Problem 1: Perform the multiplications indicated below.

Multiply the following two 8-bit unsigned binary integers into a 16-bit product: `01110010 + 10010011`.

Multiply the following two 8-bit signed 2's complement integers into a 16-bit product: `01110010 + 10010011`.

The theorem numbers in the problems below are from Dr. Skavantos' Handout 5, available via `http://www.ece.lsu.edu/alex/EE2720/EE2720_HO5.pdf`.

Problem 2: Prove the following

(a) Prove theorem T5 (called *complement* in the notes, but a better name is the damned-if-you-do,-damned-if-you-don't theorem) by perfect (finite) induction.

(b) Prove that

$$(x + y) \cdot (z + w) = x \cdot z + x \cdot w + y \cdot z + y \cdot w$$

using the axioms and theorems T1 to T11 (and their duals). **Do not** prove it using perfect induction or by otherwise substituting values for the variables. (For example, a proof like the following is not allowed: If $x = 0$ the statement becomes $y \cdot (z + w) = y \cdot z + y \cdot w$, true by T8, if $x = 1$ ....)

(c) Draw a logic diagram (a connection of logic gates) for the left- and right-hand side of the equality in the previous problem.

(d) Prove that

$$x + y_1 \cdot y_2 \cdot \;\cdots\; \cdot y_n = (x + y_1) \cdot (x + y_2) \cdot \;\cdots\; \cdot (x + y_n)$$

using induction.

Problem 3: Simplify the expressions below:

(a) Simplify:

$$i \cdot n + a \cdot n \cdot t \cdot i \cdot c + t \cdot a \cdot x \cdot i' + t \cdot a \cdot n \cdot x$$

(b) Simplify:

$$i \cdot n + (i \cdot n)' \cdot k + c \cdot a \cdot s \cdot k$$

(c) Simplify, noting that the difference in the second term in this subproblem and the previous one is significant.

$$i \cdot n + i' \cdot n' \cdot k + n' \cdot i' \cdot b$$

Problem 4: Perform the conversions indicated below:

(a) Convert the following to sum-of-products form, paying attention to opportunities for simplification.

$$(a + b) \cdot (a + c) \cdot (a' + d) + b \cdot a \cdot d$$

(b) Convert the following to product-of-sums form, paying attention to opportunities for simplification.

$$[(a+b)\cdot(a+c)\cdot(a'+d)+b\cdot a\cdot d]'$$

(c) Convert the following to sum-of-products form, paying attention to opportunities for simplification.

$$[(a+b)\cdot(a+c)\cdot(a'+d)+b\cdot a\cdot d]'$$

(d) Verify your solution to the conversions above by constructing a truth table. One output column should be for the expression $(a+b)\cdot(a+c)\cdot(a'+d)+b\cdot a\cdot d$, and there should be three more columns, one for each of the conversions.