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Fuzzy-logic based trend classification for fault diagnosis of chemical processes

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Abstract

In this paper, fault diagnosis based on patterns exhibited in the sensors measuring the process variables is considered. The temporal patterns that a process event leaves on the measured sensors, called event signatures, can be utilized to infer the state of operation using a pattern-matching approach. However, the qualitative nature of the features leads to imprecise classification boundaries at the trend-identification stage and hence at the trend-matching stage. Moreover, noise and other underlying phenomena may lead to non-reproducibility of the same trends chosen to represent an event. Thus, a crisp inference process might lead to a large knowledge-base of signatures; it could also cause misclassification. To overcome this, a fuzzy-reasoning approach is proposed to ensure robustness to the inherent uncertainty in the identified trends and to provide succinct mapping. A two-staged strategy is employed: (i) *identifying* the most likely fault candidates based on a *similarity measure* between the observed trends and the *event-signatures* in the knowledge-base and, (ii) *estimation* of the fault magnitude. The fuzzy-knowledge-base consists of a set of physically interpretable *if–then* rules providing physical insight into the process. The technique provides multivariate inferencing and is transparent. We illustrate the application of the proposed approach in the fault diagnosis of an exothermic reactor case study.

Keywords: Process monitoring; Qualitative trend analysis; Fuzzy logic; Classification; Fault diagnosis

1. Introduction

One of the important reasons for developing a trend analysis technique is the subsequent use of the classified trends in fault diagnosis. Hence, typical reasoning systems that depend on the use of 'event signatures' for fault diagnosis have three important components: (i) A *language* to represent the trends (ii) A technique to *identify* the trends and (3) A *mapping* from trends to operational conditions. This process is shown in Fig. 1 comprising of the trend extraction and trend-utilization stages. In a recent paper (Dash, Rengaswamy, & Venkatasubramanian, 2001) we presented an *interval-halving scheme to facilitate automatic extraction* of temporal features from sensor data in terms of the trend language of primitives (Janusz & Venkatasubramanian, 1991), thus covering the first two requirements of the whole process (Fig. 1). The next and final step in the pattern-recognition process is to be able to relate the characteristic trends (signatures) to process operating conditions. This is shown in Fig. 2. This paper addresses this issue thus completing the process of trend-based-reasoning.

The fundamental elements of the trend description language proposed by Janusz and Venkatasubramanian (1991) are the *primitives* i.e., A(0, 0), B(+, +), C(+, 0), D(+, −), E(−, +), F(−, 0), G(−, −) where the signs are of the first and second derivatives respectively (Fig. 3). It can be shown that these primitives can be used to qualitatively explain any reasonable continuous function. A detailed description of this trend description language can be found in Rengaswamy and Venkatasubramanian (1995). The problem of recovering these primitives from real-time noisy process data is a difficult problem due to the absence of a priori knowledge about

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Nomenclature

i	generic index with no specific connotation
q_i	identified quadratic in trend
P_i	primitive
t	time stamps of trend
t^*	time stamps of trend signature
F^*	identified fault from Stage 1
t_u	common/uniform time interval formed by $\cup[t, t^*]$
Tr	extracted sensor trend
SI^1,SI^2	similarity indices
CI	confidence index
$s_{P_1P_2}$	similarity between primitives P_1 and P_2
Superscripts	
$+$	indicates positive deviation
$-$	indicates negative deviation
level	fuzzy variable = [low, medium, high]
Greek letters	
α	observed to signature magnitude ratio
μ	fuzzy membership function
ΔM	change in steady state magnitude due to fault

sensor trend characteristics such as noise level and varying scales of evolution. For automatic trend extraction from noisy process data, we will use a novel interval-halving approach that has been proposed by Dash, Rengaswamy, and Venkatasubramanian (2001). This approach parameterizes the data as a sequence of primitives with the goodness of fit determined with respect to noise. The interval-halving approach is a recursive method, where, initially a single primitive is sought to characterize the entire data record, failing which, the interval is halved and the process is repeated on the halved length scale till success is achieved. The procedure is recursively applied until the entire data record is covered. The fuzzy trend inferencing approach presented in this paper relies on and uses such an automatic trend extraction from real-time noisy process data. In this paper, we do not provide a detailed description of both these aspects (trend language and a corresponding extraction mechanism) as they have been discussed extensively elsewhere (Rengaswamy & Venkatasubramanian, 1995; Dash, Rengaswamy, & Venkatasubramanian, 2001).

The main motivation for using fuzzy inferencing to perform fault diagnosis is to handle the impreciseness in trend representations. As we will discuss later, the same underlying events could lead to slightly different trends in sensors due to a variety of reasons such as different magnitudes of the underlying events, noise characteristics, other unknown phenomena and so on. One of the ways to handle this problem is to generate a database with a library of slightly varying signatures for each individual underlying event. Such an approach would suffer from both explosion in the size of the knowledge-base and also difficulty in subsequent maintenance. A natural way of handling this uncertainty is through the use of fuzzy reasoning which is the focus of this paper. Hence, to summarize, we will use the interval-halving strategy for trend extraction and show the utility of fuzzy logic to exploit the trend information for inferencing. We will demonstrate the utility of this approach through the use of a CSTR case study. It should be

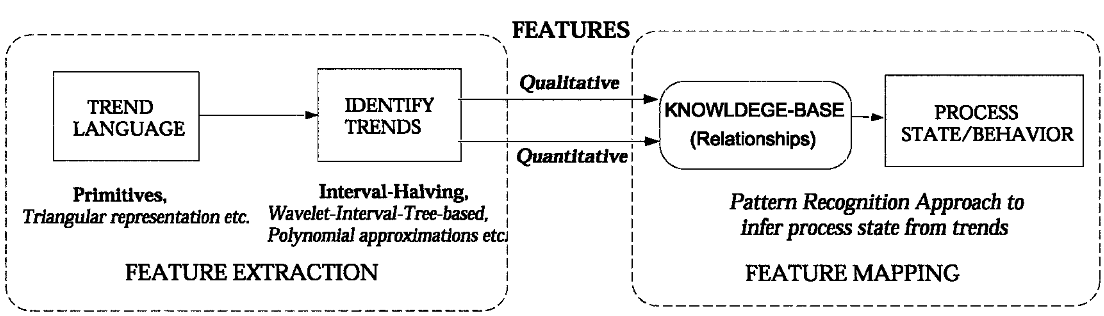

Fig. 1. Feature extraction and mapping.

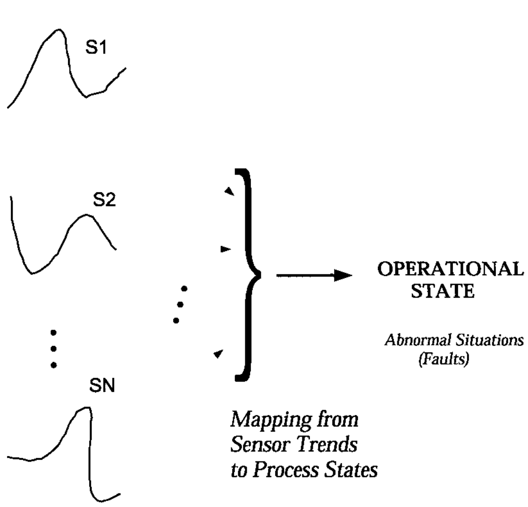

Fig. 2. Multivariate trend mapping from sensors to process states.

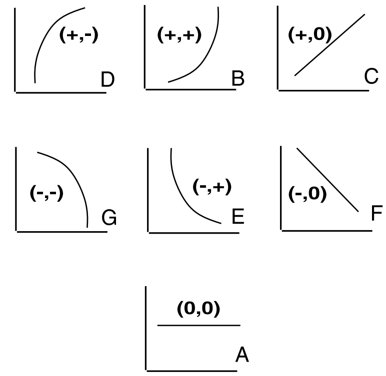

Fig. 3. Fundamental language: primitives.

noted that the application to fault diagnosis considered here is not limiting, and that the whole methodology is developed in a generic manner.

The structure of the rest of the paper is as follows. In Section 2 we discuss some of the important issues in inferencing based on sensor trends and motivate the use of fuzzy logic. Section 3 discusses the proposed approach in detail. The strategy has two stages: in Stage 1 we look for a fuzzy match in terms of a *similarity measure* between the *event signatures* in the knowledge-base and the identified trends, purely from a qualitative standpoint. Then in stage 2 we use quantitative information to get an estimate for the severity of the fault. Both these stages are based on fuzzy inferencing. We illustrate the application of the whole technique through the fault-diagnosis of an exothermic CSTR case study in Section 4. We end with conclusions in Section 5.

2. Issues in mapping

In this section we discuss some of the factors in the trend-to-event mapping process and parallely motivate the development of our approach i.e. the fuzzy trend-matching and reasoning. Some of the important issues to be addressed include:

2.1. Exact vs fuzzy identification/matching

In real-life systems, precision tends to be vague—more so, when dealing with *qualitative features* like trends. An important factor in their consideration is that unlike crisp and definitive measures such as numbers, there is some latitude i.e. *degree of fuzziness* associated with them, both in the *identification* and *matching* stages. Trends present scope of variation even for the same underlying event. It is important to understand that the concept of a *trend* as understood when *visually seen* can be different from what results when a particular *trend-identification* scheme such as the triangular representation (Cheung & Stephanopoulos, 1990) or primitives (Rengaswamy & Venkatasubramanian, 1995) is employed. Depending on the particular representation and extraction method, the final qualitative description can vary in its feature-capture and detail for the same data due to the technique i.e. trend language/extraction method as also external factors such as fault evolution, noise characteristics and other unknown phenomena. In most cases it is practical to keep the fundamental elements minimal i.e., succinct and allow complex shapes to be described in terms of these (Fu, 1982). However, in view of the challenges (Rengaswamy & Venkatasubramanian, 1995; Dash, Rengaswamy, & Venkatasubramanian, 2001) that extraction techniques face, the possibility of generating slightly different final qualitative representation is real regardless of the specific technique employed.

Given these facts, it is neither practical nor desirable to develop *exact* identification and matching algorithms from the point of view of robustness in real-life applications. The inherent *fuzzy nature* of trends might lead to slight variations in their representation and as such instead of trying to fine-tune the performance, it is in general simpler to acknowledge this and develop techniques to accommodate it. In fact, it is this qualitative nature of these features which lends it robustness compared to numerical methods and makes it appealing from an understanding and ease-of-use viewpoint. The transformation of a data record of *numbers* into *qualitative shapes* i.e. trends is in itself an abstraction aiming to retain *only* important features i.e., there is encapsulation/condensation of knowledge. This process may lead to some loss of precision, (sometimes the precision is unnecessary) but is justified considering the trade-off in terms of the succinctness gained, speed and transparency of reasoning and other benefits.

Even while dealing with numerical values in deriving conclusions in real-life situations, we almost always relax requirements of exactness close to the boundaries. Kramer (1987) presents an approach to chemical fault diagnosis that utilizes patterns of violation and satisfaction of the quantitative constraints governing the process in which the equality constraints are relaxed to approximately equal. Interpretation of the pattern of constraint violations is treated by boolean and non-boolean techniques and it is shown that non-boolean reasoning techniques increase stability and robustness of the diagnosis in the presence of noise.

2.2. Multivariate inferencing

The mapping from sensor trends to process states involves reasoning and inferencing based on matching. Clearly, we would like this evidence gathering procedure to be as comprehensive and exhaustive as possible for added confidence. Instead of using only single-sensor evidences, it would be better to collect information from multiple-sensors during the process. The multivariate inferencing refers to this issue and is shown in Fig. 2. Konstantinov and Yoshida (1992) present *if–then* rules which are simple and univariate i.e., only single sensors are used in the process. The decision tree approach of Bakshi and Stephanopoulos (1994) has a multivariate feel to it, because the tree branches on different variables.

2.3. Learning capability

Given that the technique is data-driven, it is essential that the scope of the method develops as more information becomes available. This is the issue of incremental learning. The learning process ideally should be automatic and easy. Minimal human intervention is desired.

2.4. Transparency

One of the biggest advantages of trend-based reasoning is that it is easily understood and can be well related to by operators and engineers alike. While we want it to become sophisticated when required, retaining its transparent nature should also be given importance.

All these issues need to be adequately addressed in any effective technique. Here we present a fuzzy-trend based approach which attempts to satisfy these criteria.

3. Proposed fuzzy logic-based reasoning strategy

In this section we briefly discuss fuzzy-logic, review some of the fuzzy-logic based trend applications and describe the relevance of fuzzy-logic to our trend-matching strategy by showing how to incorporate the same.

Fuzzy-logic: Zadeh in 1965 introduced the concept of a fuzzy set that is a class with unsharp boundaries, providing a basis for a qualitative approach to the analysis of complex systems in which *linguistic* rather than *numerical variables* are employed to describe system behavior and performance. A much better understanding of how to deal with uncertainty may be achieved and better models of human reasoning may then be constructed. Fuzzy logic (Zadeh, 1988) was proposed to explain classes of objects having imprecise boundaries where a set of inputs is mapped to a set of outputs using *if–then* rules. One of the attractive features of fuzzy logic is its ability to convert numeric data into linguistic variables using membership functions that define how well a variable belongs to the output i.e. degree between 0 and 1. A fuzzy model (Tsoukalas & Uhrig, 1997) maps inputs to outputs using a combination of rules, membership functions (μ) and logical operators (AND, OR etc.). A system created first by Mamdani and Assilian (1975) evaluates the inputs with membership functions, uses the rules and the logical operators to combine them and generate an output for each rule. These outputs can then be *aggregated* by unifying the outputs of each rule to generate a unified output. A crisp output can then be created as a result of aggregation using a number of defuzzification algorithms.

3.1. Fuzzy logic in trend signatures, similarity indices and mapping

Fuzzy logic has found applications in many trend-based reasoning techniques. To see how fuzzy logic fits into the framework of trend representation using primitives (Rengaswamy & Venkatasubramanian, 1995) as the language, consider Fig. 4. The overlap i.e. unsharp boundaries between the elements P which eventually might result in fuzziness in strings of P i.e., trend signatures is shown in Fig. 4. Thus trends naturally lend themselves to fuzzy treatment. At the mapping stage, the trends being compared can differ in (i) the *qualitative shape* i.e., the sequence of primitives

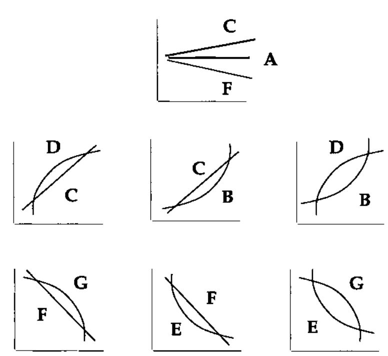

Fig. 4. Fuzziness across the primitives (Table 1).

P_i, (ii) the duration of P_i and (iii) the magnitude change accompanying P_i. Fig. 5 shows two visually close trends which differ in their P_i and their duration. There is a need to make the technique cognizant of this very real possibility so that such a scenario can be accommodated and hence does not affect the performance.

We will be utilizing the trend extraction procedure based on interval-halving (Dash, Rengaswamy, & Venkatasubramanian, 2001) in this work. For each sensor record $[y_1, y_2, \ldots, y_n]$, at the end of the application of the technique, we obtain M unimodal regions U_i over $[t_i, t_{i+1}]$ each (i) described qualitatively by a *primitive* P_i and (ii) parameterized quantitatively by a quadratic

$$q_i = \sum_{k=0}^{k=2} \beta^k t^k.$$

Thus the data is transformed into:

$$\text{Qualitative description: trend } \mathrm{Tr} = \{P_1,\ P_2,\ \ldots,\ P_M\}$$

$$\text{Quantitative description} = \{q_1,\ q_2,\ \ldots,\ q_M\} \qquad (1)$$

describing it in its entirety. We will make use of both these descriptions while concentrating more on the qualitative abstraction. The mapping of trends to states is achieved in the form of *if–then* rules relating the sensor trends to process state. This is usually a *many-to-one* mapping with many sensor trends being mapped to one fault. For example a rule might read,

If $\boxed{\text{sensor S1 shows trend Tr1 AND sensor S2 shows trend Tr2}\ldots}$
then $\boxed{\text{event is F1}}$

The need here is to develop methods to take care of imprecision in Tr, as discussed above. In this case the trends Tr are the qualitative descriptions extracted from sensor data. Exact matching procedures would necessitate a large number of rules, since we are not flexible in

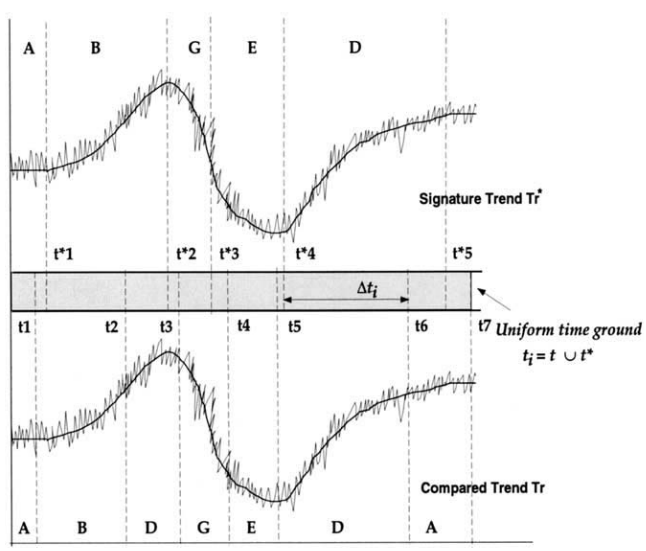

Fig. 5. Uniform time interval t_u for SI^1 and SI^2 calculation.

the comparison procedure. For example, Tr = BGE, CF, CGF are all similar to some extent and hence a strict comparison process will fail to recognize this fact. A restrictive comparison procedure would require the presence of *all* such possibilities in the *antecedent* of the rules so that at least one of them matches an observed trend, which clearly is not practical. Also, an assumption in the trend-analysis scheme is that the sensors show *qualitatively similar* trends for the same fault i.e., the severity of faults does not distort the trends to a great extent. This allows us to reason based on patterns, instead of attaching too much significance to the numerical values. Of course, this assumption is not restrictive, since we could always create new categories of faults if the trends observed are very different at different severities. Hence we relax the comparison procedure to define a *similarity* of match between two trends. This would make the knowledge-base size manageable and more insightful. The next section develops some relaxed criteria to judge the proximity of two trends.

3.1.1. Similarity indices for trend matching

A variety of similarity measures may be defined depending on the particular application. The set of primitives (A–G) are related to each other to some extent (Fig. 4). As outlined earlier, one of the ways in which identified trends could differ was in the symbol P_i i.e., primitive thus contributing to fuzziness. For example, B and C are more similar than are B and E. To quantify this similarity between individual primitives in a fuzzy manner we define a primitive similarity matrix shown in Table 1 reflecting this degree of match [0–1]. Each entry in Table 1, $s_{P_1P_2}$, gives a similarity between P_1 and P_2. Note that some primitives such as D and G are treated to be completely different as their fundamental behavior is opposite.

The similarity can also be thought of as a syntactic (structural pattern-recognition (Fu, 1982)) approach. Similar to the template-matching method in decision-theoretic approach, a similarity or a *distance measure* can be defined between a trend representing an unknown pattern and a string describing a prototype pattern. Recognition of the unknown pattern can be

Table 1
Primitive similarity matrix $s_{P_1P_2}$ (Fig. 4)

	A	B	C	D	E	F	G
A	1	0	0.25	0	0	0.25	0
B	0	1	0.75	0.5	0	0	0
C	0.25	0.75	1	0.75	0	0	0
D	0	0.5	0.75	1	0	0	0
E	0	0	0	0	1	0.75	0.5
F	0.25	0	0	0	0.75	1	0.75
G	0	0	0	0	0.5	0.75	1

carried out on the basis of the maximum-similarity or minimum-distance criterion. A distance between two strings x, y can be defined in terms of the minimum number of *error-transformations* used to derive one from the other. The error transformations are usually defined in terms of substitution (T_S), deletion (T_D) and insertion (T_I) errors. These errors are treated as syntax errors by defining transformations. A weighted Levenshtein distance can also be used by defining non-negative numbers σ, γ and δ to transformations T_S, T_D and T_I respectively. If x, y are two strings then the weighted Levenshtein distance is defined as

$$d(x,\ y) = \min_j \{\sigma k_j + \gamma m_j + \delta n_j\} \tag{2}$$

where k_j, m_j and n_j are the number of substitution, deletion and insertion transformations respectively in j. Various other algorithms for string matching can be found in Cormen, Leiserson, and Rivest (1990).

Here we define two similarity indices, which are intuitive and simple. Consider a trend in the knowledge-base $\mathrm{Tr}^* = \{P_1^*, P_2^*, \ldots, P_S^*\}$ with each P_i^* occurring over $[t_i^*,\ t_{i+1}^*]$ and a trend to be compared $\mathrm{Tr} = \{P_1, P_2, \ldots, P_M\}$ with each P_i over $[t_i,\ t_{i+1}]$. In the general case, both are of different length i.e., $M \neq S$. We define an uniform time ground as the time interval obtained by the union of both the time stamps $t_u = \cup[t,\ t^*]$. Let the number of intervals in t_u be R and its length be T_u. This is shown in Fig. 5 for the two trends. Our similarity indices are then defined on this t_u:

(1) SI^1: A simple similarity measure just based on the *sequence* can be defined as

$$\mathrm{SI}^1 = \frac{\sum_{i=1}^{i=R} s_{P_i P_i^*}}{R} \tag{3}$$

This incorporates the similarity of individual patterns, as given in Table 1. Only the order of the primitives is considered here.

(2) SI^2: The definition of this similarity measure takes into account the *interval-length* $\Delta t_i = t_{u_{i+1}} - t_{u_i}$ in addition to the *sequence*. Thus greater weightage is given to longer intervals compared to the shorter ones.

$$\mathrm{SI}^2 = \sum_{i=1}^{i=R} s_{P_i P_i^*} \frac{\Delta t_i}{T_u} \tag{4}$$

This is more restrictive and calls for a stricter matching.

We intend to maximize the similarity of the two trends Tr, Tr* under inspection i.e. desire to evaluate the trends *as closely as possible*. This means we would like to maximize their similarity and so the final similarity measure we use is the maximum of those defined above. Thus

$$\mathrm{SI} = \max[\mathrm{SI}^1,\ \mathrm{SI}^2] \tag{5}$$

Once the similarity measure is defined as above, we are in a position to describe the proposed strategy for fault diagnosis in the next section.

3.2. A two-staged strategy for fault diagnosis

We present here a two-stage strategy to detect and diagnose faults. In Stage 1, the likely fault candidates are determined (shortlisted) based *purely* on the qualitative shape responses from the measured variables. As will be shown a multivariate strategy using multiple-sensor evidences is utilized. In stage 2, we estimate the severity of the shortlisted faults using quantitative information from the change in steady state magnitudes. This is shown in Fig. 6. Thus we utilize both qualitative (P_i) and quantitative (q_i) information (Section 3.1). The advantages of utilizing both these kinds of information was realized by Yu and Lee (1991) who proposed a framework integrating quantitative process knowledge (steady-state gains) described in terms of membership functions into a qualitative model (signed directed graph). The truth value of a hypothesis i.e. a fault origin was calculated based on fuzzy logic and the diagnostic resolution was shown to improve significantly. We describe both the stages of our strategy in detail next.

3.2.1. Stage I—qualitative information

In Stage 1 we *identify* the possible fault candidates using only the qualitative shapes of sensor trends exhibited by the measurement sensors. The trend-extraction is based on the interval-halving technique presented in Dash, Rengaswamy, and Venkatasubramanian (2001). The basic idea there was to recursively keep *halving* the signal length until a quadratic could fit well within the error bounds represented by noise. The noise

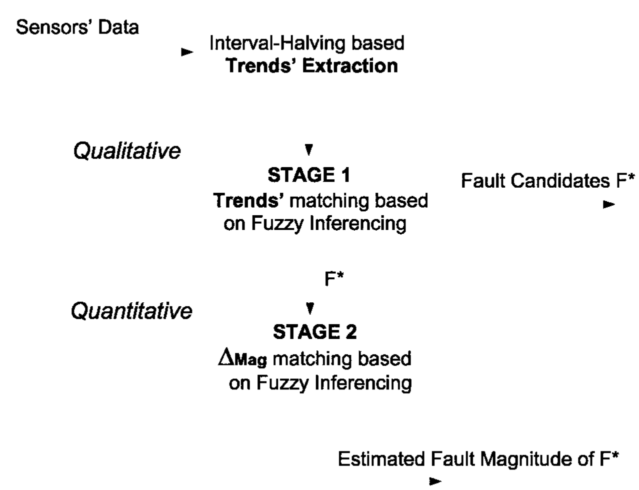

Fig. 6. Qualitative–quantitative two-stage approach.

estimation was done through a wavelet analysis. Details of the procedure along with extensive examples can be found in Dash, Rengaswamy, and Venkatasubramanian (2001).

Rule-based knowledge-base: To identify the faults in this stage we make use of a knowledge-base (Fig. 7) mapping the fault signatures to the faults in the form of *if–then* rules (Section 3.1). The fault signatures are patterns that are exhibited by the sensors in response to a fault, obtained either from dynamic simulations or historical databases. An example of the ith rule (ith fault) is shown below

$$\text{If } \boxed{\text{sensor S1} \rightarrow \text{trend } \mathrm{Tr}^*_{1i}} \text{ AND } \text{sensor S2} \rightarrow \text{trend } \mathrm{Tr}^*_{2i} \text{ AND } \ldots \text{then } \boxed{\text{Fault is } \mathrm{F_i}} \tag{6}$$

where $i = 1, \ldots, M$, i.e., there are M such rules relating sensor trends to the M fault scenarios. Tr^*_{1i} refers to the fault signature exhibited by Sensor S1 for fault F_i. Note the multivariate nature of the inferencing due to the conjunction operator AND used here.

Under the assumption that the sensor signatures are broadly the same under different fault severities, we require to store only one representative signature for each sensor for each fault i.e., the antecedents in the knowledge-base in this stage consist of *only one* fault signature representative. Thus, if faults are simulated at severity levels of low, medium and high, the medium level fault signatures should suffice. As pointed out earlier, if this is not the case (found by evaluating the effectiveness) it is always possible to augment the knowledge-base by creating additional fault classes.

Rules evaluation: To evaluate the rules for an observed trend Tr, fuzzy inferencing is employed. By comparing Tr fuzzily with the signature Tr* in each part j of the *antecedent* (shown boxed in Eq. (6)) in the ith rule, a similarity index SI^j_i is obtained. This is shown in Fig. 8. This is a measure of the degree of match between the observed trend *Tr* and that in the knowledge-base *Tr**. Once all the SI^j_is are evaluated, the overall confidence index CI_i of the ith rule's consequent (i.e. the fault) is calculated. The fuzzy logic interpretation of AND (Tsoukalas & Uhrig, 1997) as min is employed, thus CI_i is given by the minimum over all the antecedent parts j

$$\mathrm{CI}_i = \min_j[\mathrm{SI}^j_i] \tag{7}$$

This evaluation is physically intuitive. The truth value of a hypothesis takes a value between 0 and 1, and the strength of a rule is considered equivalent to the weakest link (Yu & Lee, 1991) i.e., the part of the antecedent with the smallest SI^j_i value. Once all the CI_i are obtained, the faults are ranked in decreasing order of their confidences. This process is shown in Fig. 7. Here

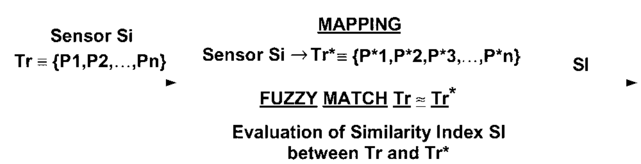

Fig. 8. Mapping and fuzzy matching.

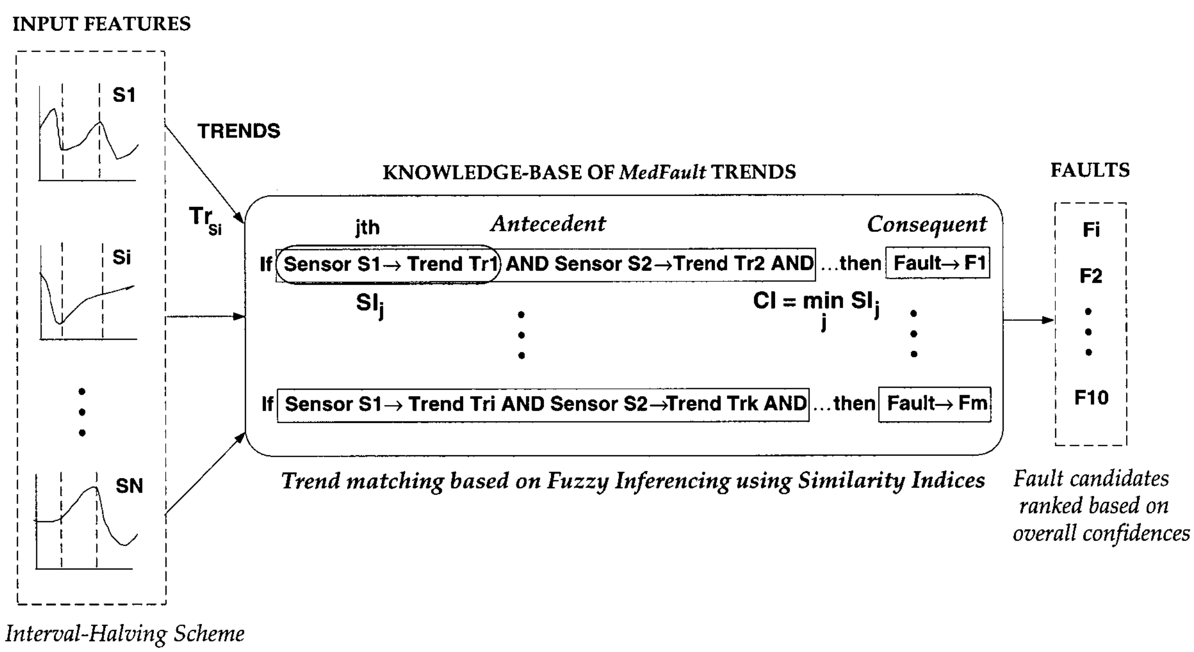

Fig. 7. *Stage 1*: qualitative fuzzy reasoning using trend-matching.

we try to exploit only the qualitative shape information and no magnitude information is used. One important issue here is that of *fault resolution*. If all the faults can be qualitatively resolved i.e., distinguishable based only on the measured sensor signatures (CI_i are well separated), then this would result in a high degree of accuracy in pinpointing the actual fault. However if this is not the case i.e., the CI_i are close, it is necessary to add more discriminating sensors to resolve the conflict or look for finer qualitative distinctions for better discrimination. Of these two options sensor selection based on fault diagnostic observability criteria using qualitative models has been researched upon (Raghuraj, Bhushan, & Rengaswamy, 1999). The second option is possible in some cases (not always) and one needs to develop tools that extract features for finer discrimination automatically. Once the most likely fault candidate F^* is identified, it is passed to the next stage to evaluate the severity (fault magnitude).

3.2.2. Stage II—quantitative information

In Stage 1 no quantitative information was used. In this stage we estimate the magnitude of the detected fault F^* using fuzzy logic. This is shown in Fig. 9. The knowledge-base in this stage consists of steady-state magnitude changes $\Delta M^{S_i}{}_{\text{level}}$ in sensor S_i accompanying faults at different levels (low, medium, high). This information can be obtained from simulations. To compare the observed magnitude change in sensor S_i, ΔM^{S_i} with that in the knowledge-base $\Delta M^{S_i}{}_{\text{level}}$, we adopt a fuzzy-approach instead of strict matching to lend robustness to the technique. Define the extent of match at a level as

$$\alpha^{S_i}_{\text{level}} = \frac{\Delta M^{S_i}}{\Delta M^{S_i}_{\text{level}}} \tag{8}$$

with membership function μ_{level}. Along the same lines as trend comparison, we also do not desire an exact magnitude match and hence use a membership function defined on a normalized magnitude scale $\alpha = [0, 2]$ making it level-independent. This is shown in Fig. 10. To be able to do so we define the fault magnitude of F^* as a fuzzy variable with fuzzy values Low$F^{\pm}$, Medium$F^{\pm}$, High$F^{\pm}$ each having their own membership functions as shown in Fig. 10. The $\pm$ signs indicate both the directions of deviation. All the membership functions used are Gaussian. The knowledge-base (Fig. 9) consists of rules such as

$$\text{If } \boxed{\text{sensor } \alpha^{S_1}_{\text{level}} \text{ is } \alpha_{\text{level}}} \text{ AND } \alpha^{S_2}_{\text{level}} \text{ is } \alpha_{\text{level}} \text{ AND } \ldots \text{then fault is level} F \tag{9}$$

Here we make use of the quantitative information in the trend i.e., magnitude change ΔM as a result of the fault. To evaluate the magnitude we again make use of standard fuzzy reasoning algorithms (Tsoukalas & Uhrig, 1997) comprising of fuzzification, inferencing and defuzzification steps (Section 3.1). We use the Mamdani–Min implication and centre of area defuzzification method to estimate the fault as shown in Fig. 9. In the next section, we illustrate the whole methodology in the fault diagnosis of a reactor. Going back to the issues discussed in Section 2, we can easily see that the proposed strategy satisfies all the criteria. It takes care of the fuzzy aspect in trends allowing inexact matching. Moreover given the 'AND' based inferencing used in the rules, it is naturally multivariate. The rules are linguistic, and thus transparent and insightful. Moreover, adding rules to increment or expand the scope of the knowledge-base as processes develop or new events are found is relatively easy. The knowledge-bases, given their human-like-reasoning nature, are thus easily related to and understood, addressing the transparency issue. This simplicity however does not limit its effectiveness.

4. Illustration

In this section, we show the application of the above mentioned strategy on an exothermic reactor case study.

4.1. CSTR case study

The exothermic CSTR system given by Luyben (1990) is simulated to obtain the manipulated and controlled variable data to be used by the algorithm. The schematic of the CSTR system is shown in Fig. 11. The process involves a liquid phase reaction $A_{(l)} \rightarrow B_{(l)}$ This reaction is highly exothermic and occurs in the reactor. The temperature controller controls the temperature of the reactor by manipulating the flow rate of the coolant flowing through the jacket. The level in the reactor is controlled by the level controller by manipulating the outlet flowrate from the reactor. Both the reactor and the jacket are modeled with perfectly mixed-tank dynamics. The reactant volume V and concentration C_A at any time is given by,

$$\frac{dV}{dt} = F_0 - F$$

$$r_A = C_A k_0 e^{-\frac{E}{RT}}$$

$$\frac{dC_A}{dt} = \frac{F_0}{V}(C_{A_0} - C_A) - r_A \tag{10}$$

Assuming constant heat capacities and densities, an overall heat balance on the reactor gives the reactant temperature as,

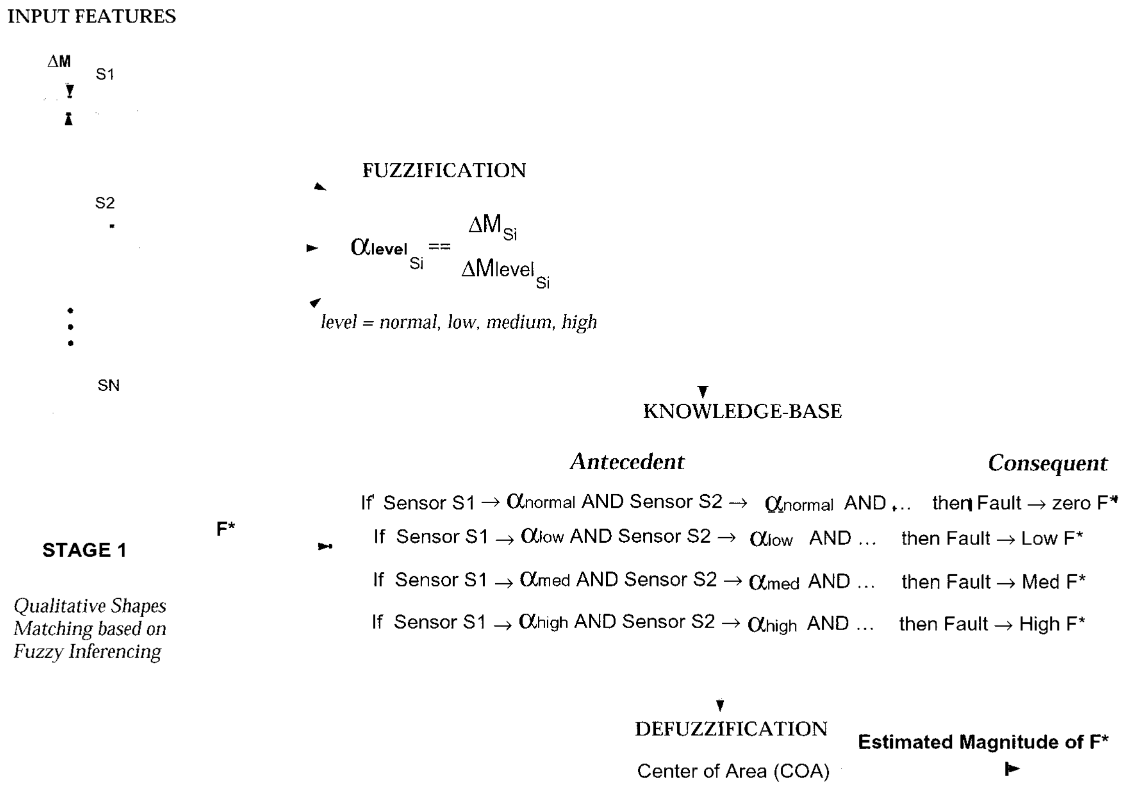

Fig. 9. *Stage 2:* quantitative fuzzy reasoning using ΔM extent matches.

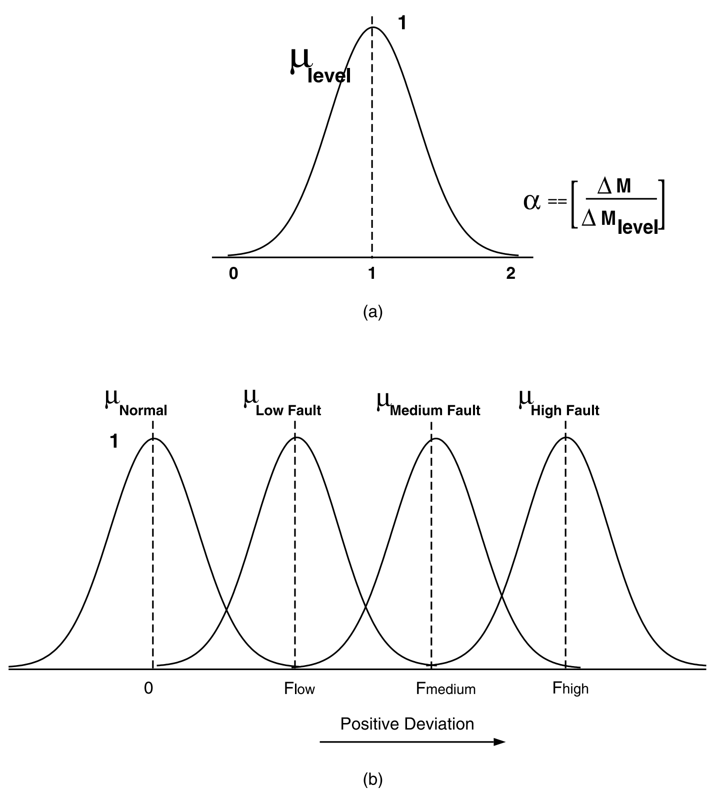

Fig. 10. Membership functions.

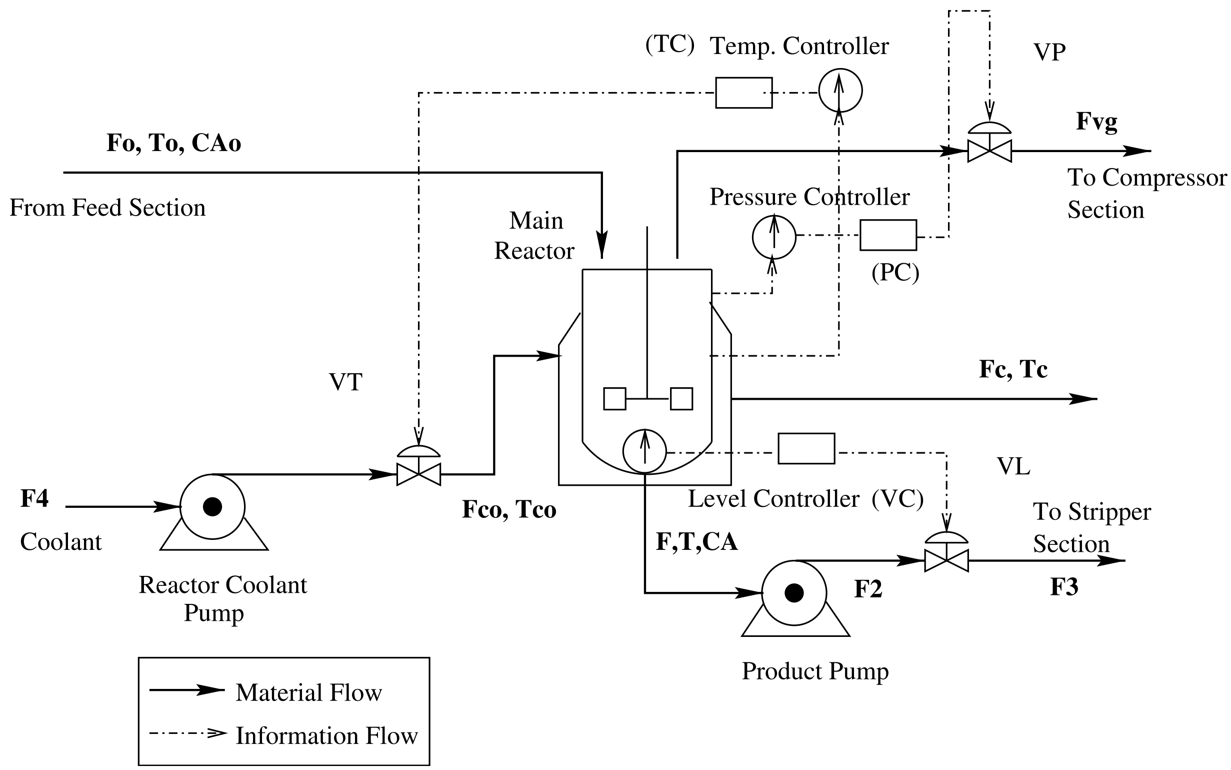

Fig. 11. Schematic of the CSTR process.

Table 2
Values for the CSTR of Fig. 11

Notation	Variable	(Steady state/constant) value
V	Volume of reactor	48 ft^3
C_A	Reactant concentration in reactor	0.2345 lb mol A/ft^3
T	Reactor temperature	600°R
F_0	Inlet feed flow rate	40 ft^3/h
C_{A0}	Inlet reactant concentration	0.50 lb.mol A/ft^3
T_c	Jacket temp.	590.51°R
F_c	Coolant flow rate	56.626 ft^3/h
T_0	Inlet feed temp.	530°R
V_j	Volume of jacket	3.85 ft^3
k_0	Frequency factor	7.08×10^{10} /h
E	Activation energy	30 000 Btu/lb mol
R	Universal gas constant	1.99 Btu/lb mol °R
U	Heat transfer coefficient	150 Btu/h ft^2 °R
A	Heat transfer area	150 ft^2
T_{c0}	Inlet coolant temperature	530°R
ΔH	Heat of reaction	−30 000 Btu/lb mol
C_p	Heat capacity (process side)	0.72 Btu/lbm °R
C_j	Heat capacity (coolant side)	1.0 Btu/lbm °R
ρ	Density of process mixture	50 lb m/ft^3
ρ_j	Density of coolant	62.3 lb m/ft^3
K_T	Proportional constant of temperature controller	4.3
K_H	Proportional constant of level controller	10

$$\frac{dT}{dt} = \frac{F_0}{V}(T_0 - T) + \frac{r_A(-\Delta H)}{\rho C_p} - \frac{UA(T - T_c)}{V\rho C_p}$$

Table 3
Measurement and fault variables for CSTR

No	Sensors
Measurement sensors	
1	Volume of reactor V
2	Outlet concentration C_a
3	Outlet temperature T
4	Outlet coolant temperature T_c
Fault variables	
1	Inlet flow rate F_o
2	Inlet Concentration C_{A0}
3	Inlet temperature T_0
4	Inlet coolant temperature T_{c0}

Overall heat balance on the jacket gives the coolant temperature,

$$\frac{dT_c}{dt} = \frac{F_c}{V_j}(T_{c0} - T_j) + \frac{UA(T - T_c)}{V_j \rho_j C_j}$$

Propotional controllers are used to control the temperature T by manipulating the outlet coolant flow rate F_j and the volume V by manipulating the outlet reactant flow F of the reactor.

$$F_j = F_{js} - K_T(T_{set} - T)$$

$$F = F_s - K_H(V_{set} - V)$$

Table 4
Simulated step fault scenarios

F_0		C_{A0}		T_0		T_{c0}	
Level	Fault	Level	Fault	Level	Fault	Level	Fault
+5	LowF_0^+	+0.05	LowC_{A0}^+	+20	LowT_0^+	+20	LowT_{c0}^+
+10	MedF_0^+	+0.10	MedC_{A0}^+	+40	MedT_0^+	+40	MedT_{c0}^+
+15	HighF_0^+	+0.15	HighC_{A0}^+	+60	HighT_0^+	+60	HighT_{c0}^+
−5	LowF_0^-	−0.05	LowC_{A0}^-	−20	LowT_0^-	−20	LowT_{c0}^-
−10	MedF_0^-	−0.10	MedC_{A0}^-	−40	MedT_0^-	−40	MedT_{c0}^-
−15	HighF_0^-	−0.15	HighC_{A0}^-	−60	HighT_0^-	−60	HighT_{c0}^-

The values of the constants and parameters used in the simulation and the notation are tabulated in Table 2.

4.2. Knowledge-base and trend fuzziness

To evaluate the proposed strategy fault scenarios are simulated for knowledge-base construction and testing. The measurement sensors and the fault variables for the case study are shown in Table 3. The normal behavior of the process is defined by the steady state, represented by the primitive ‘A’.

Knowledge-base: To construct the knowledge-base we carry out fault simulations in positive and negative deviations for all the 4 input fault variables at a noise level of 5%. Three levels of *step* faults (*low*, *medium* and *high*) are simulated in each direction for all the fault variables (totalling $4 \times 3 \times 2 = 24$ scenarios) as shown in Table 4. The level of the fault is treated as a fuzzy variable with fuzzy values low, medium and high. For Stage 1, which involves only the qualitative shapes of sensor responses, we only use the shapes from the *medium level* faults in the knowledge base. This is to enforce and evaluate the assumption that the qualitative shapes are same at different levels of the fault. We thus retain only one representative (medium) of the fault scenario. We desire to do with minimal information at this stage. This amounts to the following 8 scenarios: $F_0^{\pm}$, $C_{A0}^{\pm}$, $T_0^{\pm}$ and $T_{c0}^{\pm}$. The fault signatures corresponding to these scenarios form the Stage 1 KB as shown in Table 5. This knowledge-base is used in fuzzy-inferencing using trend shapes as shown in Fig. 7. The knowledge-base for Stage 2 on the other hand uses magnitude information and consists of all the quantitative information for *all* levels of faults i.e., all 24 scenarios (Section 3.2). The inferencing procedure for Stage 2 is shown in Fig. 9.

Trend identification and fuzziness: The trends from the measurement sensors are extracted using the interval-halving technique described in Dash, Rengaswamy, and Venkatasubramanian (2001). To illustrate the issue of variability in trend extraction as discussed in Sections 2 and 3.1, we show a sample of the identified trends at different fault and noise levels. Fig. 12(a and b) shows the sensor shapes for the F_0^+ fault at levels +5, +15 respectively. Although visually the trends look similar, one can notice slight variations in the extracted trends as shown in Table 6. Similar behavior can also be seen due to the effect of noise as shown in Table 6 where the

Table 5
Stage 1 Knowledge base: qualitative shapes of *Med* faults only

No	Fault	Variable	Fault signatures
1	MedF_0^+	V	[D]
		C_A	[C D E]
		T	[E C D E]
		T_c	[G E C D E]
2	MedF_0^-	V	[F E]
		C_A	[F E]
		T	[D E]
		T_c	[C D F E]
3	MedC_{A0}^+	V	[A]
		C_A	[B D F D]
		T	[B D E]
		T_c	[B D E]
4	MedC_{A0}^-	V	[A]
		C_A	[E]
		T	[D D G E]
		T_c	[G E]
5	MedT_0^+	V	[A]
		C_A	[G E D]
		T	[D D E]
		T_c	[D G E]
6	MedT_0^-	V	[A]
		C_A	[B D]
		T	[E D]
		T_c	[D G E D]
7	MedT_{c0}^+	V	[A]
		C_A	[D G E C D F C F A]
		T	[E B D F E E B D E D]
		T_c	[C C D F E C G]
8	MedT_{c0}^-	V	[A]
		C_A	[B D]
		T	[E D]
		T_c	[F]

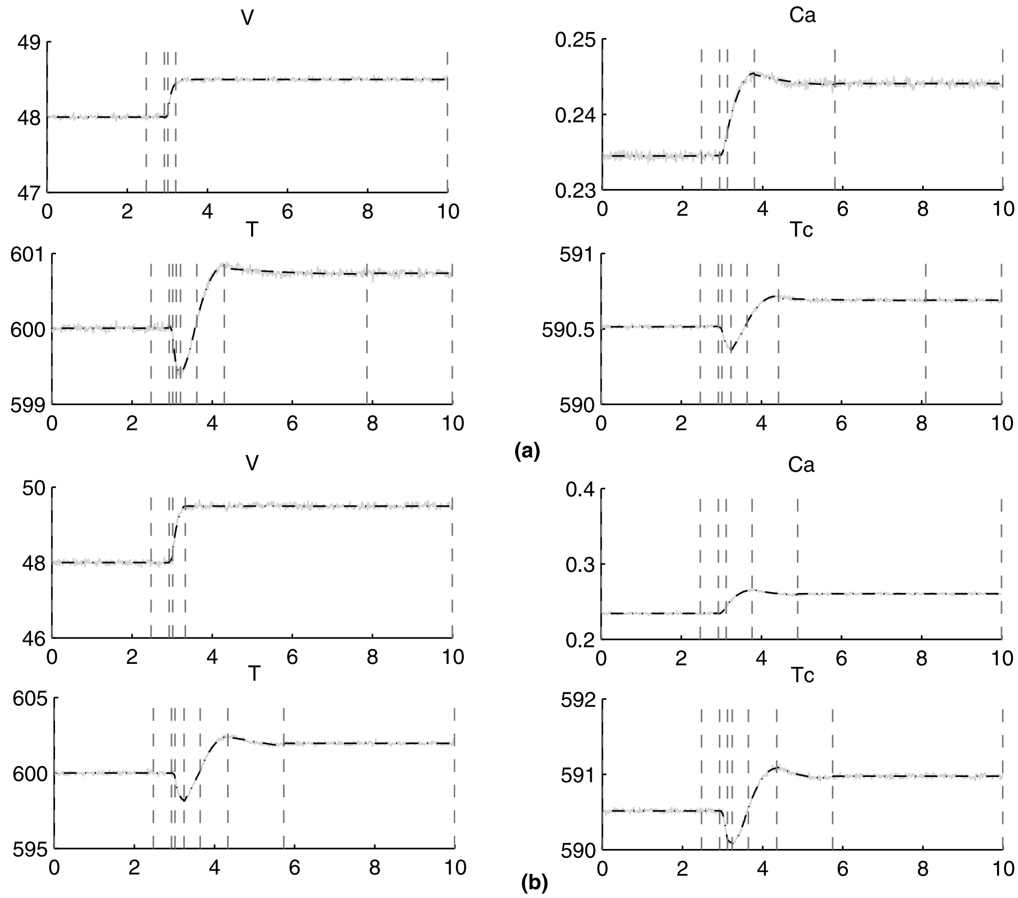

Fig. 12. Effect of fault level: sensor trends at 7.5% noise level for (a) F_0^+ (5) and (b) F_0^+ (15).

identified primitives are given for different noise levels. However the assumption that the trends are broadly the same hold true across all these scenarios allowing qualitative analysis. It should be emphasized that this difference is not a shortcoming in the identification technique or a deficiency in the trend analysis methodology, but an inevitable offshoot of the qualitative nature of the technique which leads to some latitude in preciseness. As was discussed extensively in Section 2, there is a need to accommodate such fuzziness as trends might never be exactly reproducible given the inherent fuzziness.

Table 6
Variation in identified trends due to fault level/noise

Level[a]	Variable	Trends
+5	V	[D(14)]
	C_A	[B(14) D(46) E(135)]
	T	[F(14) E(8) B(28) D(47) E(238)]
	T_c	[E(16) C(28) D(53) E(246)]
+15	V	[D(22)]
	C_A	[B(14) D(44) E(77)]
	T	[E(16) C(28) D(47) F(94)]
	T_c	[G(14) E(9) B(28) D(48) E(94)]
Noise[b]	Variable	Trends
7.5	V	[D(14)]
	C_A	[C(14) D(46) E(103)]
	T	[E(16) C(28) D(46) E(118)]
	T_c	[G(14) E(7) C(28) D(53) E(115)]
10.0	V	[D(14)]
	C_A	[B(14) D(45) E(133)]
	T	[F(14) E(8) B(28) D(53) E(232)]
	T_c	[E(17) D(56) D(25) E(371)]

[a] Fault F_0, Noise = 7.5%.
[b] Fault F_0, Level = 10%.

Similarity index: To show the way trends are proposed to be matched, we give a few examples in Fig. 13. The similarity indices between the fault signatures and the observed trends varies between 0.1 and 0.96 and is seen to follow intuition. The calculation of SI was discussed in Section 3.1. For example, when the trend of T_c for F_0^+(7) fault (Case l) is matched against the fault signature of MedF_0^- (Fig. 13(a)), the movement is fundamentally different (positive vs negative). This is manifested in the low SI of 0.1. On the other hand when comparing with the trend of MedF_0^+ (Fig. 13(d)), the trends are visually very similar and the same fact is shown by the high SI value. Similarly, the remaining two trends—T (SI = 0.96) and C_A (SI = 0.53) for fault C_{A0}^+

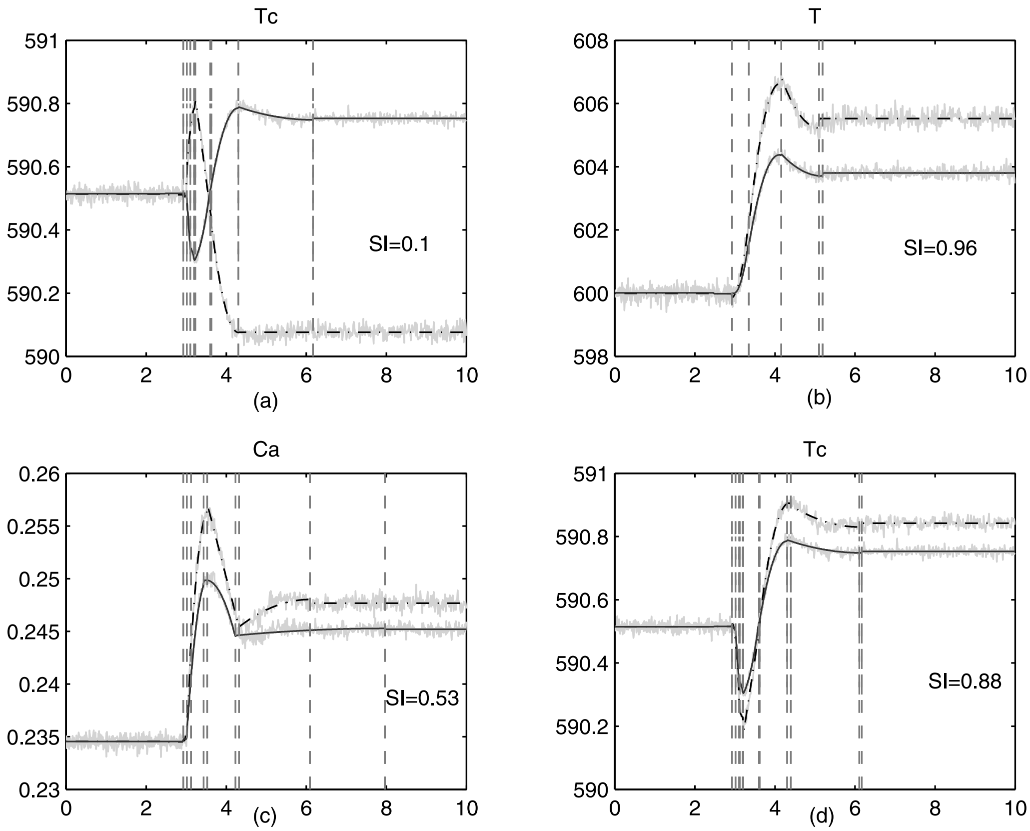

Fig. 13. Examples of SI variation.

(0.07) (Case 3) matched against signature MedC_{A0}^{+} show good matching (Fig. 13(b and c)).

4.3. Testing of fault cases

Test cases: Eight different test fault scenarios are simulated to be tested using the two-stage approach. The method was discussed in great detail in Section 3.2. All the test fault cases (Cases 1–8) are listed in Tables 7 and 8. All these faults are simulated at different levels to evaluate the robustness of the technique.

Stage 1: The extracted sensor trends from the 8 test scenarios are also shown in Tables 7 and 8. Each of the scenarios is evaluated using the fuzzy procedure as described in Section 3.2 and shown in Fig. 7. The top two fault candidates as a result, ranked by the confidence index CI, are shown. For each test scenario, the measurement sensor (S_i) trends along with the similarity index between the fault signatures of the candidate fault (SI_{S_i}) and observed trends evaluated as outlined in Section 3.1 are tabulated. The overall confidence CI is taken to be the minimum (Section 3.1) over all SI_{S_i} for the given fault candidate. For all the cases, it is seen that the correct fault is identified with good resolution. In fact, it can be seen that in all the cases except one, the first and second closest faults are comfortably separated.

One case where the separation is not obvious is one of differentiating between the inlet coolant temperature increase and inlet temperature increase (Case 7, Table 8). One can see that the top two faults show the confidence of 0.49 and 0.40, respectively. The reason for such a behavior becomes clear upon looking at the sensor responses for the T_0 and T_{c0} faults. Fig. 14 shows the response of all the 4 measurement sensors for the MedT_0^{+} and LowT_{c0}^{+} faults. Visually there is not much difference in the sensor responses for both the cases. The sensors respond in remarkably the same manner for these two *different* faults i.e., the fault signatures are close. Since the whole technique is data-driven i.e., no fundamental understanding of the process is incorporated the performance is limited by the quality of the knowledge-base. This as mentioned earlier is the issue of good fault resolution (Section 3.2).

A poor fault resolution in this technique can thus arise when trends exhibited are similar for *all* the measured sensors across different faults thus leaving little or no evidence for distinguishability. To overcome this problem either additional discriminating sensors need to be added or finer features which may help distinguish the faults needs to be emphasized. Another reason for such a performance could be high nonlinearity in the trend manifestation across fault severity levels

Table 7
Stage 1: testing of faults: Cases 1–4

Case	Simulated fault (10% noise)	Sensor S_i	Trends T_{S_i}	Fault 1st	SI_{S_i}	CI = min[SI_{S_i}]	Fault 2nd	SI_{S_i}	CI = min[SI_{S_i}]
1	F_0^+ (+7)	V	[C D]	MedF_0^+	0.76	0.76	MedC_{A0}^+	0.13	0.13
		C_A	[D E]		0.79			0.32	
		T	[F E B D E]		0.91			0.50	
		T_c	[F F B D E]		0.88			0.51	
2	F_0^- (−13)	V	[E]	MedF_0^-	0.54	0.54	None		
		C_A	[F E]		0.83				
		T	[D F E]		0.72				
		T_c	[B D G E]		0.81				
3	C_{A0}^+(+0.07)	V	[A]	MedC_{A0}^+	1.0		MedT_0^+	1.0	
		C_A	[D G D]		0.53	0.53		0.37	
		T	[B D E]		0.96			0.50	
		T_c	[B D E]		0.95			0.36	0.36
4	C_{A0}^-(−0.13)	V	[A]	MedC_{A0}^-	1.0		MedT_0^+	1.0	
		C_A	[E]		1.0			0.38	
		T	[F E]		0.64	0.64		0.47	
		T_c	[D D G E]		0.71			0.24	0.24

which would invalidate the assumption of similar patterns across fault levels. In such a case it would be necessary to store additional fault patterns at those levels i.e., create additional fault classes in Stage 1 knowledge base. The important point though in testing all these cases is that we get excellent results for situations that are different from the ones used for knowledge-base generation in terms of the fault severity levels.

Stage 2: Once the most likely fault is identified in Stage 1, its magnitude estimation is carried out in Stage 2 as described in Section 3.1. For all the Cases 1–8, we carry the top-most identified fault F^* to this stage. The severity estimation method was outlined in Section 3.2 and shown in Fig. 9. The results are shown in Table 9. It is seen that the simulated and the estimated fault magnitudes are very close.

We also tested the robustness of the approach to variations other than noise and fault severity level. These results are summarized in Table 10. In case 9, we have a large C_{A0} fault and a small T_0 fault and in the Case 10, we have a small C_{A0} fault and a large T_0 fault. In both the cases, we also let the second smaller fault manifest itself 2 h after the original larger fault. It can be seen from Table 10 that the approach is still quite robust to such variations with the approach identifying clearly the correct fault. Hence for secondary faults that have very small magnitude, the fuzzy measure is robust and addition of new rules is not necessary highlighting the utility of the proposed method. However, when there are multiple faults such as simultaneous occurrence of two faults with large magnitudes (Case 11), then the approach might not provide the correct diagnosis and this is to be expected of any data based approach.

Table 8
Stage 1: testing of faults: Cases 5–8

Case	Simulated fault (10% noise)	Sensor S_i	Trends T_{S_i}	Fault 1st	SI_{S_i}	CI = min[SI_{S_i}]	Fault 2nd	SI_{S_i}	CI = min[SI_{S_i}]
5	T_0^+ (+30)	V	[A]	MedT_0^+	1.0		MedC_{A0}^+	1.0	
		C_A	[G E D]		0.98			0.59	
		T	[B D E]		0.72			0.58	
		T_c	[E C D F]		0.62	0.62		0.43	0.43
6	T_0^- (−55)	V	[A]	MedT_0^-	1.0		MedC_{A0}^+	1.0	
		C_A	[E B D]		0.48	0.48		0.31	
		T	[E D]		0.74			0.25	0.25
		T_c	[D F E D]		0.72			0.32	
7	T_{c0}^+ (+30)	V	[A]	MedT_{c0}^+	1.0		MedT_0^+	1.0	
		C_A	[G E D E D]		0.49	0.49		0.47	
		T	[E C D G E B D F]		0.58			0.40	0.40
		T_c	[C D D G E D]		0.52			0.47	
8	T_{c0}^- (−55)	V	[A]	MedT_{c0}^-	1.0		MedT_0^-	1.0	
		C_A	[B D]		0.95			0.68	
		T	[E D]		0.67	0.67		0.50	
		T_c	[F]		1.00			0.15	0.15

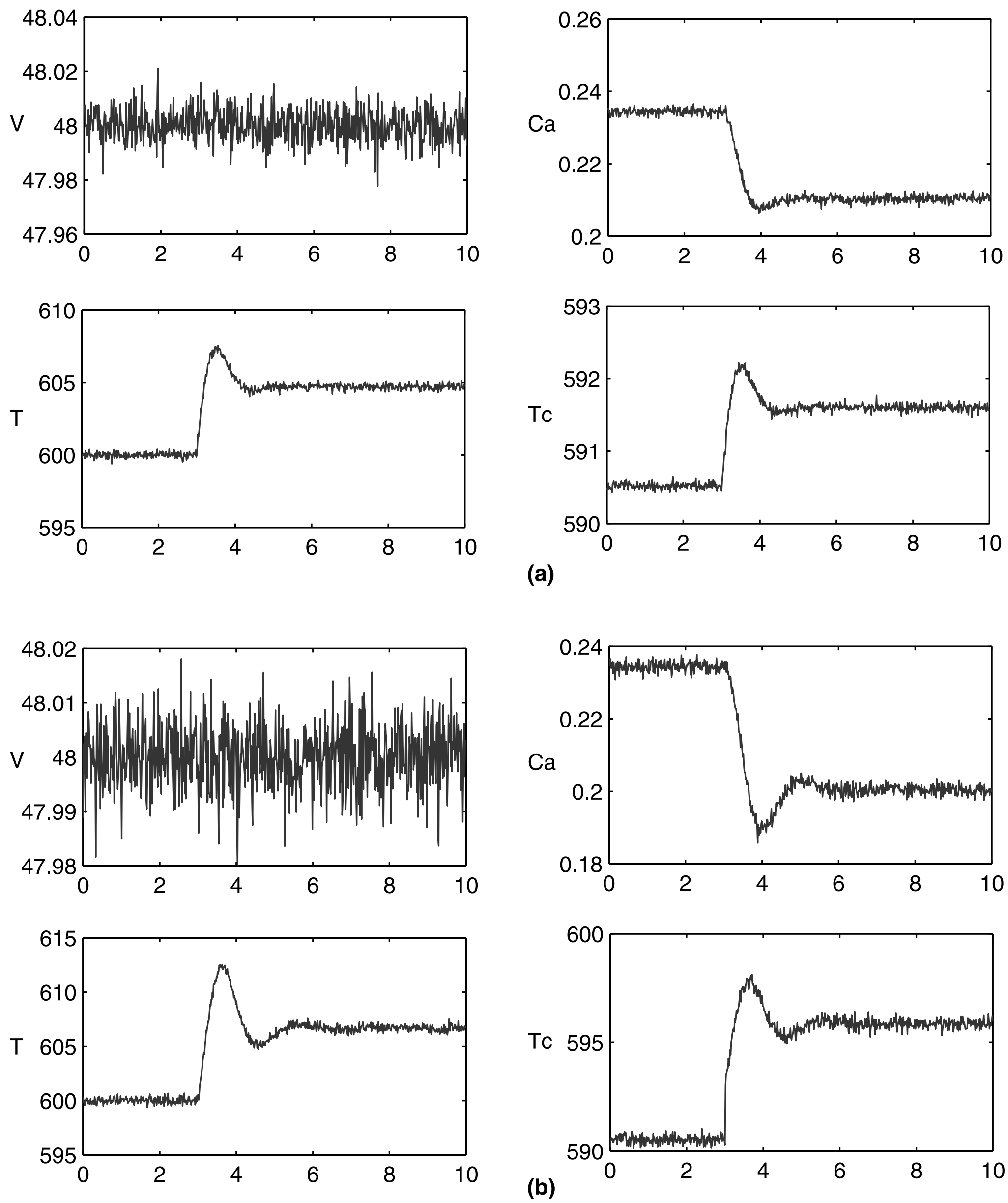

Fig. 14. Similar sensor responses for MedT_0^+ and LowT_{c0}^+.

5. Conclusions

Process trend analysis is increasingly becoming an useful tool to model and reason about process behavior. Two important components of this approach are trend extraction and mapping of trends to process states. In an earlier paper (Dash, Rengaswamy, & Venkatasubramanian, 2001) we presented a robust, efficient and simple approach to extract sensor trends based on an interval-halving technique. In this paper, we addressed the trend-analysis based reasoning strategy i.e., establishing and utilizing relationships between sensor trends and process operations. We listed the main issues involved in this task and discussed at length the issue of fuzziness in the qualitative features represented by trends. We argued

Table 9
Stage 2: testing of faults: level estimation

Case	Fault	Level	Estimated level
1	F_0^+	7	6.77
2	F_0^-	13	10.2
3	C_{A0}^+	0.07	0.0672
4	C_{A0}^-	0.13	0.1052
5	T_0^+	30	29.77
6	T_0^-	55	41.19
7	T_{c0}^+	30	27.02
8	T_{c0}^-	55	41.25

Table 10
Stage 1: testing of faults: Cases 9–11

Case	Simulated fault (10% noise)	Senso S_i	Trends T_{S_i}	Fault 1st	SI_{S_i}	CI = min[SI_{S_i}]	Fault 2nd	SI_{S_i}	CI = min[SI_{S_i}]
9	$C_{AO}^{+}(+0.07)T_{O}^{+}(+3)$	V	[A]	**Med**C_{AO}^{+}	1.0		**Med**T_{o}^{+}	1.0	
		C_A	[E C D D E G]		0.40	0.40		0.41	
		T	[B D E D]		0.70			0.33	0.33
		T_C	[E B D E A D]		0.54			0.39	
10	$T_{O}^{+}(+30)C_{AO}^{+}(+0.001)$	V	[A]	**Med**To^{+}	1.0		**Med**T_{co}^{+}	1.0	
		C_A	[G E D]		0.60	0.60		0.45	0.45
		T	[B D D E]		0.71			0.45	
		T_C	[B D E]		0.72			0.53	
11	$T_{O}^{+}(+30)C_{AO}^{+}(+0.07)$	V	[A]	MedT_{co}^{+}	1.0		**Med**T_{o}^{-}	1.0	
		C_A	[G E C A D F F D]		0.39	0.39		0.49	
		T	[B D E E B D]		0.42			0.39	0.39
		T_C	[B D E E B D]		0.44			0.39	

the use of fuzzy-logic to handle the uncertainty of these temporal features for robust inferencing. Similarity indices to perform fuzzy trend matches were then defined. Our main contribution—the two-stage strategy towards trend-based inferencing was presented in detail. Stage I dealt with only the qualitative aspect of the trends and utilized multivariate inferencing to reason about process state based on the trends. Stage II used the quantitative information in a fuzzy manner (low, medium and high) to estimate magnitude changes. An exothermic reactor case study was used to demonstrate the strategy extensively with various fault scenarios. Although the application to fault diagnosis was illustrated, the method was developed generically and thus can find use in any trend-based-reasoning process. The technique performed well in the fault scenarios considered, estimating their severities correctly. However, as was pointed out in Section 4.3, the method is data-driven and so in the absence of other knowledge the performance is limited by the quality of the knowledge-base. In particular, fault-resolution depended on the fault signatures' ability to distinguish across faults—in case of identical signatures across all sensors, poor fault resolution can be expected. It is necessary in these cases to add discriminating sensors or augment the system with additional knowledge about the process.

Future research is underway at integrating such data-driven techniques with model-based methods to overcome these shortcomings.

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