yamātārājabhānasalagām: An Interesting Combinatoric Sūtra

Subhash Kak
Department of Electrical & Computer Engineering
Louisiana State University
Baton Rouge, LA 70803-5901, USA
FAX: 225.388.5200; Email: kak@ece.lsu.edu

Indian Journal of History of Science, vol. 35, pp. 123-127, 2000

Abstract

This note considers the history of a sūtra which describes all combinations of a binary sequence of length 3 in connection with the classification of metres as sequences of *laghu* and *guru* syllables.

Keywords: Chandas, binary numbers, de Bruijn cycles, Piṅgala, Pāṇini

In his book, *Mathematics: The Man-Made Universe*[1], Sherman Stein traces the history of "memory wheels", strings of 0s and 1s whose substrings sweep out all the combinations of binary sequences of a certain length. Quoting the opinion of the composer George Perle, Stein traced the first such memory wheel to India of about 1000 AD. The next use of a memory wheel was in France in 1882, where Émile Baudot used it for 32 quintuplet telegraphy. In this past century, memory wheels, also called de Bruijn cycles, have been used in a variety of applications, ranging from probability theory, coding, and communications.

Stein quotes the Sanskrit sūtra, *yamātārājabhānasalagām*, which describes all possible triplets of short and long syllables, as evidence of the Indian knowledge of a memory wheel of length 3. Since Sanskrit metres are based on the system of short, *laghu*, and long, *guru*, syllables, represented traditionally by | and $\mathcal{S}$ (by us as 1 and 0), the sūtra may be written as:

1000101110

which represents the sequences

100, 000, 001, 010, 101, 011, 111, 110

Since the last two bits are the same as the first two, the 10-bit sequence may be written as a wheel of 8-bits, which describes all the 8 different subsequences of length 3. According to some scholars the correct form of the sūtra is *yamātārājabhānasalagam*, because the last *la* and *gam* are, by definition, *laghu* and *guru* and so there is no need to use the long vowel with the *ga*. It appears that the sūtra is usually taught in this latter form in India, but the first form was also taught by some teachers, as attested by Stein. Here we are interested in the sūtra taught in the first form, because of its mathematical implications. I add that this form, quoted by Stein, is more logical, because it assigns a long vowel to the last *guru*, consistent with the usage in the earlier part of the sūtra.

Historians of mathematics have tried very hard to find the history of the sūtra. Sherman Stein once even put an advertisement in a Mumbai newspaper soliciting information about it.

Curt Sachs, in his *Rhythm and Tempo*[2], presents the metrical context for the sūtra. He provides the corresponding Greek names for the metres of 3-syllables.

Stein does not give any reasons for assigning the date of 1000 AD to the sūtra. The composer George Perle, the person from whom he learned about the sūtra, believed it was that old not based on any textual references; his dating appears to have been on the advice of some Sanskritist pandit, who may have thought that it arose during the time of the medieval metrical texts.

This paper considers the history of the sūtra from various angles. But first, we present a brief review of the metrical tradition.

The metrical tradition

Classification schemes for metres go back very early, to the early Vedic age and their great significance in Vedic thought may be gleaned from the extensive, recurring discussion of them in the Śatapatha Brāhmaṇa (e.g. ŚB

8.6.2). Metres of different lengths are described. In *CS* two basic classificatory schemes for metres are presented. One of them is direct and it represents a regular binary number mapping; the other represents a metre in terms of groups of three syllables, which is really an octal representation of numbers. The Aitareya Brāhmaṇa (3.28) says that originally the triṣṭubh, one of the most important of the metres, was 3-syllabic, and this cryptic remark may indicate the great antiquity of the octal representation.

The classification of metres is described most comprehensively first in *Chandaḥśāstra* (CS) by Piṅgala[3], who has been assigned variously from the 5th century BC to the 2nd century AD. According to one tradition, Piṅgala was the younger brother of Pāṇini, the great grammarian,[4] and if that is correct then he should be assigned to about the 5th century BC. Metres are described in Agni Purāṇa and in several technical texts written in the past 2,000 years, the most famous of which is the *Vṛttaratnākara* of Kedāra[5].

Piṅgala's representation for the 3-syllable verse-feet (CS, Adhyāya 1) is given in Table 1. The first bit, 1 or 0, of each verse-foot determines whether it is *laghu* or *guru*, respectively.

Table 1. *The Piṅgala mapping*

m	000
y	100
r	010
s	110
t	001
j	101
bh	011
n	111

Note that the 8 sequences fall into two complementary subclasses when 0s and 1s are interchanged. Thus *m, y, r, s* are in one group and *t, j, bh, n* are in another.

The mapping of Table 1 represents a binary number coding of the 3-syllables excepting that the order of the bits is reversed from our modern representation, so the bit on the left is the least significant and the one on the right is the most significant. We also note, parenthetically, that rather than rank the sequences from 0 to $2^n - 1$, they were ranked from 1 to 2^n, but this was done by adding 1 to the computed numerical value for each sequence. But this process made it clear that Piṅgala understood perfectly the binary number system which maps the numbers from 0 on, as has been explained by van Nooten.[6]

This mapping has some similarity to the Kaṭapayādi scheme, but that question is being discussed by the author elsewhere.

Piṅgala's Table 1 also provides us the reason for the exact form of the sūtra *yamātārājabhānasalagām*: *ya* is 100; *mā* is 000; and so on.

Piṅgala in his CS 6.24 comes very close to a description of this sūtra. He presents there the *chandas svāgatā* as consisting of *r, n, bh* [as according to Table 1], followed by two *guru* syllables. This represents the sequence

01011101100

If we cyclically shift it 8 places to the left, we get 10001011101, or *y, r, n, laghu, guru*, which is the same as our sūtra except for an extra 1 at the end. We speculate that when it was realized that this sequence spanned all the 3-patterns, the last 1 was dropped.

Not surprisingly, Kedāra's *Vṛttaratnākara* presents the same *chandas* (in his 3.40).

A Forgotten Reference

The earliest mention of this sūtra that I have been able to trace is in the book *Sanskrit Prosody & Numerical Symbols Explained* by Charles Philip Brown[7]. There on page 28 Brown ascribes this sūtra to Pāṇini. But we don't have any corroborating evidence connecting it to Pāṇini. The *Aṣṭādhyāyī* does not mention it, and I have been unable to check *Pāṇinīya Śikṣā* or references to Pāṇini's tradition in other commentaries. So the question of the earliest reference to this sūtra remains open at this time. The earliest we have to anything close to it is CS 6.24 of Piṅgala that was described above. But if it is really ancient and there is no reference to it in any text, then it must have been handed down orally from generation to generation.

Since our sūtra does not represent a Vedic metre, there was no reason for it to be in any of the Sanskrit books on prosody. It is quite plausible that this sūtra was known to Piṅgala and is very ancient. But this argument does not constitute a proof of its antiquity.

All we can say at this point is that sometime before 1869, when his book was published, Brown was taught this sūtra by his instructor. That is proof of its usage prior to that by Baudot in 1882. This, although not very satisfactory, is still of some significance to the historian of mathematics.

In conclusion, I pose a further investigation of the history of the sūtra as a problem to the readers of the Journal.

Acknowledgement

This note is a response to a query by Donald Knuth of Stanford University. I am thankful to K.S. Arjunwadkar of Pune for bringing Brown's book to my attention. I am also thankful to George Cardona of the University of Pennsylvania for his comments.

Notes

1. Stein, S. *Mathematics: The Man-Made Universe.* W.H. Freeman, San Francisco, 1969, pp. 144-148; see also Stein, S., " The mathematician as an explorer." *Scientific American*, May 1961, 149-158.

2. Sachs, C.. *Rhythm and Tempo.* Norton, New York, 1953, pp. 98-101.

3. Weber, A. *Ueber die Metrik der Inder.* Ferd. Dümmler's Verlagsbuchhandlung, Berlin, 1863 (reprinted by Georg Olms Verlag, Hildesheim, New York, 1973); this contains the text of Piṅgala's *Chandaḥśāstra.*

4. Agrawala, V.S.. *India as Known to Pāṇini.* University of Lucknow, Lucknow, 1953, pp 11-12;

   Shrava, S., *A Comprehensive History of Vedic Literature: Brāhmaṇa and Āraṇyaka Works.* Pranava Prakashan, New Delhi, 1977, pp 300-304.

5. Śridharānanda (ed.), *Kedāra's Vṛttaratnākaraṃ.* Motilal Banarsidass, Delhi, 1966.

6. van Nooten, B., "Binary numbers in Indian antiquity", *Journal of Indian Philosophy*, 21, 31-50, 1993; reprinted in Rao, T.R.N. and Kak, S. (eds.), *Computing Science in Ancient India.* USL Press, Lafayette, 1998.

7. Brown, C.P. *Sanskrit Prosody & Numerical Symbols Explained.* Trübner & Co, London, 1869 (Reprinted in India, Asian Publications, New Delhi, 1981).