

*Note: This assignment is based on material covered in Brown and Vranesic 2nd Edition Sections 2.3 (truth tables), 2.4 (logic gates and networks), 2.5 (Boolean algebra), 2.6.1 (SoP and PoS forms; canonical forms), 2.7 (NAND and NOR networks), 2.8 (design examples).*

**Problem 1:** The Boolean expression below is from Midterm Exam 1.

(a) Draw a logic diagram (show the gates) corresponding to the following Boolean expression. (As written, don't simplify it first.) Use AND, OR, and NOT gates.

$$x \cdot y + x \cdot y \cdot z' + y \cdot w \cdot x$$

(b) In the expression below  $xy$  has been factored out. Draw a logic diagram corresponding to this expression. As far as the diagram is concerned, treat the 1 as an input ( $x$ ,  $y$ ,  $z$ , and  $w$  are the real inputs). *Note: If this is solved correctly then something in the diagram should look foolishly wasteful. See the next part.*

$$x \cdot y \cdot (1 + z' + w)$$

(c) The simplified form of the expression is just  $x \cdot y$ . Explain why that's much more obvious in the second diagram above (where  $xy$  was factored out) than in the first.

**Problem 2:** The expression below is also from Midterm Exam 1.

(a) Draw a logic diagram (show the gates) corresponding to the following Boolean expression. (As written, don't simplify it first.) Use AND, OR, and NOT gates.

$$x \cdot (a + b) + (a + b)' \cdot (c + e) + (c + e) \cdot x$$

(b) The expression above can be simplified by applying the consensus theorem ( $i \cdot y + i' \cdot z + y \cdot z = i \cdot y + i' \cdot z$ ). Simplify it (or just look at the test solution if you honestly don't need the practice). Indicate the change in the logic diagram from the previous part.

**Problem 3:** The expression below is also from Midterm Exam 1.

(a) Draw a logic diagram (show the gates) corresponding to the following Boolean expression. (As written, don't simplify it first.) Use AND, OR, and NOT gates.

$$(a + b + c) \cdot (a + b + e) \cdot (a + b + f)$$

(b) Simplify the expression above by applying the rule  $(x + i)(x + j) = x + ij$  several times. Note that  $x$  need not correspond to just one variable in the expression. As with the previous problem, look at the exam solution if you don't need the practice.

(c) Show a logic diagram for the simplified expression.

(d) Look back and forth at the two logic diagrams and think to yourself, "Both circuits do the same thing, but they look so different. Gee."

**Problem 4:** Consider the expression below, which did not, incidentally, appear on the exam.

$$(a \cdot b + a \cdot c) \cdot (a \cdot b \cdot c)'$$

- (a) Draw a logic diagram (show the gates) corresponding to the Boolean expression above. (As written, don't simplify it first.) Use AND, OR, and NOT gates.
- (b) Repeat the problem above, but use only NAND gates. The NAND gates can have any number of inputs.
- (c) Repeat the problem above, but use only two-input NAND gates.
- (d) Repeat the problem above, but use only NOR gates. The NOR gates can have any number of inputs.

**Problem 5:** The expression below is from the last problem, written with different notation.

$$(ab + ac)\overline{abc}$$

- (a) Construct a truth table for this expression.
- (b) Using the truth table, write the expression in minterm canonical form. (Show a Boolean expression don't use a shorthand such as  $\sum_{x,y,z,w} m(12, 15)$ .)
- (c) Using the truth table, write the expression in maxterm canonical form.

**Problem 6:** A seven-segment display consists of rectangular lights, called *segments*, arranged to form an 8. If all segments are lit an 8 will appear. If all but the middle segment is lit a zero appears, etc. Let variables  $x_3, x_2, x_1, x_0$  hold the four bits of a number represented in BCD (just one digit), with  $x_3$  being the most significant bit.

- (a) Construct a truth table for the Boolean function that indicates whether the topmost segment is lit. That is, the Boolean function should be 1 if  $x_3 \cdots x_0 = 1001_2$  because for a nine the top segment is lit. The function should be 0 if  $x_3 \cdots x_0 = 1_2$  because for the digit 1 the top segment is not lit. (Those are just two inputs, the function needs to be specified for the other eight digits.) Be creative about what to do when  $x_3 \cdots x_0 > 1001_2$ .
- (b) Based on the truth table write the Boolean function in either minterm canonical form or maxterm canonical form, whichever is shorter.