Name


## Exam Rules

Use only a pencil or pen. No calculators of any kind are allowed. Texting is out of the question.

| Problem 1 | (22 pts) |
| :---: | :---: |
| Problem 2 | (22 pts) |
| Problem 3 | (22 pts) |
| Problem 4 | (12 pts) |
| Problem 5 | (12 pts) |
| Problem 6 | (10 pts) |

Alias $\qquad$ Exam Total $\qquad$ (100 pts)

Problem 1: $(22 \mathrm{pts})$ The problems below are based on the following Boolean function:

$$
\left(a+b c+b^{\prime} c^{\prime}\right)\left(a b c^{\prime}\right)^{\prime}
$$

(a) Draw a logic diagram (using AND, OR, and NOT gates) corresponding to the Boolean function. (Do not simplify the expression.)

Logic diagram.
(b) Write the Boolean function in minterm canonical form. (Show a Boolean expression, not just a list of minterm numbers.) Hint: For most people directly constructing a truth table would be easier than algebraic manipulation.
$\square$ Expression in minterm canonical form.
(c) Write the Boolean function in maxterm canonical form.
$\square$ Expression in maxterm canonical form.
(d) Draw a Karnaugh map for the expression. (Just draw the Karnaugh map, don't use it to simplify the expression.)
$\square$ Karnaugh map, including variables and row and column numbers.

Problem 2: (22 pts) Consider the Karnaugh map below.
zw
xy

|  | 1 |  |  |
| :--- | :--- | :--- | :--- |
| 1 | 1 |  | 1 |
| 1 |  |  | 1 |
|  | 1 | 1 |  |

(a) Write in the row and column numbers.
$\square$ Row and column numbers.
(b) List all of the prime implicants both on the Karnaugh map above, and as a list below.
$\square$ Prime implicants circled on Karnaugh map.
$\square$ List prime implicant expressions below.
(c) In the list of prime implicants above, write an "E" next to each essential prime implicant.
$\square$ Write an "E" next to essential prime implicants.
(d) Provide an example of an implicant that's neither a prime implicant, nor a minterm. Circle this implicant and show the corresponding Boolean expression.
$\square$ Circle implicant that's not just a minterm but not prime either.
$\square$ Show an expression for the implicant.
(e) Based on the Karnaugh map show a minimum-cost expression for this logic function.
$\square$ Minimum-cost expression.

Problem 3: (22 pts) Consider the Boolean function below:

$$
a b^{\prime}+b^{\prime} c+a^{\prime} b c^{\prime}
$$

(a) Use a $3 \times 8$ decoder plus whatever logic gates are needed to implement this function.
$\square$ Implement using $3 \times 8$ decoder and gates.
(b) Use an 8-input multiplexer to implement this function.

Implement using an 8-input multiplexer.
(c) Use a multiplexer and additional logic, including possibly exclusive-or gates, to implement this function by performing a Shannon expansion with respect to $a$ (use $a$ as the multiplexer control input). Hint: it might be easier to eyeball a truth table than to do this by algebraic manipulation.

Implement using a multiplexer based on $a$.

Problem 4: ( 12 pts ) Show how to implement the 8 -input multiplexers described below. In each case the three select input bits should be labeled $s_{2}, s_{1}, s_{0}$, with $s_{0}$ being least significant. Label the data inputs 0 to 7 .
(a) Implement an 8 -input multiplexer using two 4 -input multiplexers and a 2 -input multiplexer.

Eight-input mux using two 4-input multiplexers.
(b) Implement an 8-input multiplexer using four 2-input multiplexers and one 4-input multiplexer.

Eight-input mux using four 2-input multiplexers and a 4 -input mux.

Problem 5: (12 pts) Implement the devices as described below.
(a) Show the logic gates needed to implement a $2 \times 4$ decoder, include an enable input.
$\square$ Logic diagram for a $2 \times 4$ decoder, just use gates.
$\square$ Include logic for enable input.
(b) Show how to implement an 8-input multiplexer using a decoder and logic gates.
$\square$ Logic diagram for an 8-input multiplexer using gates and a decoder.

Problem 6: (10 pts) Answer each question below.
(a) Consider five seats, numbered 0 to 4, arranged in a circle and described by Boolean variables $i_{0}$ to $i_{4}$. Boolean variable $i_{0}$ is true if seat 0 is occupied and $i_{0}$ is false if the seat is not occupied (no one is sitting in the seat), likewise for $i_{1}, i_{2}, i_{3}$, and $i_{4}$.

Write a Boolean expression that's true if at least two people are sitting next to each other and at least one seat is not occupied. (Note: Just write one Boolean expression.) Hint: This can easily be solved without a truth table.
$\square$ Boolean expression.
(b) The statement below is not true. Explain why and correct it.
"By implementing a sum-of-products expression using only NAND gates (in place of AND and OR gates) we expose additional opportunities for simplification."

Statement is incorrect because ...

The real reason for using NAND gates is ...

