Exam Rules
Use only a pencil or pen. No calculators of any kind are allowed. Texting is out of the question.

Problem 1 ________ (22 pts)
Problem 2 ________ (22 pts)
Problem 3 ________ (22 pts)
Problem 4 ________ (12 pts)
Problem 5 ________ (12 pts)
Problem 6 ________ (10 pts)

Alias ______________________ Exam Total ________ (100 pts)

Good Luck!
Problem 1: (22 pts) The problems below are based on the following Boolean function:

\[(a + bc + b'c')(abc)'\]

(a) Draw a logic diagram (using AND, OR, and NOT gates) corresponding to the Boolean function. (Do not simplify the expression.)

☐ Logic diagram.

(b) Write the Boolean function in minterm canonical form. (Show a Boolean expression, not just a list of minterm numbers.) Hint: For most people directly constructing a truth table would be easier than algebraic manipulation.

☐ Expression in minterm canonical form.

(c) Write the Boolean function in maxterm canonical form.

☐ Expression in maxterm canonical form.

(d) Draw a Karnaugh map for the expression. (Just draw the Karnaugh map, don’t use it to simplify the expression.)

☐ Karnaugh map, including variables and row and column numbers.
Problem 2: (22 pts) Consider the Karnaugh map below.

\[
\begin{array}{c|c|c|c|c}
\hline
& zw & & \\
\hline
xy & & & \\
\hline
& 1 & & \\
\hline
& 1 & 1 & 1 \\
\hline
& 1 & & 1 \\
\hline
& 1 & 1 & \\
\hline
\end{array}
\]

(a) Write in the row and column numbers.

☐ Row and column numbers.

(b) List all of the prime implicants both on the Karnaugh map above, and as a list below.

☐ Prime implicants circled on Karnaugh map.

☐ List prime implicant expressions below.

(c) In the list of prime implicants above, write an “E” next to each essential prime implicant.

☐ Write an “E” next to essential prime implicants.

(d) Provide an example of an implicant that’s neither a prime implicant, nor a minterm. Circle this implicant and show the corresponding Boolean expression.

☐ Circle implicant that’s not just a minterm but not prime either.

☐ Show an expression for the implicant.

(e) Based on the Karnaugh map show a minimum-cost expression for this logic function.

☐ Minimum-cost expression.
Problem 3: (22 pts) Consider the Boolean function below:

\[ ab' + b'c + a'bc' \]

(a) Use a 3 \times 8 decoder plus whatever logic gates are needed to implement this function.

☐ Implement using 3 \times 8 decoder and gates.

(b) Use an 8-input multiplexer to implement this function.

☐ Implement using an 8-input multiplexer.

(c) Use a multiplexer and additional logic, including possibly exclusive-or gates, to implement this function by performing a Shannon expansion with respect to \( a \) (use \( a \) as the multiplexer control input). \textit{Hint: it might be easier to eyeball a truth table than to do this by algebraic manipulation.}

☐ Implement using a multiplexer based on \( a \).
Problem 4: (12 pts) Show how to implement the 8-input multiplexers described below. In each case the three select input bits should be labeled \( s_2, s_1, s_0 \), with \( s_0 \) being least significant. Label the data inputs 0 to 7.

(a) Implement an 8-input multiplexer using two 4-input multiplexers and a 2-input multiplexer.

□ Eight-input mux using two 4-input multiplexers.

(b) Implement an 8-input multiplexer using four 2-input multiplexers and one 4-input multiplexer.

□ Eight-input mux using four 2-input multiplexers and a 4-input mux.
Problem 5: (12 pts) Implement the devices as described below.

(a) Show the logic gates needed to implement a $2 \times 4$ decoder, include an enable input.

☐ Logic diagram for a $2 \times 4$ decoder, just use gates.

☐ Include logic for enable input.

(b) Show how to implement an 8-input multiplexer using a decoder and logic gates.

☐ Logic diagram for an 8-input multiplexer using gates and a decoder.
Problem 6: (10 pts) Answer each question below.

(a) Consider five seats, numbered 0 to 4, arranged in a circle and described by Boolean variables \(i_0\) to \(i_4\). Boolean variable \(i_0\) is true if seat 0 is occupied and \(i_0\) is false if the seat is not occupied (no one is sitting in the seat), likewise for \(i_1\), \(i_2\), \(i_3\), and \(i_4\).

Write a Boolean expression that’s true if at least two people are sitting next to each other and at least one seat is not occupied. (Note: Just write one Boolean expression.) *Hint: This can easily be solved without a truth table.*

☐ Boolean expression.

(b) The statement below is not true. Explain why and correct it.

“By implementing a sum-of-products expression using only NAND gates (in place of AND and OR gates) we expose additional opportunities for simplification.”

☐ Statement is incorrect because . . .

☐ The real reason for using NAND gates is . . .