Problem 1: Consider the following logic function in canoncial form:
$\sum_{a,b,c,d} m(0, 2, 5, 8, 10, 12, 13)$.

(*a*) Draw a truth table for this logic function.

(*b*) Draw a logic circuit for this function. Do not simplify.

(*c*) Draw a Karnaugh map for the logic function.

(*d*) List the prime implicants.

(*e*) List the essential prime implicants.

(*f*) List all of the minimum cost sum-of-product expressions.

(*g*) Draw a logic diagram for your favorite one.

Problem 2: Consider again the logic function from the previous problem,
$\sum_{a,b,c,d} m(0, 2, 5, 8, 10, 12, 13)$. This time however suppose the outputs are *don't care* for two sets of inputs, $a = 0$, $b = 1$, $c = 0$, $d = 0$ (corresponding to row (minterm) 4) and $a = 0$, $b = 1$, $c = 1$, $d = 1$ (corresponding to row (minterm) 7).

(*a*) Draw a Karnaugh map, include the don't cares.

(*b*) Find a minimum-cost sum-of-products expression making the best use of the don't cares.

(*c*) Draw a logic diagram corresponding to the minimum-cost expression.

Problem 3: The *population* of an n-bit quantity is the number of bits with value 1. For example, the population of 4-bit quantity `0101` is 2, the population of `1101011` is 5.

(*a*) Show a truth table for a Boolean function with an output that's logic 1 if the population of 2-bit input a_1a_0 is the same as the population of 2-bit input b_1b_0. (The function has four inputs, a_1, a_0, b_1, and b_0.)

(*b*) Derive a Boolean algebraic expression for the same function without using the truth table. Use the following approach: derive an expression that's logic 1 when the population of a_1a_0 is zero. Derive similar expressions for when the population is 1 and when the population is 2. Then pair such expressions for a and b.

(*c*) Draw a logic diagram for either the hand-derived expression (the previous part) or if you couldn't do the previous part, an expression based on the truth table.

(*d*) Try simplifying the Boolean expressions using the exclusive or ($\oplus$) operator ($a \oplus b = ab' + a'b$). If successful, draw a logic diagram based on the simplified expressions.