Fault-tolerant Control System Design and Analysis

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Outline of the presentation

- Overview of two approaches to fault-tolerant control system design and analysis
- Redundancy in fault-tolerant control systems
- Trade-offs among redundancy, performance and integrity
- An example of passive fault-tolerant control design
- An example of active fault-tolerant control design
- Some open problems
Fault-tolerant control: An overview

- Passive fault-tolerant control systems
  - Robust fixed structure controller
  - Faults have been considered at the controller design stage

- Active fault-tolerant control systems
  - Explicit fault detection/diagnosis schemes
  - Real-time decision-making and controller reconfiguration

- The key to any fault-tolerant control system
  - Redundancy
Passive fault-tolerant control systems

Controller $L$ Actuator System

Sensor faults

Outputs faults
Active fault-tolerant control systems
Features and limitations

- **Passive fault-tolerant control systems**
  - Simple to implement
  - Difficult to account for large number of fault scenarios
  - Unable to deal with unforeseen faults

- **Active fault-tolerant control systems**
  - Potentially be able to deal with a large number of fault scenarios
  - Can deal with certain number of unforeseen faults
  - More complex to implement
  - Real challenge is real-time decision-making
Redundancies

- **Actuator redundancies**
  - Multiple physical actuators
    - They usually act on the system at different locations

- **Sensor redundancies**
  - Multiple physical sensors
    - They usually measure the same physical quality

- **Analytical redundancies**
  - Rely on mathematical models (FDI)
Actuator redundancies
For a multi-input linear system with the following state space representation:

\[
\dot{x} = Ax + Bu \\
y = Cx
\]

where \( x \in \mathbb{R}^{n \times 1}, \ y \in \mathbb{R}^1 \) are the system state and the output, respectively. The system and the output matrices, \( A \) and \( C \), are assumed to have appropriate dimensions. The input matrix \( B \in \mathbb{R}^{n \times p} \) can be represented by

\[
B = \begin{bmatrix} b_1 & b_2 & \ldots & b_p \end{bmatrix}
\]

with each column being \( b_i \in \mathbb{R}^{n \times 1} \quad 1 \leq i \leq p \). The system input vector associated with the multiple actuators is given by \( u = \begin{bmatrix} u_1 & u_2 & \ldots & u_p \end{bmatrix}^T \). Three types of redundancy can be defined.
Definitions of actuator redundancies

Definition 2.1:

The system of (EQ 1) is said to have \((p-1)\) degree of actuator redundancy, if the pair \((A, b_i)\) is completely controllable \(\forall i \quad (1 \leq i \leq p)\).

Definition 2.2:

The system of (EQ 1) is said to have \((p-1)\) degree of non-uniform actuator redundancy, if \((A, b_i)\) is completely controllable \(\forall i \quad 1 \leq i \leq p\) and the \(\text{Rank}[B] = p\).

Definition 2.3:

The system of (EQ 1) is said to have \((p-1)\) degree of uniform actuator redundancy if \((A, b_i)\) is completely controllable \(\forall i \quad 1 \leq i \leq p\) and the \(\text{Rank}[B] = 1\).
Sensor redundancies
Analytical Redundancies

- Inputs
- System
- Sensors
- Intermediate system variables
- Redundant system output
- Outputs
- Analytical model
Performance trade-offs

Three main factors to consider in any fault-tolerant control system design:

- System Integrity (safety requirements)
- Performance (design specifications)
- Redundancy (physical and financial constraints)

Problem: How to design a control system, under a given degree of redundancy such that the integrity of the system is guaranteed and the performance is satisfactory.

**Issue 1**: System integrity should always be maintained

**Issue 2**: Faults should result in reduction of the degree of redundancy first

**Issue 3**: One should consider performance degradation with available redundancies.
Example of passive fault-tolerant control system

The system used in this example represents a bank-angle control system for a jet transport aircraft flying at the speed of 0.8 Mach, and the attitude of 40,000 ft. There are two manipulated variables: the aileron, and the rudder. The variable being controlled is the bank-angle of the aircraft.

The transfer function matrix for this system is given as follows:

\[
G(s) = \begin{bmatrix}
\frac{1.1476s^2 - 2.0036s - 13.7264}{s^4 + 0.6358s^3 + 0.9389s^2 + 0.5116s + 0.0037} & \frac{10.7290s^2 + 2.3169s + 10.237}{s^4 + 0.6358s^3 + 0.9389s^2 + 0.5116s + 0.0037}
\end{bmatrix}
\]

To convert the non-uniform actuator redundancy to a uniform one, the following dynamic pre-compensator is used:

\[
D(s) = \text{diag} \begin{bmatrix}
\frac{(s + 0.108)^2 + 0.9708^2}{s^2} & \frac{(s - 4.4399)(s + 2.6940)}{s^2}
\end{bmatrix}
\]
Description of system

With such a pre-compensator, the augmented system can be represented in the following state-space form:

\[
\dot{x}_a(t) = \begin{bmatrix}
0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \\
0.0 & 0.0 & -0.0037 & -0.5116 & -0.9389 & -0.6358 \\
\end{bmatrix} x_a(t) + \begin{bmatrix}
0.0 \\
0.0 \\
0.0 \\
0.0 \\
0.0 \\
0.0 \\
\end{bmatrix} \nu_a(t) + \begin{bmatrix}
i_1 \\
i_2 \\
\end{bmatrix}
\]

and the output equation becomes:

\[
y(t) = \begin{bmatrix}
-11.4125 \\
-4.2488 \\
-11.3838 \\
-1.53 \\
\end{bmatrix} x_a(t)
\]

The following state feedback gain matrix is obtained:

\[
K = \begin{bmatrix}
1.525 \times 10^{-2} & 0.14012 & 0.5257 & 1.05 & 1.1154 & 0.9106 \\
2.256 \times 10^{-3} & 2.051 \times 10^{-2} & 7.736 \times 10^{-2} & 0.1211 & 0.1585 & 0.1241 \\
\end{bmatrix}
\]
Control system performance

**TABLE 1.** The eigenvalues of the closed-loop system under three modes of operation

<table>
<thead>
<tr>
<th>Normal Operation</th>
<th>Aileron Failure</th>
<th>Rudder Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.2937+j0.6001</td>
<td>-0.2606+j1.0475</td>
<td>-0.5052+j0.9191</td>
</tr>
<tr>
<td>-0.2937-j0.6001</td>
<td>-0.2606-j1.0475</td>
<td>-0.5052-j0.9191</td>
</tr>
<tr>
<td>-0.3468</td>
<td>-0.1068+j0.3094</td>
<td>-0.2040+j0.4738</td>
</tr>
<tr>
<td>-1.0782</td>
<td>-0.1068-j0.3094</td>
<td>-0.2040-j0.4738</td>
</tr>
<tr>
<td>-0.5</td>
<td>-0.7621</td>
<td>-0.2745+j0.0858</td>
</tr>
<tr>
<td>-0.5</td>
<td>-0.1840</td>
<td>-0.2745-j0.0858</td>
</tr>
</tbody>
</table>
Control system performance

Fig. 3. Step responses of the system for different actuator operating modes.
Example of active fault-tolerant control system
Simulation results
Simulation results

[Graphs showing simulation results for control signal over time with different configurations: normal system, with reconfiguration, without reconfiguration.]
Simulation results
Simulation results
Some open problems

- Reliability analysis of fault-tolerant control systems
- Stability analysis of fault-tolerant control systems
- Graceful performance degradation
- Integration of passive and active approaches
- Industrial applications of fault-tolerant control system technologies
Thank You!