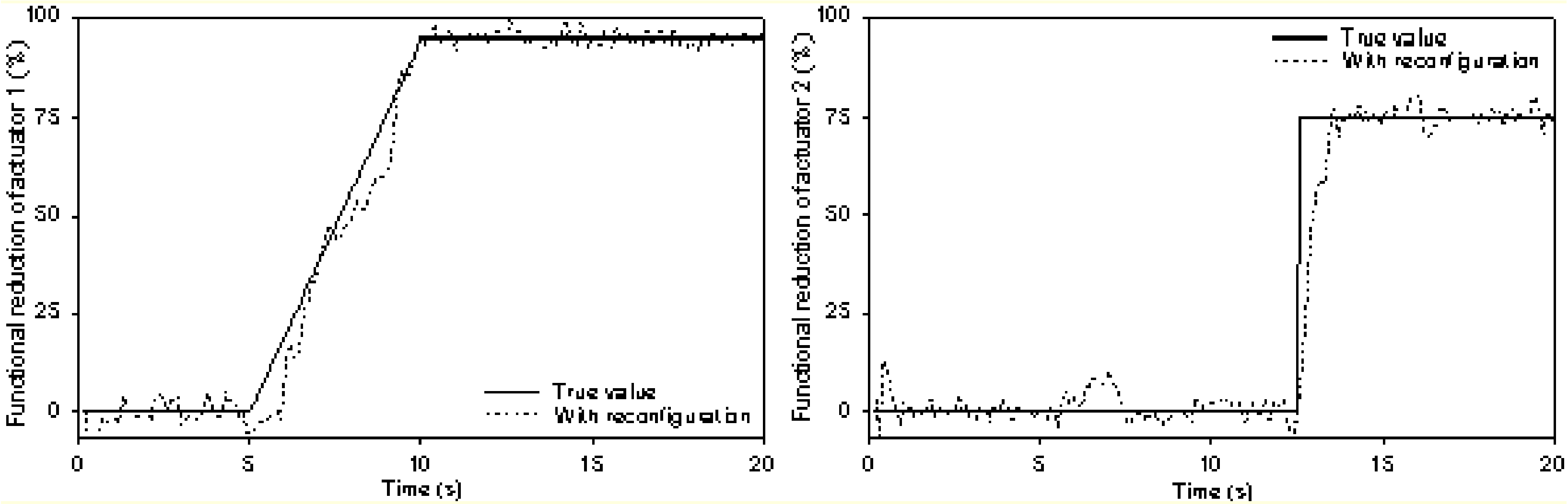


Simulation results



Some open problems

- Reliability analysis of fault-tolerant control systems
- Stability analysis of fault-tolerant control systems
- Graceful performance degradation
- Integration of passive and active approaches
- Industrial applications of fault-tolerant control system technologies

Thank You !