Homework 2 (EE7600 MIMO Systems for Wireless Communications)

1. Consider the following MIMO channel with a random constant-channel-matrix

$$X_k = \sqrt{\rho} S_k H + W_k, \qquad k = 0, 1, 2, 3, \ldots .$$

We have two cases: the realization of H known for the receiver but unknown for transmitter (*coherent* case) and the realization of H unknown for both transmitter and receiver (*noncoherent* case).

Assume that $M = 2$, $T = 2$, and $N = 1$ and we have the following two unitary signal constellations having four signal points

$$\Omega_1 = \left\{ U_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad U_2 = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}, \quad U_3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad U_4 = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix} \right\}$$

and

$$\Omega_2 = \left\{ V_1 = \begin{pmatrix} 0 & b - ia \\ -b - ia & 0 \end{pmatrix}, \quad V_2 = \begin{pmatrix} 0 & -b - ia \\ b - ia & 0 \end{pmatrix}, \right.$$
$$\left. V_3 = \begin{pmatrix} -ib & ia \\ ia & ib \end{pmatrix}, \quad V_4 = \begin{pmatrix} ib & ia \\ ia & -ib \end{pmatrix} \right\}$$

where $a = \sqrt{1/3}$, $b = \sqrt{2/3}$, $i = \sqrt{-1}$.

(a) Simulate and plot the uncoded block error rates of Ω_1 and Ω_2 for *coherent* modulation in the coherent case for SNR ρ of 0dB–30dB with increment 2dB (or 3dB, or 5dB).

(b) Simulate and plot the uncoded block error rates of Ω_1 and Ω_2 for *differential* modulation in the noncoherent case for SNR ρ of 0dB–30dB with increment 2dB (or 3dB, or 5dB).

(c) Calculate and compare the diversity product ξ and diversity sum η of the two unitary constellations Ω_1 and Ω_2 (Hint: The definitions of ξ and η in the coherent modulation are *the same* as those in the differential modulation).

(d) Draw some conclusions from the obtained results in (a), (b), and (c).

2. Consider the following MIMO channel

$$X_k = \sqrt{\rho} S_k H_k + W_k, \qquad k = 1, 2, 3, \ldots,$$

where H_k is an i.i.d. random-channel-matrix sequence.

We have two cases: the realizations of H_k known for the receiver but unknown for transmitter (*coherent* case) and the realizations of H_k unknown for both transmitter and receiver (*noncoherent* case).

Assume that $M = 2$, $T = 4$, and $N = 1$ and we have the following unitary signal constellations having two signal points

$$\Omega = \left\{ V_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad V_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{pmatrix} \right\}$$

(a) Simulate and plot the uncoded bit error rate of Ω for *noncoherent* modulation in the noncoherent case for SNR ρ of 0dB–30dB with increment 2dB (or 3dB, or 5dB).

(b) Simulate and plot the uncoded bit error rate of Ω for *coherent* modulation in the coherent case for SNR ρ of 0dB–30dB with increment 2dB (or 3dB, or 5dB).

(c) Draw some conclusions from the simulation results in (a) and (b).

(d) Calculate the diversity product ξ and diversity sum η of the unitary constellation Ω for both coherent and noncoherent cases (Hint: The definitions of ξ and η in the coherent modulation are *not* the same as those in the noncoherent modulation).

Note:

M, N denote the numbers of transmit and receive antennas, respectively. T is the length of block of channel uses. Random variables in H, H_k and W_k are i.i.d. $\mathcal{CN}(0, 1)$.